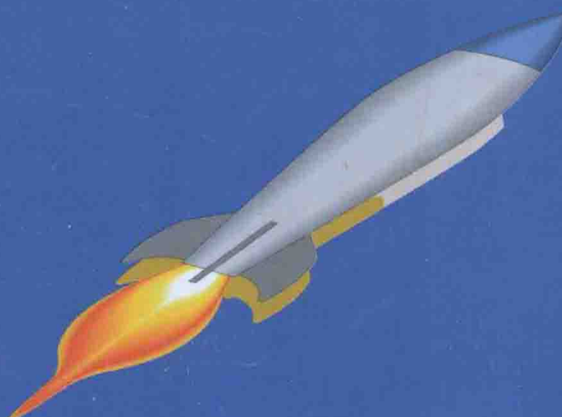
高等学校“十一五”规划教材

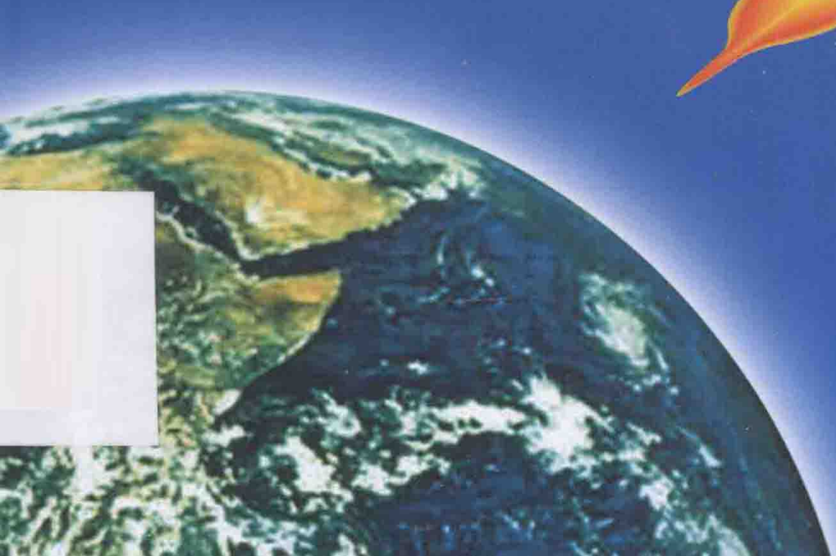
现代控制理论

Modern Control Theory

（中英文对照）

李道根 主编





哈爾濱工業大學出版社

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Brief of contents

This book states the elementary principle of modern control theory and discusses its analysis and synthesis techniques based on "state space method". The book is divided in five chapters, including state space description of linear systems, dynamic analysis of linear systems, controllability and observability of linear systems, state-space synthesis of linear systems, and Lyapunov stability analysis.

This book is a product of the bilingual teaching practice and experience of the Editors in Jiangsu University of Science and Technology. It can be used as a textbook of automation, electrician engineering and automation specialties, and so on, as well as a reference book for relevant engineers.

内 容 简 介

全书以状态空间法为基础阐述了现代控制理论的基本原理及其分析和综合方法。全书分五章,内容包括线性系统的状态空间描述、线性系统的运动分析、线性系统的能控性和能观测性、线性系统的状态综合以及李雅普诺夫稳定性分析。

本书是在江苏科技大学近年来现代控制理论双语教学的基础上编写的。本书可作为高等院校自动化、电气工程及其自动化等控制类专业的双语教学教材,也可作为相关工程技术人员的参考书。

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Preface

Bilingual teaching is the inevitable trend of education reform, also the important action to improve undergraduate education quality, and the requirement of cultivate compound-type talent with high quality who can adapt social development in the 21-the century. Finding one adapted way to ordinary colleges will be helpful to enhance the student' s ability of English application and understanding of the latest scientific achievements.

National Ministry of Education calls for actively promotion to develop bilingual teaching in colleges and universities, early of 2007, National Ministry of Education and Ministry of Finance jointly introduced the "Opinion on the implementation of quality of undergraduate education and teaching reform", encouraging colleges and universities to build 500 bilingual teaching model curriculums within five years.

So it is significant to research the special feature, method and mode of bilingual teaching and improve level of special English of students as soon as possible. The compilers of this book conforms to the trend of the development of higher education, carried through bilingual teaching of "Automatic control principle" and "Modern control theory" in automation and electrical engineering and automation specialties in Jiangsu university of science and technology. This book is one of the achievements of their bilingual teaching practice.

In despite of these or those difficulties, bilingual teaching will be developed rapidly with the update of education idea and make contribution to enhance the bilingual ability and compositive human culture quality of students.

Wang Jianhua
February, 2009

序

双语教学是我国高等教育改革发展的必然趋势，是提高我国本科教学质量的重要举措，也是培养适应21世纪社会发展高素质复合型人才的需要。寻找到一套适合普通高校双语教学的方法，对于提高学生的英语实际应用能力和了解世界科技最新成果的能力，无疑会具有很大的帮助。

为此，教育部要求各高校积极推动使用英语等外语进行公共课和专业课的教学。2007年初，教育部和财政部又联合出台了《关于实施高等学校本科教学质量与教学改革工程的意见》，提出在五年内要建设500门双语教学示范课程。

因此，研究“双语”教学的特点，积极探索“双语”教学的教学方法、教学模式，以尽快提高学生的专业英语水平，对教学研究具有十分重要的意义。编著者顺应高等教育的发展潮流，对江苏科技大学的自动化和电气工程及其自动化专业的“自动控制原理”和“现代控制理论”课程进行了双语教学，本书正是编著者开展双语教学研究的成果之一。

尽管双语教学在我国还存在着这样或那样的问题，但随着我国教育理念的更新，双语教学必定会取得长足的进展，为提高学生的“双语”能力和综合人文素质作出贡献。

王建华

2009年2月

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Foreword

As one of the basic curricula of control specialties, "Modern control theory" is significant for studying other subjects of control theory, such as Optimal control, System identification, Robust control and Intelligent control, and it is also widely used in so many fields, such as industry, agriculture, shipbuilding and aviation.

In recently years the Compilers carried through bilingual teaching of "Modern control theory" in automation and electrical engineering and automation specialties, this book is a product of our bilingual teaching practice and experience. The book takes bilingual form to provide convenience for readers.

Based on "state space method", this book systematically expatiates the elementary theory and methods of linear systems. We try our best to explain the deep things in a simple way and apply rigorous theory. We expect this book would play an active role in improving English application ability of the readers, cultivating correction ideation and the ability to connect theory with practice of students.

This book can be used as a professional teaching material of automation, electrician engineering and automation specialties, and so on, as well as a reference book for graduate students and teachers.

This book is edited by Li Daogen. Dr. Zhu Zhiyu and Liu Weiting make contribution to this book. We would especially like to thank Hao Peng, Wang Fang and Zong Yang for drawing figures and tables in this book. This book is reviewed by Professor Jiang Changsheng. And we would like to thank all the teachers and students for their help in the production and editing of this book.

Some mistakes are inevitable in this book because of our limited level, any criticizing correction will be appreciated.

Compiler
February, 2009

前　言

“现代控制理论” 是控制类专业的一门专业基础课程，本门课程对于学习控制理论的许多学科分支，如最优控制、系统辨识、鲁棒控制、智能控制等具有重要的作用，同时该理论也广泛应用于工业、农业、船舶、航空航天等诸多领域。

近几年来，编者对自动化和电气工程及其自动化等专业的“现代控制理论”课程进行了双语教学尝试，本书是编者在开展“双语”教学实践的基础上，结合多年讲授“现代控制理论”课程的心得和体会，并在参考了大量原版教材的基础上编写而成的。全书采用中英文对照的形式出版，以方便读者阅读和学习。

全书以状态空间描述法为核心，系统地阐述了线性系统的基本控制理论和方法。全书力求做到深入浅出，理论严谨。本书对提高读者的英语应用能力，培养学生的辩证思维能力和理论联系实际的能力，都具有一定的作用。

本书可用于工科自动化、电气工程及其自动化专业的教材，也可作为非本专业研究生和教师的参考书。

本书由李道根主编，朱志宇、刘维亭等参与了部分编写工作，郝鹏、王芳、宗阳等同学绘制了书中的图表。全书由南京航空航天大学姜长生教授审定。在此，向曾参与和关心本书编写工作的各位教师和同学表示感谢。

作者水平有限，对于书中存在的错误和不妥之处，敬请读者批评指正。

编　者

2009 年 2 月

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Chapter 1 State-Space Description of Linear Systems

1.1 Introduction

Nowadays, one of the challenges to control engineers is the modeling and control of modern, complex, large-scale and interrelated systems, such as chemical processes, robotic systems, and traffic control systems. Although, in classical control theory, the transfer function description of physical systems provides a practical approach to analysis and design and allows us to use block diagrams to interconnect subsystems, it has certain basic limitations. This is due to the fact that it is an external description of control systems based on the input-output relation, and is only applicable to the linear, time-invariant single-variable systems, i.e. systems with one input and one output only.

On the other hand, the time-domain method based on the state-space description is more powerful. Its principle advantages are that it is easily implemented on a digital computer, it is applicable to nonlinear systems and time-varying systems, it is readily extended to multivariable systems, i.e. systems with several inputs and outputs, and it provides additional insight into system behavior that transfer function analysis does not.

1.1.1 State Variable

The basis of modern control theory is the concepts of state and state variable. As the word implies, the state of a system refers to the past, present, and future conditions of the system. In general, the state can be described by a set of numbers, a curve, an equation, or something that is more abstract in nature. For dynamic system, however, the state of a system is described in terms of a set of state variables.

The state variables of a dynamic system is defined as the minimum set of variables such that the initial values of these variables at time t_0, together with any input to the system for $t \geqslant t_0$, will allow the behavior of the system to be determined uniquely for $t \geqslant t_0$. It should be emphasized, however, that the selection of state variables is not unique; there are many different sets of state variables that can be available for a given system.

A simple example of the state variable description of a dynamic system is the RLC circuit shown in Fig. 1.1. For a passive RLC network, the number of state variables required is equal to the number of independent energy-storage elements. The state of this system can be described in terms of a set of state variables: x_1, x_2, where x_1 is the inductor current $i_L(t)$ and x_2 is the capacitor voltage $v_C(t)$. This choice of state variables is intuitively satisfactory because the stored energy of the network can be described in terms of these variables as

$$E = \frac{1}{2}Li_L^2 + \frac{1}{2}Cv_C^2$$

Using Kirchhoff's circuit laws, we obtain following equations describing the change rate of capacitor voltage and that of inductor current, respectively

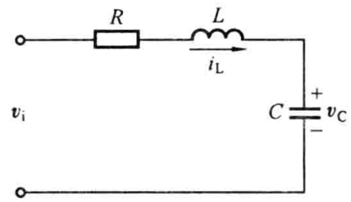

Figure 1.1 State variable model of RLC network

$$L\frac{di_L}{dt} = -Ri_L - v_C + v_i$$

$$C\frac{dv_C}{dt} = i_L$$

We can rewrite these equations as a set of first-order differential equations in terms of the state variables x_1 and x_2 as follows

$$\frac{dx_1}{dt} = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}v_i$$

$$\frac{dx_2}{dt} = \frac{1}{C}x_1$$

Using above equations and the initial conditions of the network represented by $x_1(t_0)$ and $x_2(t_0)$, we can determine the system's future behavior.

For a particular system, the number of state variables is fixed and is equal to the system order. Or, we can say that the number of state variables may be determined as the number of initial conditions needed to solve the differential equation or the number of first-order differential equations needed to define the system.

1.1.2 State-space Representation

The general form of a dynamic system is shown in Fig. 1.2.

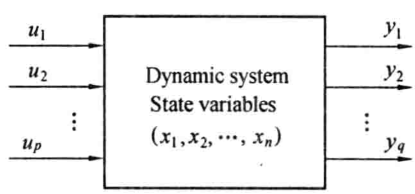

Figure 1.2 Dynamic system described with state variables

Since the system behavior can be completely described with state variables, the state of a nth-order linear time-invariant system can be described by a set of n first-order differential equations, written in terms of the state variables $x_1, x_2, \cdots, x_n$, in general form, as

$$\left.\begin{aligned}
\dot{x}_1(t) &= a_{11}x_1(t) + a_{12}x_2(t) + \cdots + a_{1n}x_n(t) + b_{11}u_1(t) + \cdots + b_{1p}u_p(t)\\
\dot{x}_2(t) &= a_{21}x_1(t) + a_{22}x_2(t) + \cdots + a_{2n}x_n(t) + b_{21}u_1(t) + \cdots + b_{2p}u_p(t)\\
&\vdots\\
\dot{x}_n(t) &= a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots + a_{nn}x_n(t) + b_{n1}u_1(t) + \cdots + b_{np}u_p(t)
\end{aligned}\right\} \quad (1.1)$$

where $\dot{x} = \frac{dx}{dt}$. This set of differential equations can be written in the matrix form as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} \tag{1.2}$$

The column vector consisting the state variables is called a state vector and is written as

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The vector of input signals is denoted as $\boldsymbol{u}$. Then the system can be represented with the vector-matrix form of the state differential equation as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1.3}$$

Eq. (1.3) is also commonly called the state equation. The $n \times n$ square matrix $\boldsymbol{A}$ is called the system matrix; the $n \times p$ matrix $\boldsymbol{B}$ is called the input matrix. In this text, bold uppercase characters refer to matrices, while bold lowercase characters refer to vectors.

In general, the outputs of a linear system can be related to the state variables and the input signals by the output equation

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{q1} & c_{q2} & \cdots & c_{qn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1p} \\ d_{21} & d_{22} & \cdots & d_{2p} \\ \vdots & \vdots & & \vdots \\ d_{q1} & d_{q2} & \cdots & d_{qp} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix}$$

i.e.

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{1.4}$$

where $\boldsymbol{y}$ is the set of output signals expressed in column vector form, the $q \times n$ matrix $\boldsymbol{C}$ is called the output matrix, and the $q \times p$ matrix $\boldsymbol{D}$ is called the feedforward matrix or direct transfer matrix. In many practical examples we have $\boldsymbol{D} = \boldsymbol{0}$.

The combination of state-equation and output equation is called the state-space representation or state-space description. The block diagram shown in Fig. 1.3 is helpful for visualizing the relationship between the various elements and signals. The double lines represent vectors, rather than single variable, and the integral block symbolizes n integrators, one for each state variable.

As for the linear time-varying systems, the standard state-space representation is of the form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}(t)\boldsymbol{x}(t) + \boldsymbol{D}(t)\boldsymbol{u}(t)$$

where the matrixes $\boldsymbol{A}(t)$, $\boldsymbol{B}(t)$, $\boldsymbol{C}(t)$, and $\boldsymbol{D}(t)$ are continuous functions of time.

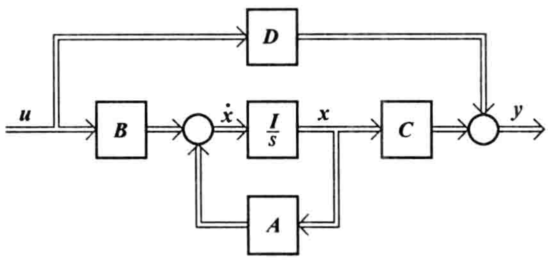

Figure 1.3 Block diagram of general state-space description

1.2 State-Space Representation from Input-Output Representation

In practice, it is often required to establish the space-state representation from the system input-output representation, i.e. differential equation describing system's dynamics. For linear time-invariant systems it is possible to develop the same result from the corresponding transfer function. In this section, we will consider only linear time-invariant single-variable systems, so the inputs and outputs will be scalars. Since the state variables of a given system are not unique, in general, we prefer to establish some "special" or canonical forms.

1.2.1 Input without Differential Items

Generally, a nth-order linear time-invariant system, for input without differential items, can be described by a differential equation

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + \cdots + a_{n-1}\dot{y}(t) + a_n y(t) = u(t) \tag{1.5}$$

where $y(t)$ is the output variable and $u(t)$ is the input. The corresponding transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \tag{1.6}$$

1. Controllable Canonical Form

For the present case, we define the state variables as

$$\left.\begin{aligned} x_1(t) &= y(t) \\ x_2(t) &= \dot{y}(t) \\ &\vdots \\ x_n(t) &= y^{(n-1)}(t) \end{aligned}\right\} \tag{1.7}$$

Then, we can obtain a set of first-order differential equations

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ &\vdots \\ \dot{x}_n(t) &= -a_n x_1(t) - a_{n-1}x_2(t) - \cdots - a_1 x_n(t) + u(t) \end{aligned}\right\} \tag{1.8}$$

In Eq. (1.8) the last equation is obtained by equating the highest-ordered derivative term to the rest of Eq. (1.5). The output equation is simply

$$y(t) = x_1(t) \tag{1.9}$$

In the vector-matrix form, the state-space representation is written as

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \left[\begin{array}{c|ccc} 0 & & & \\ \vdots & & \boldsymbol{I}_{n-1} & \\ 0 & & & \\ \hline -a_n & -a_{n-1} & \cdots & -a_1 \end{array}\right]\boldsymbol{x}(t) + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1 \quad 0 \quad \cdots \quad 0]\boldsymbol{x}(t) \end{aligned}\right\} \tag{1.10}$$

For reasons to be discussed later, a state-space representation is said to be in the controllable canonical form if it has such a pair of system matrix and input matrix as shown in Eq. (1.10). Note that the system matrix is a companion matrix, and the coefficients that appear in the differential equation or transfer function also appear directly in the system matrix. In fact, it is evident that this canonical form can be written directly from the given differential equation or transfer function.

The block diagram shown in Fig. 1.4 is drawn according to the state-space representation given by Eq. (1.10), and is called the state variable diagram. This diagram is consists of integrators, multiplicators, summators, and signal lines, and the output of each integrator is a state variable.

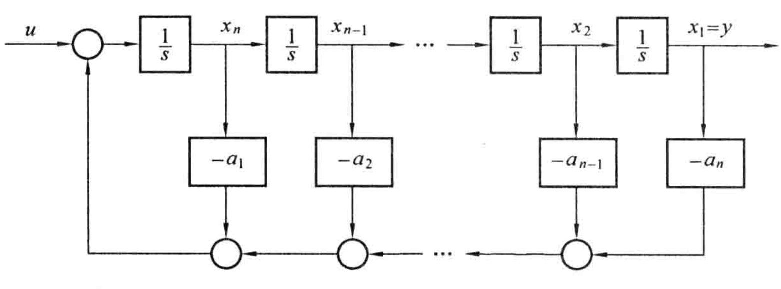

Figure 1.4 State variable diagram

A linear time-invariant state-space representation that has a prescribed rational transfer function is called a realization of this transfer function. The term "realization" is justified by the fact that, by using the state-space representation, the system with the transfer function can be built in the real world by using an operational amplifier circuit.

2. Observable Canonical Form

Taking Laplace transform of Eq. (1.5) under nonzero initial conditions and grouping the terms associated with the same power of s, we get

$$\begin{aligned}&(s^n + a_1 s^{n-1} + \cdots + a_n)Y(s) = \\ &U(s) + y(0)s^{n-1} + [\dot{y}(0) + a_1 y(0)]s^{n-2} + \cdots + \\ &[y^{(n-2)}(0) + a_1 y^{(n-3)}(0) + \cdots + a_{n-2}y(0)]s + \\ &[y^{(n-1)}(0) + a_1 y^{(n-2)}(0) + \cdots + a_{n-1}y(0)]\end{aligned}$$

By inspection, based on above equation, the state variables can be chosen as

$$\left.\begin{aligned}x_n &= y \\ x_{n-1} &= \dot{y} + a_1 y \\ &\vdots \\ x_2 &= y^{(n-2)} + a_1 y^{(n-3)} + \cdots + a_{n-2}y \\ x_1 &= y^{(n-1)} + a_1 y^{(n-2)} + \cdots + a_{n-1}y\end{aligned}\right\} \tag{1.11}$$

Thus, from Eq. (1.5) and Eq. (1.11), we get another set of first-order differential equations

$$\left.\begin{aligned}\dot{x}_1 &= u - a_n y = u - a_n x_n \\ \dot{x}_2 &= x_1 - a_{n-1}y = x_1 - a_{n-1}x_n \\ &\vdots \\ \dot{x}_{n-1} &= x_{n-2} - a_2 x_n \\ \dot{x}_n &= x_{n-1} - a_1 x_n\end{aligned}\right\} \tag{1.12}$$

The output equation is

$$y = x_n \tag{1.13}$$

In the matrix form, the state-space representation is

$$\left.\begin{aligned}\dot{\boldsymbol{x}} &= \left[\begin{array}{ccc:c} 0 & \cdots & 0 & -a_n \\ \hdashline & & & -a_{n-1} \\ & \boldsymbol{I}_{n-1} & & \vdots \\ & & & -a_1 \end{array}\right]\boldsymbol{x} + \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}u \\ y &= [0 \ \cdots \ 0 \ 1]\boldsymbol{x}\end{aligned}\right\} \tag{1.14}$$

For reasons to be discussed later, a state-space representation is said in the observable canonical form if it has such a pair of system matrix and output matrix as shown in Eq (1.14). Furthermore, this canonical form can be written directly from the given differential equation or transfer function. The state variable diagram of a state-space representation in observable canonical form is shown in Fig. 1.5.

Example 1.1 Consider the following differential equation

$$\ddot{y} + 3\dot{y} + 2y = u$$

Write its state-space representation in controllable canonical form and observable canonical form, respectively.

Solution Defining $x_1 = y$ and $x_2 = \dot{y}$ yields the state-space representation in controllable canonical form

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