Ya. PANOVKO

ELEMENTS OF THE APPLIED THEORY OF ELASTIC VIBRATION

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ОСНОВЫ ПРИКЛАДНОЙ ТЕОРИИ УПРУГИХ КОЛЕБАНИЙ

ИЗДАТЕЛЬСТВО «МАШИНОСТРОЕНИЕ»
МОСКВА

Ya. Panovko

ELEMENTS OF THE APPLIED THEORY OF ELASTIC VIBRATION

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CONTENTS

NOTATION

A = amplitude, area
$A_1, A_2, \ldots$ = coefficients
a = amplitude, constant, distance
a, b = coefficients, dimensions
$B_1, B_2, \ldots$ = coefficients, constants of integration
$b_1, b_2, \ldots$ = coefficients
$C_1, C_2, \ldots$ = constants of integration
c = damping constant
c_* = equivalent damping constant
$c_1, c_2, \ldots$ = coefficients
D = flexural rigidity of plate
$D_1, D_2, \ldots$ = constants of integration
D, d = diameter
$d_1, d_2, \ldots$ = coefficients
E = modulus of elasticity
EA = tensile stiffness
EI = flexural rigidity of beam
e = eccentricity, exponential constant (2.71828...)
F = force
f = deflection
f_s = static displacement
G = shear modulus
GI_p = torsional rigidity
g = acceleration of gravity
h = height, thickness
I = moment of inertia of an area
I_p = polar moment of inertia of an area
i = gear ratio
J = moment of inertia of mass
k = constant in general or spring constant in particular
L, l = length
M = moment or torque
M_b = bending moment
m = mass
N = longitudinal force, number of cycles, power
n = constant, damping factor, number in general, number of cycles per second in particular
P = force
P_{cr} = critical force
p = natural circular frequency
p_* = natural circular frequency of damped vibration
Q = generalized force, shearing force, weight
q = generalized co-ordinate, load per unit length, pressure
R = resisting force in general or dry friction force in particular

R, r	=	radius
r	=	deflection
r, θ	=	polar co-ordinates
S	=	impulse
T	=	friction force, kinetic energy, period of vibration
t	=	time
U	=	potential energy
u	=	longitudinal displacement, radial displacement
v	=	deflection, velocity
v_0	=	initial velocity
W	=	weight, work
w	=	deflection
x	=	displacement
x_0	=	initial displacement
x_{st}	=	static displacement
x, y, z	=	rectangular co-ordinates
y	=	displacement
y_0	=	initial displacement
α (alpha)	=	angular acceleration, phase angle or some other angle
β (beta)	=	repetition factor
γ (gamma)	=	phase angle or some other angle, shear angle, specific weight
δ (delta)	=	logarithmic decrement
δ_{ik}	=	unit displacement or influence coefficient
ε (epsilon)	=	angle, strain
ε_r	=	strain in radial direction
ε_θ	=	strain in circumferential direction
θ (theta)	=	angle, angular displacement
$\varkappa$ (kappa)	=	amplitude ratio, correction factor
λ (lambda)	=	characteristic number
μ (mu)	=	amplification (magnification) factor, Poisson's ratio
μ_*	=	transmissibility
π (pi)	=	3.14159...
ρ (rho)	=	mass density, radius of curvature, radius of gyration
σ (sigma)	=	stress
σ_r	=	radial stress
σ_θ	=	circumferential stress
τ (tau)	=	shearing stress, time
φ (phi)	=	angle of rotation or angular displacement
Ψ (psi)	=	energy dissipated per cycle
ψ (psi)	=	angle
ω (omega)	=	angular velocity, circular frequency
ω_{cr}	=	critical speed

INTRODUCTION

The periodic nature of operation of most machines determines the periodicity of loading and deformation of both their individual members and those structures which serve as supports or foundations; it may be said that elastic vibrations accompany the operation of each machine.

In some cases, however, vibrations occur in the absence of periodic excitation. These are, for instance, relatively simple processes of free vibration developing upon a sudden disturbance of the state of equilibrium of a mechanical system as well as more complicated and, at the same time, less studied processes such as self-excited vibrations.

It is difficult to indicate a domain of engineering in which the study of elastic vibrations would not be an urgent problem. Much attention is given by investigators to vibration of structures of widely differing purposes: turbine rotors, internal-combustion engine shafts, turbine blades, propellers, automobiles and railroad cars, ships and aircraft, engineering structures, industrial-building floors, parts worked on metal-cutting machines, jigging conveyors, etc.

In certain cases vibrations impede the normal service or even directly endanger the strength by gradually promoting fatigue failure; in such cases the theory may indicate ways of reducing detrimental vibrations. At the same time it enables one to substantiate and optimize the manufacturing processes which use vibrations purposefully (as in jigging conveyors).

For all the variety of problems treated in the theory of elastic vibration there is a deep intrinsic connection between outwardly different problems. The existence of common laws forms the fundamental basis for the general theory which enables one to consider simultaneously wide classes of phenomena covering a host of particular problems.

We may distinguish at least the following five sufficiently independent categories of vibratory processes differing in their nature:

free vibrations, i.e., vibrations which are performed by a mechanical system having no energy supply from outside if the system is disturbed from its position of equilibrium and then released;

critical states of rotating shafts and rotors which consist in a sudden increase in the deflections of their axes at definite speeds of rotation (or in definite ranges of speeds);

forced vibrations which result when the mechanical system is acted on by fluctuating external forces (driving forces);

parametric vibrations caused by periodic variations of the parameters of a system (for example, its stiffness);

self-excited vibrations, i.e., vibratory processes which are maintained by constant sources of energy of a nonvibratory nature.

Each of these categories of vibratory processes is discussed in the appropriate chapter.

CHAPTER I

FUNDAMENTALS

1. Number of Degrees of Freedom of an Elastic System

The complexity of a theoretical analysis of vibration depends largely on the number of degrees of freedom of the mechanical system in question. The *number of degrees of freedom* of a mechanical system is defined as the number of independent co-ordinates which determine uniquely the positions of all particles of the system.

In dynamic problems, particularly in vibration problems, the positions of the particles of a system vary with time so that the above co-ordinates are functions of time. The fundamental problem of dynamic analysis is to find these functions, i.e., to determine the motion of the system. It is an easy matter then to find strains, stresses and internal forces in the constraints of the system.

Every mechanical system involves an infinite number of particles and consequently the number of degrees of freedom is always infinite. However, in solving practical problems use is generally made of simplified schemes which are characterized by a finite number of degrees of freedom. In such design schemes some (the lightest) parts of the system are assumed to be massless and are represented as deformable inertialess constraints; the bodies for which the property of inertia is retained in the design scheme are then considered to be particles ("concentrated masses") or absolutely rigid bodies.

In an endeavour to simplify the design scheme one should remember that the neglect of all inertia properties of a given system deprives the problem of dynamic specific features.

Consider, for instance, a massless spring (Fig. 1*a*) with a force $P(t)$, given as a function of time, applied to its end. If k is the stiffness of the spring, the displacement x of its end is defined by the ordinary static formula

$$x = \frac{P}{k}. \tag{1.1}$$

This formulation of the problem is not really dynamic though the displacement thus found is not constant but represents a function of time. The true dynamics of processes in real mechanical systems is associated with *the property of inertia* and this property must be reflected in one way or another in the design scheme.

A simple example of a dynamic system with one degree of freedom is represented in Fig. 1*b*. Here we can no longer work with purely static relationships; thus, we observe that the reaction R of the spring is not equal to the external force P.

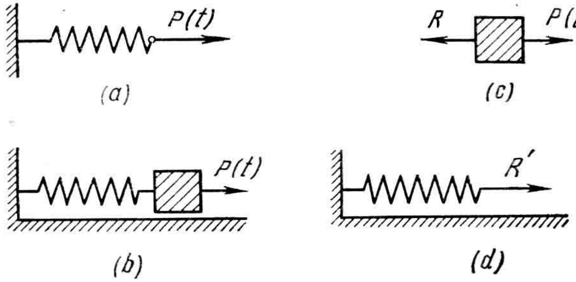

Fig. 1

The differential equation of motion of the mass in the x direction (Fig. 1*c*) is of the form

$$P+R_x=m\ddot{x}, \quad (1.2)$$

where $R_x=-kx$ is the projection of the reaction of the spring on the x axis. Thus we obtain

$$m\ddot{x}+kx=P(t). \quad (1.3)$$

Unlike expression (1.1) which serves to calculate x directly, relation (1.3) represents a *differential equation* in the function x. To find the form of this function it is necessary to integrate the differential equation (1.3). After solving Eq. (1.3) the function $x=x(t)$ is used to find internal forces, stresses, etc.

It may be said that in the above example the function x alone defines completely all the elements of the state of strain at any instant. Such systems possess *one degree of freedom*.

The systems shown in Fig. 2 fall into this type. The characteristic co-ordinate for the diagram of Fig. 2*a* is the ordinate y of the mass, while for the diagram of Fig. 2*b* it is the angle of rotation φ of the rigid body (in both cases the elastic constraints are assumed to be massless).

The systems represented in Fig. 2*c*, *d*, *e*, *f* each have *two degrees of freedom*. In the diagram of Fig. 2*c* there are two concentrated masses and the motion of the system is completely determined by two func-

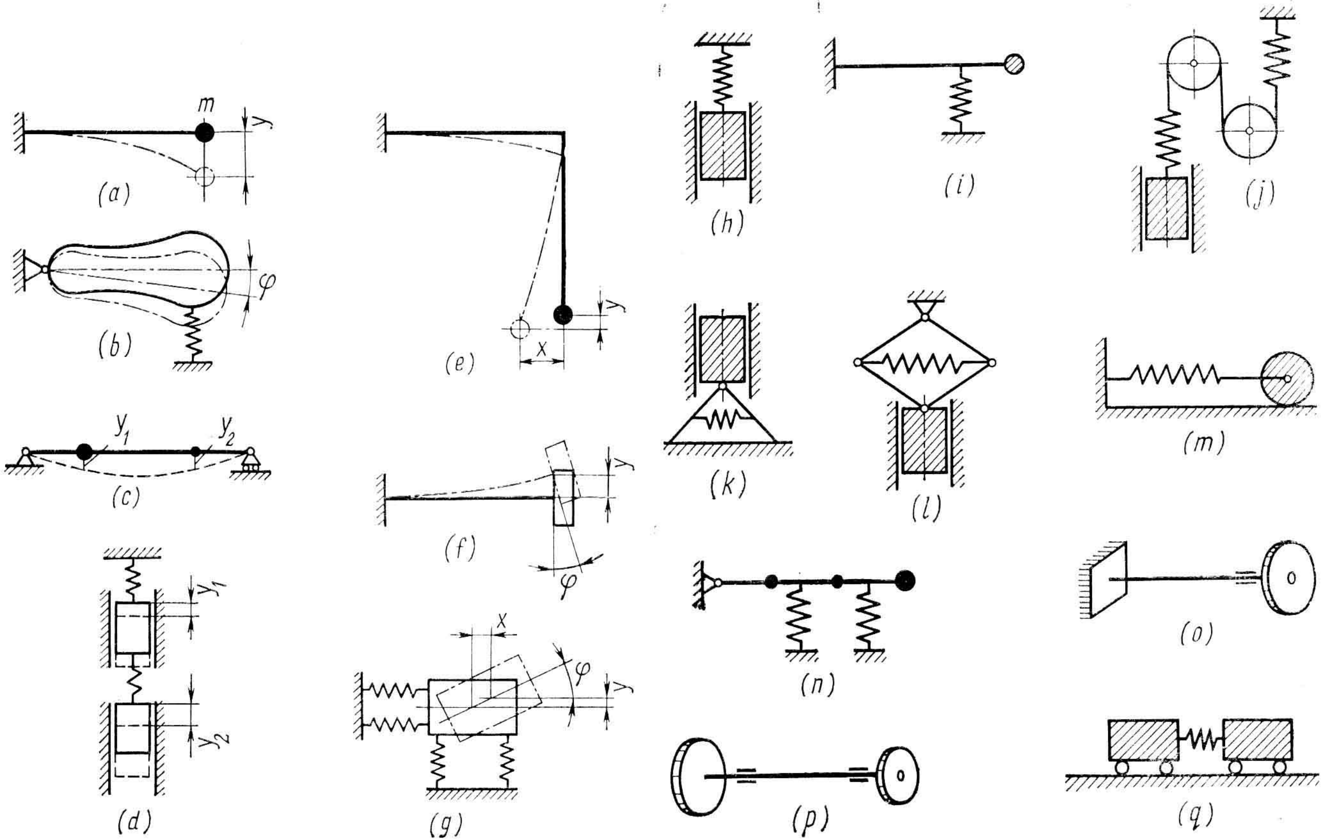

Fig. 2