Wiley Series in Probability and Statistics

Examples and Problems in Mathematical Statistics

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WILEY SERIES IN PROBABILITY AND STATISTICS

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To my wife Hanna,
our sons Yuval and David,
and their families, with love.

Preface

I have been teaching probability and mathematical statistics to graduate students for close to 50 years. In my career I realized that the most difficult task for students is solving problems. Bright students can generally grasp the theory easier than apply it. In order to overcome this hurdle, I used to write examples of solutions to problems and hand it to my students. I often wrote examples for the students based on my published research. Over the years I have accumulated a large number of such examples and problems. This book is aimed at sharing these examples and problems with the population of students, researchers, and teachers.

The book consists of nine chapters. Each chapter has four parts. The first part contains a short presentation of the theory. This is required especially for establishing notation and to provide a quick overview of the important results and references. The second part consists of examples. The examples follow the theoretical presentation. The third part consists of problems for solution, arranged by the corresponding sections of the theory part. The fourth part presents solutions to some selected problems. The solutions are generally not as detailed as the examples, but as such these are examples of solutions. I tried to demonstrate how to apply known results in order to solve problems elegantly. All together there are in the book 167 examples and 431 problems.

The emphasis in the book is on statistical inference. The first chapter on probability is especially important for students who have not had a course on advanced probability. Chapter Two is on the theory of distribution functions. This is basic to all developments in the book, and from my experience, it is important for all students to master this calculus of distributions. The chapter covers multivariate distributions, especially the multivariate normal; conditional distributions; techniques of determining variances and covariances of sample moments; the theory of exponential families; Edgeworth expansions and saddle-point approximations; and more. Chapter Three covers the theory of sufficient statistics, completeness of families of distributions, and the information in samples. In particular, it presents the Fisher information, the Kullback–Leibler information, and the Hellinger distance. Chapter Four provides a strong foundation in the theory of testing statistical hypotheses. The Wald SPRT is discussed there too. Chapter Five is focused on optimal point estimation of different kinds. Pitman estimators and equivariant estimators are also discussed. Chapter Six covers problems of efficient confidence intervals, in particular the problem of determining fixed-width confidence intervals by two-stage or sequential sampling. Chapter Seven covers techniques of large sample approximations, useful in estimation and testing. Chapter Eight is devoted to Bayesian analysis, including empirical Bayes theory. It highlights computational approximations by numerical analysis and simulations. Finally, Chapter Nine presents a few more advanced topics, such as minimaxity, admissibility, structural distributions, and the Stein-type estimators.

I would like to acknowledge with gratitude the contributions of my many ex-students, who toiled through these examples and problems and gave me their important feedback. In particular, I am very grateful and indebted to my colleagues, Professors A. Schick, Q. Yu, S. De, and A. Polunchenko, who carefully read parts of this book and provided important comments. Mrs. Marge Pratt skillfully typed several drafts of this book with patience and grace. To her I extend my heartfelt thanks. Finally, I would like to thank my wife Hanna for giving me the conditions and encouragement to do research and engage in scholarly writing.

SHELEMYAHU ZACKS

List of Random Variables

$B(n, p)$	Binomial, with parameters n and p
$E(\mu)$	Exponential with parameter μ
$EV(\lambda, \alpha)$	Extreme value with parameters λ and α
$F(\nu_1, \nu_2)$	Central F with parameters ν_1 and ν_2
$F(n_1, n_2; \lambda)$	Noncentral F with parameters ν_1, ν_2, λ
$G(\lambda, p)$	Gamma with parameters λ and p
$H(M, N, n)$	Hyper-geometric with parameters M, N, n
$N(\mu, V)$	Multinormal with mean vector μ and covariance matrix V
$N(\mu, \sigma)$	Normal with mean μ and σ
$NB(\psi, \nu)$	Negative-binomial with parameters ψ, and ν
$P(\lambda)$	Poisson with parameter λ
$R(a, b)$	Rectangular (uniform) with parameters a and b
$t[n; \lambda]$	Noncentral Student's t with parameters n and λ
$t[n; \xi, V]$	Multivariate t with parameters n, ξ and V
$t[n]$	Student's t with n degrees of freedom
$W(\lambda, \alpha)$	Weibul with parameters λ and α
$\beta(p, q)$	Beta with parameters p and q
$\chi^2[n, \lambda]$	Noncentral chi-squared with parameters n and λ
$\chi^2[n]$	Chi-squared with n degrees of freedom

List of Abbreviations

a.s.	Almost surely
ANOVA	Analysis of variance
c.d.f.	Cumulative distribution function
$\text{cov}(x, y)$	Covariance of X and Y
CI	Confidence interval
CLT	Central limit theorem
CP	Coverage probability
CR	Cramer Rao regularity conditions
$E\{X \mid Y\}$	Conditional expected value of X, given Y
$E\{X\}$	Expected value of X
FIM	Fisher information matrix
i.i.d.	Independent identically distributed
LBUE	Linear best unbiased estimate
LCL	Lower confidence limit
m.g.f.	Moment generating function
m.s.s.	Minimal sufficient statistics
MEE	Moments equations estimator
MLE	Maximum likelihood estimator
MLR	Monotone likelihood ratio
MP	Most powerful
MSE	Mean squared error
MVU	Minimum variance unbiased
OC	Operating characteristic
p.d.f.	Probability density function
p.g.f.	Probability generating function
$P\{E \mid A\}$	Conditional probability of E, given A
$P\{E\}$	Probability of E
PTE	Pre-test estimator
r.v.	Random variable
RHS	Right-hand side
s.v.	Stopping variable

SE	Standard error
SLLN	Strong law of large numbers
SPRT	Sequential probability ratio test
$\text{tr}\{A\}$	trace of the matrix A
UCL	Upper control limit
UMP	Uniformly most powerful
UMPI	Uniformly most powerful invariant
UMPU	Uniformly most powerful unbiased
UMVU	Uniformly minimum variance unbiased
$V\{X \mid Y\}$	Conditional variance of X, given Y
$V\{X\}$	Variance of X
w.r.t.	With respect to
WLLN	Weak law of large numbers

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