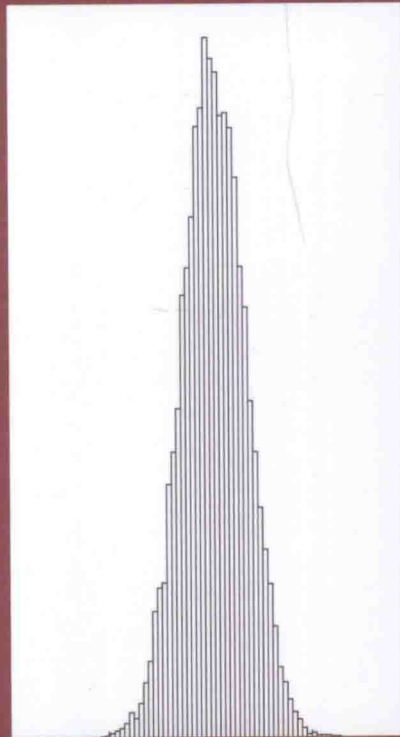
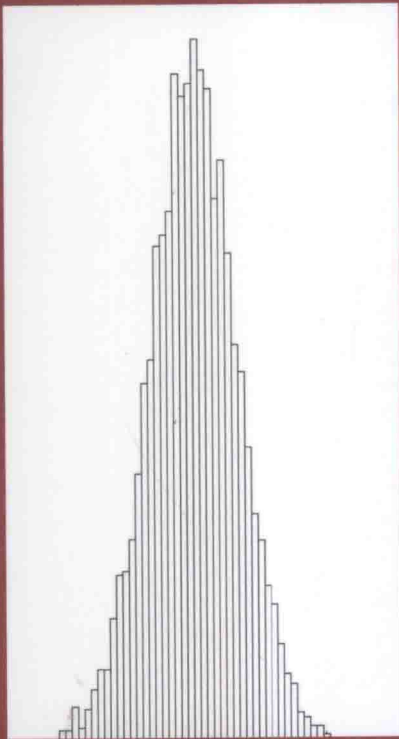


Introduction to

BAYESIAN ECONOMETRICS

Second Edition



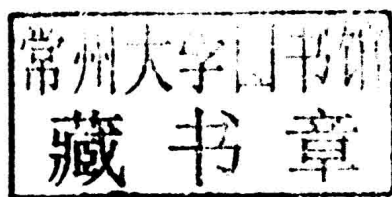
Edward Greenberg

Introduction to Bayesian Econometrics

Second Edition

EDWARD GREENBERG

Washington University in St. Louis



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Introduction to Bayesian Econometrics

Second Edition

This textbook, now in its second edition, is an introduction to econometrics from the Bayesian viewpoint. It begins with an explanation of the basic ideas of subjective probability and shows how subjective probabilities must obey the usual rules of probability to ensure coherency. The book then turns to the definitions of the likelihood function, prior distributions, and posterior distributions. It explains how posterior distributions are the basis for inference and explores their basic properties. The Bernoulli distribution is used as a simple example. Various methods of specifying prior distributions are considered, with special emphasis on subject-matter considerations and exchangeability. The regression model is examined to show how analytical methods may fail in the derivation of marginal posterior distributions, which leads to an explanation of classical and Markov chain Monte Carlo (MCMC) methods of simulation. The latter is preceded by a brief introduction to Markov chains. The remainder of the book is concerned with applications of the theory to important models that are used in economics, political science, biostatistics, and other applied fields. New to the second edition is a chapter on semiparametric regression and new sections on the ordinal probit, item response, factor analysis, ARCH-GARCH, and stochastic volatility models. The new edition also emphasizes the R programming language, which has become the most widely used environment for Bayesian statistics.

Edward Greenberg is Professor Emeritus of Economics at Washington University, St. Louis, where he served as a Full Professor on the faculty from 1969 to 2005. Professor Greenberg also taught at the University of Wisconsin, Madison, and has been a Visiting Professor at the University of Warwick (UK), Technion University (Israel), and the University of Bergamo (Italy). A former holder of a Ford Foundation Faculty Fellowship, Greenberg is the author of the first edition of *Introduction to Bayesian Econometrics* (Cambridge University Press, 2008) and the co-author of four books: *Wages, Regime Switching, and Cycles* (1992); *The Labor Market and Business Cycle Theories* (1989); *Advanced Econometrics* (1983, revised 1991); and *Regulation, Market Prices, and Process Innovation* (1979). His published research has appeared in leading journals such as the *American Economic Review*, *Econometrica*, *Journal of Econometrics*, *Journal of the American Statistical Association*, *Biometrika*, and the *Journal of Economic Behavior and Organization*. Professor Greenberg's current research interests include dynamic macroeconomics as well as Bayesian econometrics.

Preface to the Second Edition

THE MOTIVATION FOR this edition is the same as for the first: to provide a concise introduction to the main ideas of Bayesian statistics and econometrics. The changes, however, have made the book somewhat less concise. In particular, I have added a chapter on Bayesian nonparametrics and new sections on the ordinal probit model, item response models, factor analysis models, and time-varying variances. I believe that these additional materials make the book more useful to readers. Another difference is that this edition adopts the R statistics environment as the primary tool for computing.

In addition to those thanked in the preface to the first edition, without implicating them in any errors or omissions, I offer my sincere gratitude to John Burkett, Stephen Haptonstahl, Alejandro Jara, Kyu Ho Kang, Xun Pang, Jong Hee Park, Srikanth Ramamurthy, Richard Startz, and Ghislain Vieilledent.

I am grateful for the continued support of Lisa, Aida, my grandchildren, and Sylvia Silver and her family.

With sadness, I note the recent passing of my friends and colleagues Peter Steiner, Arthur Goldberger, and Arnold Zellner, and of my dear son Arthur, to whom I dedicate this edition.

Preface to the First Edition

To Instructors and Students

THIS BOOK IS a concise introduction to Bayesian statistics and econometrics. It can be used as a supplement to a frequentist course by instructors who wish to introduce the Bayesian viewpoint or as a text in a course on Bayesian econometrics supplemented by readings in the current literature.

While the student should have had some exposure to standard probability theory and statistics, the book does not make extensive use of statistical theory. Indeed, because of its reliance on simulation techniques, it requires less background in statistics and probability than most books that take a frequentist approach. It is, however, strongly recommended that the student become familiar with the forms and properties of the standard probability distributions collected in Appendix A.

Since the advent of Markov chain Monte Carlo (MCMC) methods in the early 1990s, Bayesian methods have been extended to a large and growing number of applications. This book limits itself to explaining in detail a few important applications. Its main goal is to provide examples of MCMC algorithms to enable students and researchers to design algorithms for the models that arise in their own research. More attention is paid to the design of algorithms for the models than to the specification and interpretation of the models themselves because I assume that the student has been exposed to these models in other statistics and econometrics classes.

The decision to keep the book short has also meant that I have taken a stand on some controversial issues rather than discussing a large number of alternative methods. In some cases, alternative approaches are discussed in end-of-chapter notes.

Exercises have been included at the end of chapters, but the best way to learn the material is for students to apply the ideas to empirical applications of their choice. Accordingly, even though it is not explicitly stated, the first exercise at the end of every chapter in Part III should direct students to formulate a model, collect

data, specify a prior distribution on the basis of previous research, design and, if necessary, program an algorithm, then present the results.

A link to the Web site for the course may be found at <http://edg.wustl.edu/>. The site contains errata, links to data sources, some computer code, and other information.

Acknowledgments

I would like to acknowledge and offer my sincere gratitude to some of the people who have helped me over my career. On the professional side, I start with my undergraduate years at the business school of New York University, where Abraham Gitlow awakened my interest in economics. My first statistics course was with F. J. Viser and my second with Ernest Kurnow, who encouraged me to continue my studies and guided me in the process.

At the University of Wisconsin, Madison, I was mentored by, among others, Peter Steiner and Guy Orcutt. Econometrics was taught by Jack Johnston, who was writing the first edition of his pathbreaking book, and I was fortunate to have Arthur Goldberger and Arnold Zellner as teachers and colleagues. My first mathematical statistics course was with Enders Robinson, and I later audited George Box's class, where I received my first exposure to Bayesian ideas. Soon afterward, Zellner began to apply the methods to econometrics in a workshop that I attended.

My interest in Bayesian methods was deepened at Washington University first by E. T. Jaynes and then by Siddhartha Chib. Sid Chib has been my teacher, collaborator, and friend for the last fifteen years. His contributions to Bayesian statistics, econometrics, and MCMC methods have had enormous impact. I have been extremely fortunate to have had the opportunity to work with him. The students in my courses in Bayesian econometrics contributed to my understanding of the material by their blank stares and penetrating questions. I am most grateful to them.

My colleagues and the staff of the Economics Department at Washington University have always been extremely helpful to me. I am delighted to thank them for their support.

I am most grateful to my editor at Cambridge University Press, Scott Parris, for suggesting the book and for his continuing encouragement and support

I am pleased to acknowledge the comments of Andrew Martin, James Morley, and two anonymous reviewers on various drafts of this book and, especially, those of Ivan Jeliazkov, who read it most carefully and thoughtfully and tested it on his students. All remaining errors are, of course, mine.

I am grateful to Professor Chang-Jin Kim for permission to utilize his software to compute some of the examples in Chapter 11.

On the personal side, I thank Arthur and Aida, Lisa and Howard, my grandchildren, and my colleagues and friends, particularly Sylvia Silver, Karen Rensing, Ingrid and Wilhelm Neufeind, Maureen Regan and Sid Chib, Jasmine and Steve Fazzari, and Camilla and Piero Ferri.

In December 2005 my wife of more than 46 years passed away. I dedicate this book to Joan's memory.

Contents

<i>List of Figures</i>	page xi
<i>List of Tables</i>	xiii
<i>Preface to the Second Edition</i>	xv
<i>Preface to the First Edition</i>	xvii

Part I Fundamentals of Bayesian Inference

1 Introduction	3
1.1 Econometrics	3
1.2 Overview of the Book	4
1.3 Historical Note and Further Reading	5
2 Basic Concepts of Probability and Inference	7
2.1 Probability	7
2.1.1 Frequentist Probabilities	8
2.1.2 Subjective Probabilities	9
2.2 Prior, Likelihood, and Posterior	12
2.3 Summary	18
2.4 Further Reading and References	19
2.5 Exercises	19
3 Posterior Distributions and Inference	21
3.1 Properties of Posterior Distributions	21
3.1.1 The Likelihood Function	21
3.1.2 Vectors of Parameters	23
3.1.3 Bayesian Updating	25
3.1.4 Large Samples	27
3.1.5 Identification	29

3.2	Inference	30
3.2.1	Point Estimates	30
3.2.2	Interval Estimates	32
3.2.3	Prediction	33
3.2.4	Model Comparison	34
3.3	Summary	39
3.4	Further Reading and References	39
3.5	Exercises	40
4	Prior Distributions	43
4.1	Normal Linear Regression Model	43
4.2	Proper and Improper Priors	45
4.3	Conjugate Priors	46
4.4	Subject-Matter Considerations	49
4.5	Exchangeability	52
4.6	Hierarchical Models	54
4.7	Training Sample Priors	55
4.8	Sensitivity and Robustness	56
4.9	Conditionally Conjugate Priors	56
4.10	A Look Ahead	59
4.11	Further Reading and References	60
4.12	Exercises	60
Part II	Simulation	
5	Classical Simulation	65
5.1	Probability Integral Transformation Method	65
5.2	Method of Composition	67
5.3	Accept-Reject Algorithm	68
5.4	Importance Sampling	72
5.5	Multivariate Simulation	74
5.6	Using Simulated Output	75
5.7	Further Reading and References	77
5.8	Exercises	77
6	Basics of Markov Chains	79
6.1	Finite State Spaces	79
6.2	Countable State Spaces	84
6.3	Continuous State Spaces	88
6.4	Further Reading and References	89
6.5	Exercises	90

7	Simulation by MCMC Methods	93
7.1	Gibbs Algorithm	94
7.1.1	Basic Algorithm	94
7.1.2	Calculation of Marginal Likelihood	97
7.2	Metropolis-Hastings Algorithm	99
7.2.1	Basic Algorithm	99
7.2.2	Calculation of Marginal Likelihood	104
7.3	Numerical Standard Errors and Convergence	105
7.4	Further Reading and References	108
7.5	Exercises	109
Part III	Applications	
8	Linear Regression and Extensions	115
8.1	Continuous Dependent Variables	115
8.1.1	Normally Distributed Errors	115
8.1.2	Student- t Distributed Errors	118
8.2	Limited Dependent Variables	121
8.2.1	Tobit Model for Censored Data	121
8.2.2	Binary Probit Model	126
8.2.3	Binary Logit Model	130
8.2.4	Ordinal Probit Model	133
8.3	Latent Variable Models	138
8.3.1	Item Response Model	138
8.3.2	Factor Analysis Models	141
8.4	Further Reading and References	143
8.5	Exercises	146
9	Semiparametric Regression	148
9.1	Flexible Forms for the Conditional Mean Function	148
9.1.1	LS Basis	149
9.1.2	Identification	152
9.1.3	Prior Distributions	153
9.1.4	MCMC Algorithm for the Cubic Spline Approximation	155
9.1.5	Marginal Likelihood Calculation	156
9.1.6	Examples	158
9.2	Flexible Error Distributions	161
9.2.1	Introduction to Dirichlet Process Mixtures	161
9.2.2	Inference for Dirichlet Process Mixtures	164
9.2.3	Examples	165

9.3	Further Reading and References	167
9.4	Exercises	168
10	Multivariate Responses	169
10.1	SUR Model	169
10.2	Multivariate Probit Model	174
10.3	Panel Data	178
10.4	Further Reading and References	184
10.5	Exercises	186
11	Time Series	187
11.1	Autoregressive Models	187
11.2	Regime-Switching Models	193
11.3	Time-Varying Parameters	196
11.4	Time Series Properties of Models for Panel Data	199
11.5	Time-Varying Variances	201
11.5.1	ARCH–GARCH Models	201
11.5.2	Stochastic Volatility Models	202
11.6	Further Reading and References	204
11.7	Exercises	205
12	Endogenous Covariates and Sample Selection	206
12.1	Treatment Models	206
12.2	Unobserved Covariates	212
12.3	Incidental Truncation	215
12.4	Further Reading and References	221
12.5	Exercises	222
A	Probability Distributions and Matrix Theorems	223
A.1	Probability Distributions	223
A.1.1	Bernoulli	223
A.1.2	Binomial	223
A.1.3	Negative Binomial	224
A.1.4	Multinomial	224
A.1.5	Poisson	224
A.1.6	Uniform	225
A.1.7	Gamma	225
A.1.8	Inverted or Inverse Gamma	225
A.1.9	Beta	226
A.1.10	Pareto	227
A.1.11	Dirichlet	227
A.1.12	Normal or Gaussian	227

A.1.13 Truncated Normal	227
A.1.14 Multivariate Normal or Gaussian	228
A.1.15 Matricvariate Normal or Matrix Normal	229
A.1.16 Univariate Student- t	230
A.1.17 Multivariate t	230
A.1.18 Wishart	231
A.1.19 Inverted or Inverse Wishart	231
A.1.20 Multiplication Rule of Probability	232
A.2 Matrix Theorems	232
B Computer Programs for MCMC Calculations	234
<i>Bibliography</i>	237
<i>Author Index</i>	245
<i>Subject Index</i>	247

List of Figures

2.1	Beta distributions for various values of α and β .	<i>page</i> 16
2.2	Prior, likelihood, and posterior for coin-tossing example.	18
4.1	Coefficient of union membership.	52
5.1	Target and proposal density to sample from Beta(3, 3).	70
5.2	Target and proposal density to sample from $N(0, 1)$.	71
5.3	Simulation results for $Z = XY$.	75
6.1	Random walk, $p = q = 0.5$.	85
6.2	Random walk, $p = 0.55, q = 0.45$.	86
7.1	Simulation results for MH sampling of Beta(3, 4).	103
7.2	Autocorrelations of $X^{(g)}$.	107
8.1	Posterior distributions of $\beta_U y$ and $\sigma^2 y$, Gaussian errors.	117
8.2	Autocorrelations of β_U and σ^2 , Gaussian errors.	117
8.3	Posterior distributions of β_U and σ^2 , Student- t errors.	119
8.4	Autocorrelations of β_U and σ^2 , Student- t errors.	120
8.5	Posterior distributions of β : Tobit model, Mroz data.	125
8.6	Predictive distributions of hours worked, Tobit model.	126
8.7	Posterior distributions of β , probit model.	130
8.8	Posterior distributions of β , logit model.	133
8.9	Traceplots of β : computer ownership example, logit model.	134
8.10	Autocorrelations of β , logit model.	135
9.1	True and estimated functions and derivatives, simulated data.	159
9.2	Predictive distribution for y , simulated data.	160

9.3	Estimated functions and derivatives, LIDAR data.	162
10.1	Posterior distributions of β_U and $\text{Mean}(b_2)$.	183
11.1	Densities of parameters, Phillips curve data.	192
11.2	Probability of recession.	196
11.3	Time varying slope.	200
11.4	Phillips curve, GARCH model.	203
12.1	Distributions of coefficients, incidental truncation model.	221

List of Tables

2.1	Coherency: Restrictions on p .	<i>page</i> 10
2.2	Coherency: Determination of $P(A_1 \cup A_2)$.	11
2.3	Coherency: Determination of $P(A B)$.	12
3.1	Jeffreys guidelines.	36
3.2	Bayes factors for selected possible outcomes.	39
4.1	β_U as a function of hyperparameters β_{U0} and $B_{UU,0}$.	56
8.1	Summary of posterior distribution: Tobit model, Mroz data.	125
8.2	Posterior distribution, probit model.	130
8.3	Posterior distribution, logit model.	133
8.4	Posterior distribution, ordinal probit model.	138
8.5	Summary of ideal points: IRT model, Senate data.	141
8.6	Posterior distribution, factor analysis model.	144
9.1	Marginal likelihoods for numbers of knots, simulated data.	159
10.1	Posterior distributions of β_C and β_F , SUR model.	173
10.2	Posterior distribution of correlations, SUR model.	173
10.3	Posterior distribution of β , MVP model.	178
10.4	Posterior distributions, panel data, Gaussian errors.	183
11.1	Parameter estimates for GDP data.	195
11.2	Summary of posterior distribution: time-varying parameter model.	199
11.3	Summary of posterior distribution: GARCH model.	202
12.1	Posterior distribution, instrumental variable model.	216
12.2	Posterior distribution, probit selection model.	220

Part I

Fundamentals of Bayesian Inference