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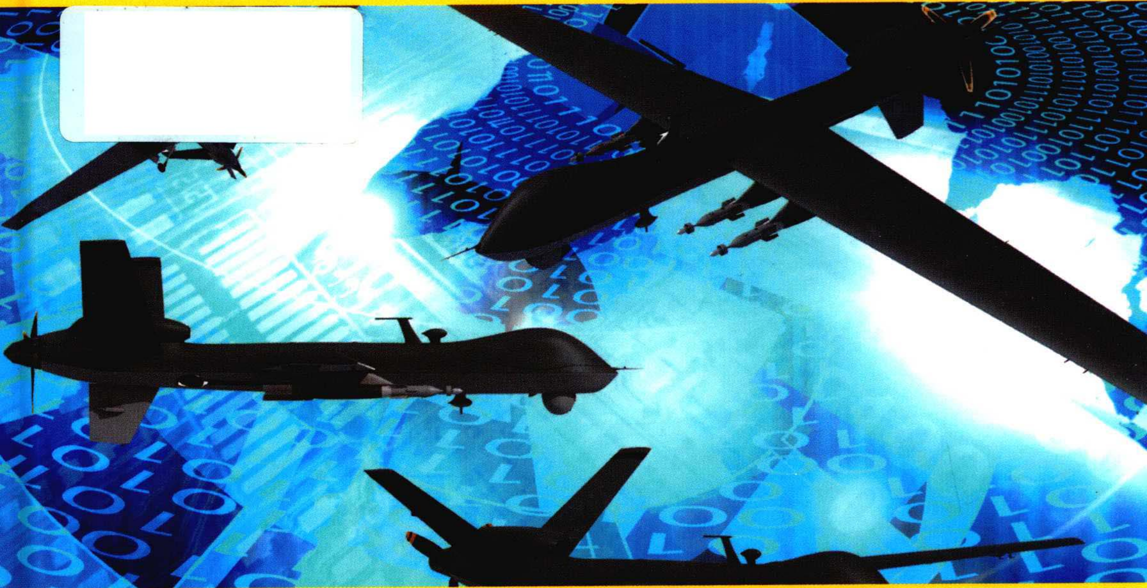

Modeling of Complex Systems

Application to Aeronautical Dynamics

Emmanuel Grunn and Anh Tuan Pham

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Introduction

The development of an airplane project closely follows a design process which defines the characteristics required to achieve the performance expectations and the desired flying criteria.

The principal purpose of this book is to offer readers a design tool which enables them to solve the different problems that can occur during the planning phase of a private project. As such, this book may be useful for design engineers as well as for keen airplane amateurs.

For the purpose of this book, we will assume that the preliminary design has been established and that an overall drawing, detailing the general shape of the airplane (i.e. surfaces, wingspan, aspect ratios, different levers, wing and stabilizer sections, as well as engine power requirements), has been completed.

In the following chapters, we will focus our attention on the dynamic behavior of the plane.

The study of flying qualities makes use of a special mathematical tool known as MATLAB/SIMULINK, outlined in Chapter 1, entitled "0D Analytical Modeling".

We shall now list the various phases which occur during the design process of an airplane.

Chapter 1

– Building a design tool under the form of a 0D mathematical model (dimensionless, time dependent only);

– describing the behavior of the plane;

– helping to optimize the flying qualities.

As validation of this model, a numerical result is given with data from an existing plane (Supersonic airliner CONCORDE, with data supplied by *Office National de Recherche Aerospatiale* (ONERA, French Aerospace Research Office and Sud-Aviation).

Chapter 2

Dimensional process, leading to presumed principal characteristics of the plane:

– mass distribution, inertial matrix;

– air loads, wing and stabilizer areas, wingspan, pitch and yaw levers, all data conditioning the flying qualities of the plane;

– selection of wing and stabilizer airfoils;

– outlining all necessary aerodynamic coefficients or derivatives, as well as all coupling terms;

– application to a realistic drone project.

Chapter 3

– Tuning the balancing equilibrium state of the plane to reach the previous objectives;

– flight tests.

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Chapter 1

0D Analytical Modeling of Airplane Motions

The 0D modeling process tries to obtain variations as functions of time for all parameters of the motions of the plane.

The plane is considered here as a solid body moving freely through open space and therefore includes six degrees of freedom (DOF):

– three translational motions by three rectangular directions;

– three rotational motions classically described by Euler angles.

The plane is also under the influence of three external force systems which are:

– aerodynamic forces;

– propulsion forces;

– gravitational forces.

1.1. References: axis systems on use

In order to define the spatial motion of the airplane, we make use of two geometrical references.

1.1.1. *Galilean reference: R_0*

This geometrical reference has its origin center matched with the center of mass G of the airplane. The three principal rectangular axes are:

– Gx_0: horizontal, generally oriented to the West;

– Gy_0: horizontal, oriented to the North;

– Gz_0: vertically downward.

Gx_0, Gy_0 and Gz_0 form a direct rectangular reference.

NOTE.– Gz_0 is directed downward, due to the natural tendency of the airplane to descend when left to the effects of gravity.

This Galilean reference is in accordance with Newton's first principle which makes use of the absolute components of the accelerations to be equal to the components of external forces.

1.1.2. *Airplane reference: R_B (body) also called "linked reference"*

This geometrical reference also has its center matched with G, the center of mass of the plane, but is physically linked to the airframe. Its three principal axes are: GX, GY and GZ.

GX, GY, GZ are preferably the principal axes of inertia of the plane and (GXYZ) is direct.

GXZ is the plane of symmetry of the airplane, with the exception of a few particular airplanes with asymmetric engine setups (Blohm and Voss, for instance; see Figure 1.1).

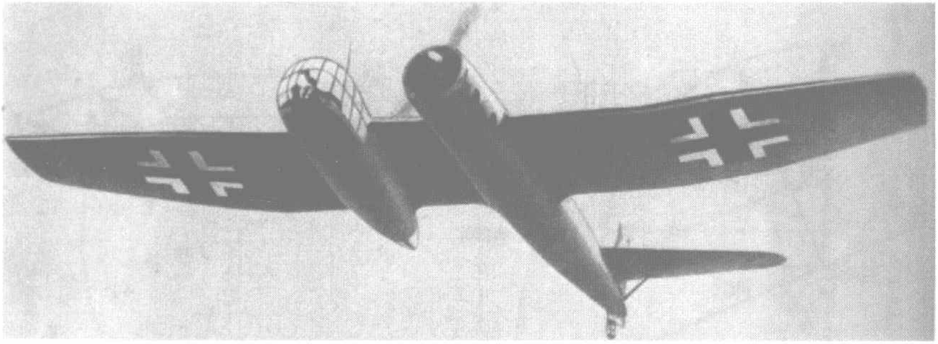

Figure 1.1. *Blohm and Voss BV 141*

(GXYZ), also called R_B, is the preferred reference for use with torque equations due to the fact that the inertias remain constant.

We can move from the Galilean reference to the body reference by making three Eulerian rotations, which are:

– Ψ (Psi): Yaw angle;

– Θ (Theta): Pitch angle;

– Φ (Phi): Roll angle.

a) Yaw rotation (ψ)

This first Euler rotation is made around the Gz_0 axis.

$$\Psi = \text{Yaw angle}$$

The associated angular velocity is: $\dot{\psi} \,.\, Z_0$

$$(Gx_0\, y_0\, z_0) \underset{(\psi)}{\longrightarrow} (Gx'\, y'\, z_0)$$

The relationship between the cosine directors are:

$$x' = \cos\psi \,.\, x_0 + \sin\psi \,.\, y_0$$

$$y' = -\sin\psi \,.\, x_0 + \cos\psi \,.\, y_0 \qquad [1.1]$$

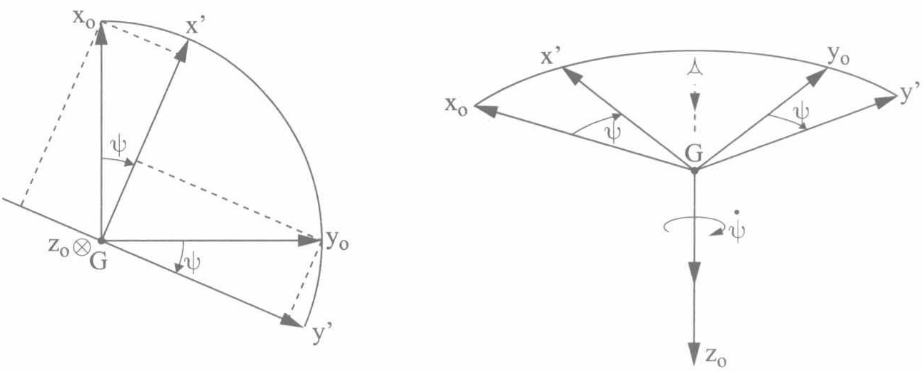

Figure 1.2. *First Euler rotation ψ around the Gz_0 axis*

b) Pitch rotation (θ)

This second rotation is made around the Gy' axis.

Θ = Pitch angle

The associated angular velocity is: $\dot{\theta}\ .\ y$

$$(Gx'y'z_0) \underset{(\theta)}{\rightarrow} (GXy'z')$$

The relationships between the cosine directors are:

$$X = \cos\theta\ .x' - \sin\theta\ .\ z_0$$

$$z' = \sin\theta\ .\ x' + \cos\theta\ .\ z_0 \qquad [1.2]$$

And as obtained before:

$$z_0 = \cos\theta\ .\ z' - \sin\theta\ .\ X$$

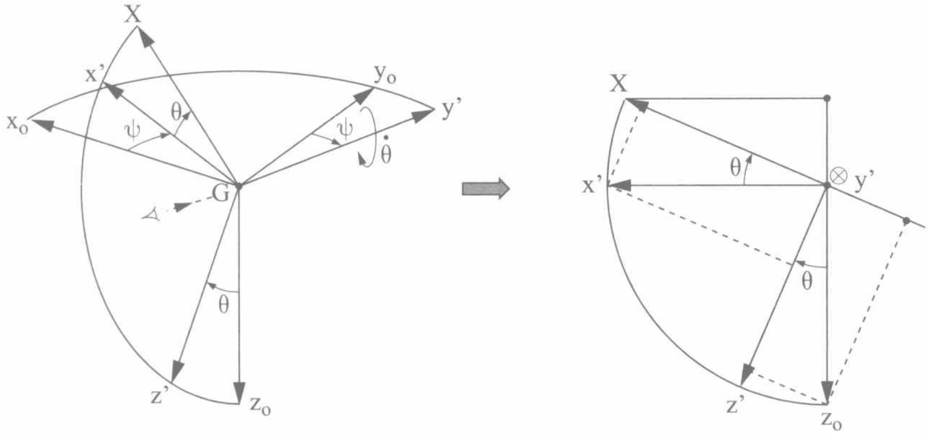

Figure 1.3. *Second Euler rotation θ around the Gy' axis*

c) Roll rotation (φ)

The third rotation (φ) is made around the GX axis.

Φ is the roll angle.

The associated angular velocity is: $\dot{\varphi}$ .X

$$(GXy'z') \underset{(\varphi)}{\rightarrow} (GXYZ)$$

The relationships between cosine directors are:

$$Y = \cos\varphi\ .y' + \sin\varphi\ .\ z'$$

$$Z = -\sin\varphi\ .\ y' + \cos\varphi\ .\ z' \qquad [1.3]$$

therefore:

$$z' = \sin\varphi\ .Y + \cos\varphi .Z$$

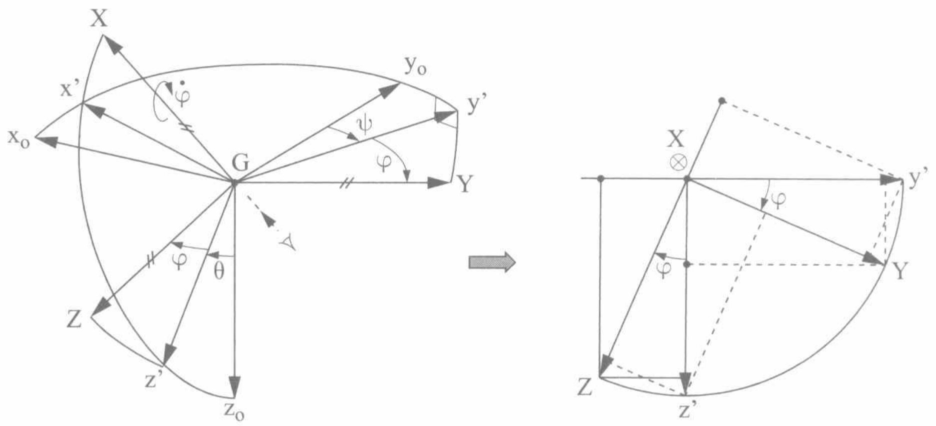

Figure 1.4. *Third Euler rotation φ around the GX axis*

1.1.3. *Resultant angular velocity*

The resultant angular velocity can be expressed as:

$$\vec{\Omega} = \dot{\psi} \cdot \vec{z}_0 + \dot{\theta}\vec{y} + \dot{\varphi}\vec{x} \quad [1.4]$$

These rotations are made around three axes which do not form a rectangle.

We can now express the components of $\vec{\Omega}$ by the reference R_B linked to the airplane:

$$\vec{\Omega} = \dot{\psi} . \left(\cos\theta . \vec{z'} - \sin\theta . \vec{X} \right) + \dot{\theta} . \left(\cos\varphi . \vec{Y} - \sin\varphi . \vec{Z} \right) + \dot{\varphi} . \vec{X} .$$

therefore:

$$\vec{\Omega} = \dot{\psi} . \left[\cos\theta . \left(\sin\varphi . \vec{Y} + \cos\varphi . \vec{Z} \right) - \sin\theta . \vec{X} \right] + \cos\varphi . \dot{\theta} . \vec{Y} - \sin\varphi . \dot{\theta} . \vec{Z} + \dot{\varphi} . \vec{X} .$$

$$\vec{\Omega} = \left(\dot{\varphi} - \sin\theta . \dot{\psi}\right) . \vec{X} + \left(\cos\theta . \sin\varphi . \dot{\psi} + \cos\varphi . \dot{\theta}\right) . \vec{Y} + \left(\cos\theta . \cos\varphi . \dot{\psi} - \sin\varphi . \dot{\theta}\right) . \vec{Z}$$

Now expressed by R_B (GXYZ) reference:

$$\Omega/(GXYZ) = \begin{bmatrix} p = \dot{\varphi} - \sin\theta\, .\dot{\psi} \\ q = \cos\theta\, .\sin\varphi\, .\dot{\psi} + \cos\varphi\, .\dot{\theta} \\ r = \cos\theta\, .\cos\varphi\, .\dot{\psi} - \sin\varphi.\dot{\theta} \end{bmatrix} \qquad [1.5]$$

NOTE.– p, q and r are the components of the vector resultant angular velocity expressed by the airplane body reference R_B. They can be measured on a real airplane by angular velocity sensors, commonly called "gyrometers".

The Euler parameters: ψ, θ, φ are actually more difficult to obtain.

This point will be covered in more detail hereafter.

Equations [1.5] can be written under matricial form:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\sin\theta \\ \cos\varphi & 0 & \cos\theta.\sin\varphi \\ -\sin\varphi & 0 & \cos\theta.\cos\varphi \end{bmatrix} * \begin{bmatrix} \theta \\ \varphi \\ \psi \end{bmatrix} \qquad [1.6]$$

It is then possible to solve this matricial equation in order to extract the derivatives of Euler angles which figure in the column vector $[\theta \;\; \varphi \;\; \psi]^T$ of [1.6].

The characteristic determinant of this equation is called Δ:

$$\Delta = \begin{vmatrix} 0 & 1 & -\sin\theta \\ \cos\varphi & 0 & \cos\theta.\sin\varphi \\ -\sin\varphi & 0 & \cos\theta.\cos\varphi \end{vmatrix} \qquad [1.7]$$

$$\Delta = -(\cos^2\varphi.\cos\theta + \sin^2\varphi.\cos\theta) = -\cos\theta.$$

The minor $N_{\dot{\theta}}$ associated with $\dot{\theta}$ is:

$$N_{\dot{\theta}} = \begin{vmatrix} p & 1 & -\sin\theta \\ q & 0 & \cos\theta.\sin\varphi \\ r & 0 & \cos\theta.\cos\varphi \end{vmatrix}$$

$$N_{\dot{\theta}} = -(\cos\theta.\cos\varphi.q - \cos\theta.\sin\varphi.r)$$

Then: $\dot{\theta} = N_{\theta} / \Delta = \cos\varphi.\ q\ - \sin\varphi\ .r$ [1.8]

In the same way, the minor $N_{\dot{\varphi}}$ associated with $\dot{\varphi}$ is written as:

$$N_{\dot{\varphi}} = \begin{vmatrix} 0 & p & -\sin\theta \\ \cos\varphi & q & \cos\theta.\sin\varphi \\ -\sin\varphi & r & \cos\theta.\cos\varphi \end{vmatrix}$$

$$N_{\dot{\varphi}} = -p.(\cos^2\varphi\ .\cos\theta\ + \sin^2\varphi.\cos\theta\ ) - \sin\theta.(\cos\varphi.r + \sin\varphi.q)$$

$$N_{\dot{\varphi}} = -p.\cos\theta\ - \sin\theta.(\ \cos\varphi\ .r + \sin\varphi.q\ )$$

Then: $\dot{\varphi} = N_{\dot{\varphi}} / \Delta = p + tg\theta.(\ \cos\varphi.r\ + \sin\varphi.q)$ [1.9]

The last minor N_{ψ} associated with $\dot{\psi}$ is:

$$N_{\dot{\psi}} = \begin{vmatrix} 0 & 1 & p \\ \cos\varphi & 0 & q \\ -\sin\varphi & 0 & r \end{vmatrix}$$

$$N_{\dot{\psi}} = -(\cos\varphi.r + \sin\varphi.q)$$

Then: $\dot{\psi} = N_{\dot{\psi}} / \Delta = (\cos\varphi\ /\cos\theta\ ).r\ + (\sin\varphi\ /\cos\theta\ )\ .\ q$ [1.10]

See the *Euler block* as shown in Figure 1.5.

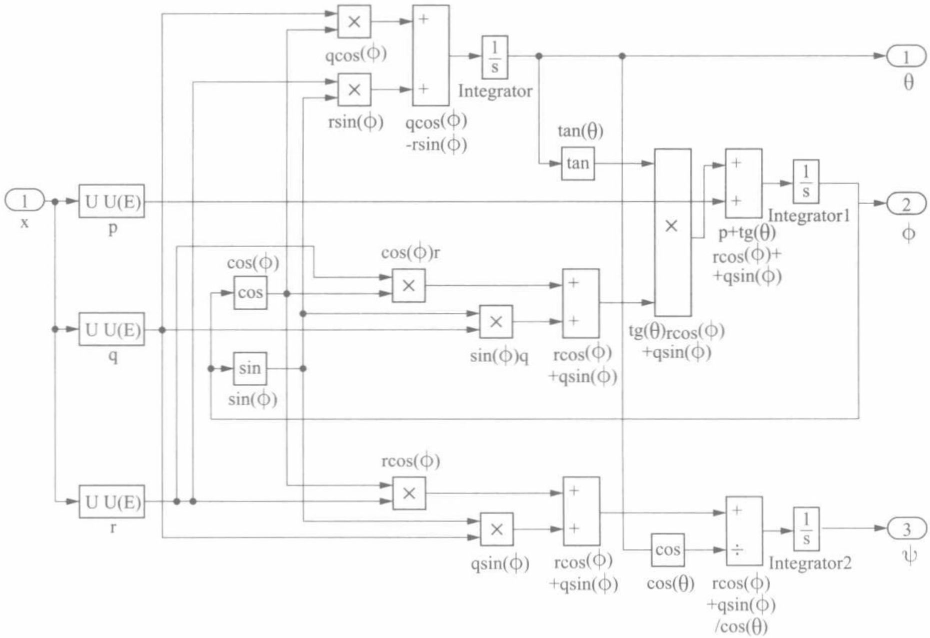

Figure 1.5. *Euler block*

This MATLAB/SIMULINK operator converts the linked parameters (p, q, r) to Euler angles:

(Theta, Phi, Psi)

1.2. Equations of motion of the airplane

The equations of motion translate the two fundamental principles of solid mechanical bodies:

1) The "quantity of acceleration" of the solid body in translation is equal to the resultant vector of all external forces (Newton's principles).

NOTE.– The quantity of acceleration is exactly the opposite value of the inertial force.

As a result, we obtain three equations for equilibrium forces.

2) The "dynamic momentum" of the solid body is equal to the resultant torque of the external forces.

As a result, we obtain three equations for torque equilibrium.

1.2.1. *Expression of Newton's principle*

Two vectors define the motion of the solid body as functions of time:

– $\vec{V}$ (Uvw) and $\vec{\Omega}$ (p q r) at any time.

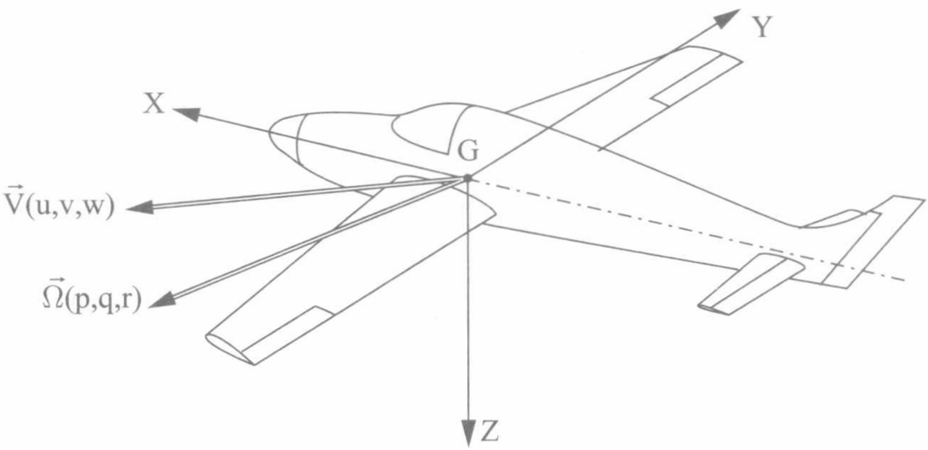

Figure 1.6.

Let us call the expression of the quantity of acceleration of the solid body Q_a

$$\overrightarrow{Q_a} = m.(\overrightarrow{dV} / dt) / R_0 \qquad [1.11]$$

In this formula:

m = mass of the airplane (considered as a solid body).

$\overrightarrow{dV}$ /dt = time derivative of the velocity vector by a Galilean reference (velocity).