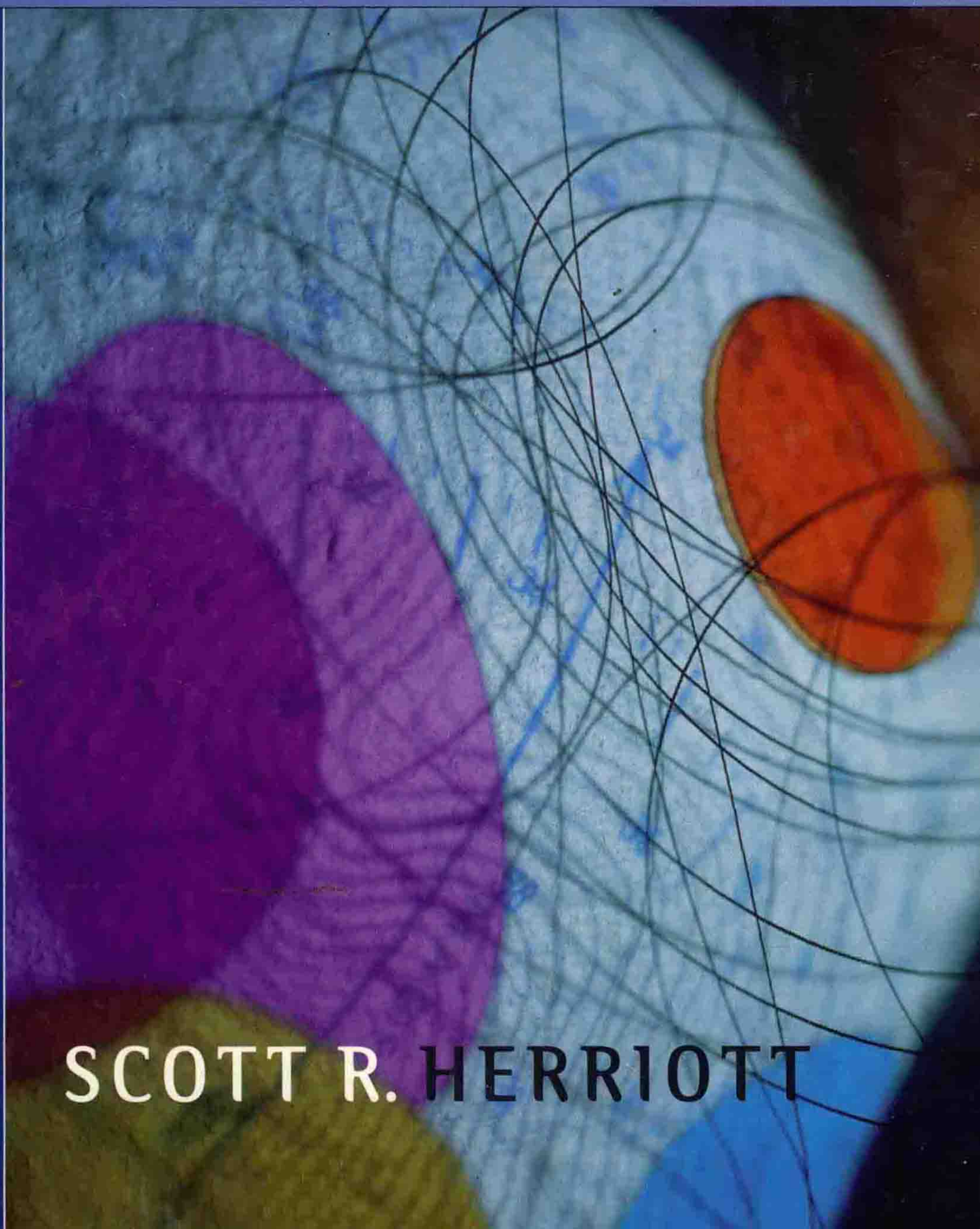
COLLEGE ALGEBRA

THROUGH FUNCTIONS AND MODELS



SCOTT R. HERRIOTT

College Algebra

Through Functions and Models

Scott R. Herriott

Maharishi University of Management

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Cover Printer: *The Lehigh Press, Inc.*
Compositor: *G&S Typesetters, Inc.*
Printer: *R.R. Donnelley (Willard)*

Printed in the United States of America
1 2 3 4 5 6 7 08 07 06 05 04

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Library of Congress Control Number: 2003110621

Student Edition: ISBN 0-534-37749-1
Instructor's Edition: ISBN 0-534-40419-7

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10 Davis Drive
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To the Student

The Usefulness of Mathematics

Welcome to the study of college algebra and its applications. Most of the students who take this course will go on to major in business or economics, in the life or health sciences, in the social sciences, or in education. Whether you know your future major or not, you will gain a lot from this course. Why? Because the examples and exercises chosen for this text are drawn from your future major field of study. You'll learn college mathematics from this book, but you will also learn a lot about the practical issues and problems that you will encounter soon enough in your courses and your career.

The study of math does more than show you how to solve practical problems. It also cultures orderliness and precision in thinking. The study of mathematics helps you to express ideas in a very orderly and precise way. Mathematics is a language. It is a very precise language through which a person can describe the order and pattern in nature. Industries and business firms, people and plants, and cities and nations are all elements of nature. So there are many useful applications of mathematics in the management, social, and life sciences, just as there are applications of mathematics in the physical sciences. As you learn to express ideas in the language of mathematics, you will sharpen your own thinking.

There is another aspect of the study of mathematics that relates to the orderliness of this language and its applications to natural phenomena. As you use mathematical concepts to express the orderliness of phenomena in the world of business and society, you will notice that some mathematical concepts appear again and again in different applications or different contexts.

As you notice these different instances of common mathematical ideas, you will learn to recognize the simple, more universal concepts that underlie the apparently diverse phenomena of the world. When you recognize the pattern, the mathematical concept, then the specific examples that you encounter will no longer seem foreign to you. They will seem familiar. You will think to yourself, "Aha! That's just a linear relationship. I know how to use that."

These two attributes of mathematics – its precision of language and its orderliness of content – help you to develop the two extremes of mental ability. Your thinking must be sharp and precise on the one hand, yet broad and integrative on the other. I have worked with students to develop these qualities of mind, and I have seen that it is possible and valuable (see www.tm.org).

The applications of mathematics that you will find in this book will be both interesting and informative. Of course, they have to be interesting, or else most students would forget the mathematics that the applications are supposed to illustrate. But the examples in this text will be informative as well. They will give you a sneak preview of the topics that you will soon be studying in your major.

Study carefully the examples in this book. As you read the exercises, you will learn some new vocabulary of the discipline from which the example was taken. Don't be afraid of unfamiliar words like "demand curve" or "cost function." In so many cases, you will see that a new word from a discipline merely expresses an old, familiar idea from mathematics. When you later take courses in economics, sociology, ecology, and so on, you will remember back to the applications of mathematics that you learned in this course, and they will not seem new to you.

Best wishes,
Scott Herriott

To the Teacher

Teaching Useful Mathematics

College Algebra is often a challenging course to teach. The students, faculty, and client departments across campus seem to have different purposes for the course. To state the point in extremes, for the students it is a required course that appears to be a repetition of what they already saw — and learned to some degree — in high school. To the mathematics faculty, it is a precalculus course that should develop students' analytic skills. And the client departments require it as a foundation for the mathematical content of their majors.

The key to satisfying all parties is to look carefully at why the client departments require college algebra and to use a textbook that develops the skills of thinking and analysis that students will need in their future majors. The development of this text benefited from data on 3,300 college algebra students at a dozen schools in the Midwest. The data showed that it is not uncommon for 40%–45% of the college algebra students to be headed toward a business major. Life/health sciences, social sciences, and education account for another 40%–45% of the intended majors of these students. At most schools, only about 10%–15% of the college algebra students even *think* they are headed toward a mathematically intensive major. On average across schools, this means that most of the students will later be taking statistics, that less than one-half will take a one-term course in calculus for business and the life sciences, and that about one-tenth of the class will attempt a full-year sequence of calculus.

The students want a college algebra course that will help them in their future study. This text motivates these students with numerous interesting, practical applications that they will encounter in their majors. In preparing the exercise sets, I scoured the fields of business, psychology, biology, health, human physiology, and sociology for new applications that will convince the students they really need to study this math.

This approach can be very satisfying to the instructor, because it develops in the students some deep skills of thinking and analysis. The most fundamental of these is mathematical modeling – the ability to take a verbal or numerical description of a phenomenon and to express it in a graph or equation. The students in this course learn to identify the distinctive mathematical characteristics of the linear, exponential, power, and quadratic functions. Working through many examples, they learn how to see these characteristics in the natural-language descriptions of phenomena. As they develop the ability to generalize, they come to appreciate the power of mathematics.

The modeling is just a context for the study of algebra. Algebra comes into play when the students have to solve their model equations to answer a practical question. The techniques for solving equations, and the mathematical principles behind those techniques, are the core of this text.

I have enjoyed the process of researching and writing this book. I hope that you and your students have a similar experience in using it.

Scott Herriott
Fairfield, Iowa

Acknowledgments

I would like to acknowledge the contributions of many people who have read or reviewed this book during its development.

Lee La Rue, Paris Junior College

Bernadette Baker, Drake University

Carl Anderson, Johnson County Community College

David Longshore, Victor Valley College

David McCormack, Minot State University

Dennis Spence, University of Phoenix

Jack Escover, University of Phoenix

Ronald Moore, University of Phoenix

Jayanthi Ganapathy, University of Wisconsin at Oshkosh

Jim Sandefur, Georgetown University

Jim Vandever, South Dakota State University

Linda O'Brien, Johnson County Community College

Mary Martin, Middle Tennessee State University

Maryann Justinger, SUNY Erie

Phoebe Rouse, Louisiana State University

Richard Nadel, Florida International University

Thomas Kline, University of Northern Iowa

Linda Hutchison, Belmont University

Wil Boykin, Georgia College & State University

Will Clark, La Sierra University

Diana McCoy, Florida International University

Greg Rosik, Ridgewater College

Joseph Ferrar, Ohio State University

Dan Shapiro, Ohio State University

Michael Rosenthal, Florida International University

Patricia Gary, Northern Virginia Community College

Ricardo Alfaro, University of Michigan at Flint

Doug Shaw, University of Northern Iowa

Russ Campbell, University of Northern Iowa

Dave Wells, Penn State University

Kim Robinson, Clayton College & State University

Susan White, Georgia Perimeter College

Christine Wise, University of Louisiana–Lafayette

Special thanks goes to Donna Stein (University of Tennessee) for creating the Test Bank and Tracie McLemore Salinas (University of Tennessee) for checking its accuracy. Thanks also to Jon Weerts (Triton College) for his work on the Student Solutions Manual.

My colleagues at Maharishi University of Management deserve thanks for their editorial help—Eric Hart, Anne Dow, David Streid, Robin Ticciati, and Cathy Gorini. Joseph Rienstra, a student in our art program, gave us 3-D drawings for Chapter 4.

I would like to honor Ernst Snapper at Dartmouth College. His courses on abstract algebra showed me the extraordinary beauty of this subject. The higher mathematics he taught does not appear in the present text, but I hope that something of his inspired teaching does. Jim March at Stanford deserves recognition for the best text on mathematical modeling that I have seen in twenty-five years. With his *Introduction to Models in the Social Sciences* he taught the higher-order thinking skills that good modeling requires. My book is about algebra more than it is about modeling, so it doesn't do justice to March's goals. But it is inspired by the quality of teaching that March put into his courses and into his book. I owe a special debt to Maharishi Mahesh Yogi. Observing his skill in teaching subtle ideas and experiences has taught me a lot about education.

Finally, I'd like to thank the people who worked at or with Brooks/Cole to make this book possible. Early on, my acquisitions editor, Margot Hanis, had the vision to see the need for this book, and Bob Pirtle and John-Paul Ramin kept up the momentum. Leslie Lahr, my developmental editor, deserves thanks beyond words. She worked with me fluidly, almost like a coauthor, giving great ideas and testing my own as we made the transition from the preliminary version to the first edition. Thank you to Darlene Amidon-Brent and Lisa Chow for their invaluable assistance in the editorial process, as well as their work on the book's supplemental materials. Phil Embree checked and double-checked the manuscript for accuracy, clarity, and usability. He also created many of the skill exercises. His input and enthusiasm have been invaluable. The production of the book was guided by Andy Marinkovich at Brooks/Cole with Gretchen Otto at G&S Typesetters as the driving force. Her professionalism and patience kept everything on track. Copyeditor Karen Boyd and project editor Jessie Hunnicutt polished the text. Terri Wright found photos that are both eye-catching and relevant. The graphic artists at G&S Typesetters took my rudimentary graphics and made them into works of art. Marlene Veach designed the visual preface to show the outstanding features of this book in glowing fashion. Lucas Tomasso, national sales manager, deserves a special nod. He was quick to see the special features of this book and helped us tailor it to students' needs. I owe all these people more than I can express and thank them for a job well done.

Brief Contents

Detailed Contents

1 Review of Elementary Concepts

When people hear the word "mathematics," they tend to think of addition, subtraction, multiplication, and division of numbers. But mathematics is not really about numbers. Genuine mathematics, the kind of "higher" mathematics that you will learn from this text, is about *relationships* among various aspects of ourselves, phenomena in our society, and all that exists in our environment. Mathematics provides a language that we can use to describe these relationships.

This first chapter is a review of a few key ideas in elementary algebra. Some students will find nothing new here, but for others – especially those who have been away from mathematics for a while – this chapter will be a nice way to wade back into the pool slowly. If these ideas have grown somewhat faded in your memory, take the time to read through the chapter, work the tasks suggested in the "Practice" boxes as you are reading, and do some of the exercises at the end of each section.

1.1 Tables, Graphs, and Equations

LEARNING OBJECTIVES

- **Understand what variables are and how they are used to express relationships.**
- **Distinguish among various elements of the real number system: natural, rational, and irrational numbers and integers.**
- **Express a relationship between variables using tables, graphs, and equations.**

In this section, we look at three different ways to describe the relationship between two variables: as a table of numbers, as a graph, and as a mathematical equation. You will also learn how to transform one type of description — table, graph, or equation — into another.

Variables and Their Relationships

When we understand how two things vary in relation to each other, we can use knowledge about one thing to help predict the other. For example, a small business owner may want to know how the *price* charged for a product affects the *quantity* that will sell. This knowledge can help the businessperson to determine the best price to charge for the product. An exercise physiologist may want to know the way in which an athlete's *performance* varies with the amount of *training* the athlete undertakes. A biologist might want to know how the *reproduction rate* of bacteria in a biogas production facility depends on *temperature* in order to determine if the facility should invest in a heater.

In each of these examples, a person is trying to describe the relationship between one variable and another. A **variable** is almost always an attribute of some object. It is an attribute that can be measured or recorded, such as the *weight* of a textbook, the *distance* to a wall, or the *gender* of a person. "Time" is an exception to this rule: it is used as a variable, but it isn't an attribute of an object.

The object on which you might take a measurement of a variable is called a **case,** and the recorded quantity or type is the **value** of the variable for that case. Variables whose values can be expressed as numbers, such as weight or distance, are called ***numerical* variables.** Variables such as gender that can only be described by category names, "male" and "female," are referred to as ***categorical* variables.**

For example, if a medical researcher suspects that alcohol reduces the effectiveness of the immune system, a research study might investigate a hypothesized relationship between alcohol consumption and the incidence of disease, taking measurements on other variables that might influence this relationship, such as gender and weight. If the data were taken on 100 college students, each student would be one case of observation. Each student would be measured on those four variables. Alcohol consumption and weight are numerical variables. Gender is categorical. The incidence of disease could be measured either as a categorical variable ("yes" or "no" in the semester) or as a numerical variable (number of days ill during the semester).

PRACTICE

What type of variable (numerical or categorical) is each of the following?

1. Bank account balance
2. Marital status
3. Student's semester at college
4. Student's year of college
5. Level of pain experienced by a person brought into an emergency room

Answers: (1) numerical, (2) categorical, (3) numerical, (4) either, (5) categorical

The variable in (1) is numerical because the value of a bank account balance is measured in dollars. The variable in (2) is categorical because it is either "single" or "married." Sometimes "widowed" and "divorced" are also added as subcategories of "single." The variable in (3) is numerical because a student's semester can be denoted with the numbers 1 through 8, signifying the number of semesters that a person has been enrolled at college. The variable in (4) could be measured either as numerical or categorical: as a count of years or by category "freshman," "sophomore," "junior," and "senior." The actual measurement will depend on the purposes for which the information is to be used. The variable in (5) will probably be measured categorically in any practical usage. An admitting nurse may ask a patient, "Is your pain low, moderate, or severe?" This question gives three categories that can indicate the level of pain. But such a measurement system could be made numerical if the nurse were instructed to ask everyone, "On a scale of 1 to 10, how severe is your pain?" This example shows that some variables can be given a numerical measurement, even if it is only a subjective measure.

Store	*Price ($)*	*Quantity sold per week (lb.)*
Store A	3.40	150
Store B	2.85	300
Store C	3.00	265
Store D	3.35	170
Store E	3.20	210

Information in Tabular Form

Let's say that a grocery chain wants to find out what the relationship is between the *price of butter* and the number of *pounds sold per week.* One way to show the relationship between these two variables is to use a table such as the one here. Each row represents a case that is observed, namely the individual stores in the Garzolli grocery chain. The columns describe two attributes of the stores' sale of butter, the variables price and quantity.

Look a little closer, and you will notice that the table also shows the *relationship* between the two variables, price and quantity. By examining the table, we can see that stores that charge a higher price of butter tend to have lower weekly sales.

Now, let's consider how the information in this table can be presented graphically.

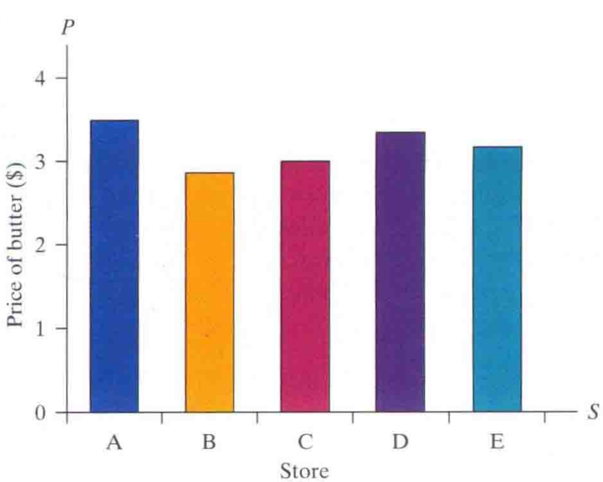

FIGURE 1.1
Bar graph showing the prices at stores A to E

The Bar Graph

A bar graph enables you to compare the level of a *single* variable across several categories. In the case of the Garzolli stores, we can use a bar graph (Figure 1.1) to compare the *price* of butter across different stores. The various cases (here, the stores) are arrayed left to right on the horizontal axis. This permits a person to compare visually the heights of the different bars: a bar that is twice as tall as another signifies a quantity that is twice as large, and so on.

The order in which the cases (e.g., stores) are presented left to right does not matter in a bar graph. Like categories, they are not logically ordered in any way. However, it is not unusual to find bar graphs in which the instances are ordered so that the bars are increasing in height, or decreasing in height, from left to right.

Bar graphs can be used to represent both positive and negative values. The table below lists the monthly profits of five Garzolli stores during May 2004. As you can see, some stores posted a loss for the month, a negative profit.

Store	***Monthly profits ($)***
Store A	120,000
Store B	450,000
Store C	−50,000
Store D	200,000
Store E	−25,000

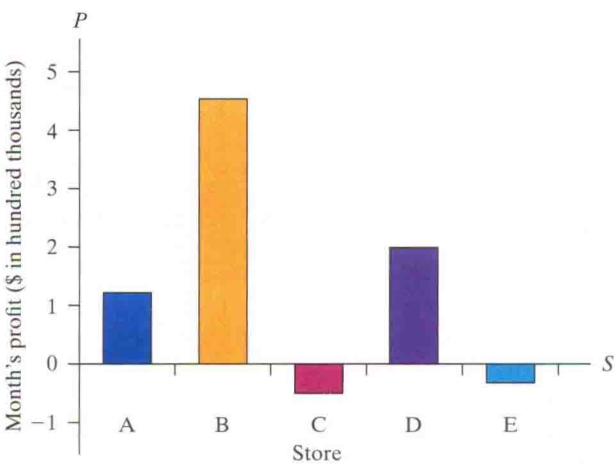

FIGURE 1.2

Bar graph showing both positive and negative numbers

In a bar graph, such as Figure 1.2, the profits are shown as the length of the bars, which are measured against a number line on the vertical axis. As you can see, the bar graph is useful for showing the relative magnitude of different quantities. The **magnitude**— or **absolute value**— of a number is its distance from zero along the number line.

The length of the bar *downward* shows the magnitude of a negative number in the bar graph. Comparing the bars for stores C and E in Figure 1.2, we see that the magnitude of the loss experienced by store C was twice as great as the magnitude of the loss at store E. The graph also tells us that store A has a profit whose magnitude is just over twice as large as the magnitude of the loss at store C.

The Real Number System

This is a good time to pause and reflect about the numbers on the vertical axis of a bar graph. *Any measurable quantity* can be represented as a distance up (positive quantities) or down (negative quantities) along the vertical scale. The markers at intervals of $100,000 along the vertical axis are just a few of the numbers that are represented on the line. *Every point* on that line, as far up and as far down as one can imagine, represents some number. This is a very important idea.

All the numbers on the line represent a measurable distance. They are part of the **real number system** or, put another way, the set of real numbers. The real numbers can be pictured as a line. In a bar graph, the real numbers appear as a vertical line with positive numbers in the upward direction and negative numbers in the downward direction. However, the real number line is