

# FILTERING, CONTROL AND FAULT DETECTION WITH RANDOMLY OCCURRING INCOMPLETE INFORMATION

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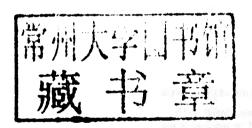
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This edition first published 2013 © 2013 John Wiley & Sons, Ltd

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John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data

Dong, Hongli, 1977-

Filtering, control and fault detection with randomly occurring incomplete information / Hongli Dong, Zidong Wang, Huijun Gao.

pages cm

Includes bibliographical references and index.

ISBN 978-1-118-64791-2 (cloth)

1. Automatic control. 2. Electric filters, Digital. 3. Fault tolerance (Engineering) I. Wang, Zidong, 1966–II. Gao, Huijun. III. Title.

TJ213.D655 2013

003'.75-dc23

2013007460

A catalogue record for this book is available from the British Library

ISBN 9781118647912

Typeset in 10/12pt Times by Aptara Inc., New Delhi, India

Printed and bound in Singapore by Markono Print Media Pte Ltd

# FILTERING, CONTROL AND FAULT DETECTION WITH RANDOMLY OCCURRING INCOMPLETE INFORMATION

The time is boring without random occurrence

The research is monotonous without incomplete information

The life is tedious without fault detection

The living is tough without noise filtering

The power is nothing without control

This book is dedicated to the Dream Dynasty, consisting of a group of simple yet happy people who are falling in love with both the random incompleteness and the incomplete randomness by detecting the faults, filtering the noises, and controlling the powers...

# Preface

In the context of systems and control, incomplete information refers to a dynamical system in which knowledge about the system states is limited due to the difficulties in modeling complexity in a quantitative way. The well-known types of incomplete information include parameter uncertainties and norm-bounded nonlinearities. Recently, in response to the development of network technologies, the phenomenon of randomly occurring incomplete information has become more and more prevalent. Such a phenomenon typically appears in a networked environment. Examples include, but are not limited to, randomly varying nonlinearities (RVNs), randomly occurring mixed time-delays (ROMDs), randomly occurring multiple time-varying communication delays (ROMTCDs), and randomly occurring quantization errors (ROQEs). Randomly occurring incomplete information, if not properly handled, would seriously deteriorate the performance of a control system.

In this book, we investigate the filtering, control, and fault detection problems for several classes of nonlinear systems with randomly occurring incomplete information. Some new concepts are proposed which include RVNs, ROMDs, ROMTCDs, and ROQEs. The incomplete information under consideration mainly includes missing measurements, time delays, sensor and actuator saturations, quantization effects, and time-varying nonlinearities. The content of this book can be divided into three parts. In the first part, we focus on the filtering, control, and fault detection problems for several classes of nonlinear stochastic discrete-time systems with missing measurements, sensor and actuator saturations, RVNs, ROMDs, and ROQEs. Some sufficient conditions are derived for the existence of the desired filters, controllers, and fault detection filters by developing new techniques for the considered nonlinear stochastic systems. In the second part, the theories and techniques developed in the previous part are extended to deal with distributed filtering issues over sensor networks, and some distributed filters are designed for nonlinear time-varying systems and Markovian jump nonlinear time-delay systems. Finally, we apply a new stochastic  $H_{\infty}$  filtering approach to study the mobile robot localization problem, which shows the promising application potential of our main results.

The book is organized as follows. Chapter 1 introduces some recent advances on the analysis and synthesis problems with randomly occurring incomplete information. The developments of the filtering, control, and fault detection problems are systematically reviewed, and the research problems to be addressed in each individual chapter are also outlined. Chapter 2 is concerned with the finite-horizon filtering and control problems for nonlinear time-varying stochastic systems where sensor and actuator saturations, variance-constrained and missing measurements are considered. In Chapters 3 and 4, the  $H_{\infty}$  filtering and control problems are addressed for several classes of nonlinear discrete systems where ROMTCDs and multiple

# Acknowledgments

We would like to express our deep appreciation to those who have been directly involved in various aspects of the research leading to this book. Special thanks go to Professor Daniel W. C. Ho from City University of Hong Kong, Professor James Lam from the University of Hong Kong, Professor Xiaohui Liu from Brunel University in the UK, Professor Steven X. Ding from the University of Duisburg–Essen in Germany, and Professor Ligang Wu from Harbin Institute of Technology of China for their valuable suggestions, constructive comments, and support. We also extend our thanks to the many colleagues who have offered support and encouragement throughout this research effort. In particular, we would like to acknowledge the contributions from Bo Shen, Liang Hu, Jun Hu, Yurong Liu, Jinling Liang, Guoliang Wei, Xiao He, Lifeng Ma, Derui Ding, Yao Wang, Xiu Kan, Sunjie Zhang, and Nianyin Zeng. Finally, we are especially grateful to our families for their encouragement and never-ending support when it was most required.

The writing of this book was supported in part by the National 973 Project under Grant 2009CB320600, the National Natural Science Foundation of China under Grants 61273156, 61134009, 61004067, and 61104125, the Engineering and Physical Sciences Research Council (EPSRC) of the UK, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany. The support of these organizations is gratefully acknowledged.

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packet dropouts are taken into account. Chapter 5 investigates the robust  $H_{\infty}$  filtering and fault detection problems for nonlinear Markovian jump systems with sensor saturation and RVNs. In Chapter 6, the fault detection problem is considered for two classes of discrete-time systems with randomly occurring nonlinearities, ROMDs, successive packet dropouts and measurement quantizations. Chapters 7, 8, and 9 discuss the distributed  $H_{\infty}$  filtering problem over sensor networks. In Chapter 10, a new stochastic  $H_{\infty}$  filtering approach is proposed to deal with the localization problem of the mobile robots modeled by a class of discrete nonlinear time-varying systems subject to missing measurements and quantization effects. Chapter 11 summarizes the results of the book and discusses some future work to be investigated further.

This book is a research monograph whose intended audience is graduate and postgraduate students and researchers.

# List of Abbreviations

CCL cone complementarity linearization

DFD distributed filter design
DKF distributed Kalman filtering
FHFD finite-horizon  $H_{\infty}$  filter design

HCMDL  $H_{\infty}$  control with multiple data losses

HFDL  $H_{\infty}$  filtering with data loss

HinfFC  $H_{\infty}$  fuzzy control HinfF  $H_{\infty}$  filtering

LMI linear matrix inequality
MJS Markovian jump system
NCS networked control system

OFDFD optimized fault detection filter design

RFD robust filter design RHFD robust  $H_{\infty}$  filter design

RLMI recursive linear matrix inequality
RMM randomly missing measurement
ROMD randomly occurring mixed time-delay

ROMTCD randomly occurring multiple time-varying communication delay

ROPD randomly occurring packet dropout
ROQE randomly occurring quantization error
ROSS randomly occurring sensor saturation

RVN randomly varying nonlinearity
RDE Riccati difference equation
SAS sensor and actuator saturation
SPD successive packet dropout
TP transition probability

T-S Takagi-Sugeno

## List of Notations

```
\mathbb{R}^n
                   the n-dimensional Euclidean space
\mathbb{R}^{n \times m}
                   the set of all n \times m real matrices
R+
                   the set of all nonnegative real numbers
11+
                   the set of all nonnegative integers
\mathbb{Z}^-
                   the set of all negative integers
                   the class of all continuous nondecreasing convex functions \phi: \mathbb{R}^+ \to \mathbb{R}^+ such
OL
                   that \phi(0) = 0 and \phi(r) > 0 for r > 0
                   the norm of matrix A defined by ||A|| = \sqrt{\operatorname{tr}(A^{T}A)}
||A||
A^{\mathrm{T}}
                   the transpose of the matrix A
A^{\dagger} \in \mathbb{R}^{n \times m}
                   the Moore–Penrose pseudo inverse of A \in \mathbb{R}^{m \times n}
                   the identity matrix of compatible dimension
I
0
                   the zero matrix of compatible dimension
Prob(·)
                   the occurrence probability of the event "."
\mathbb{E}\{x\}
                   the expectation of the stochastic variable x
\mathbb{E}\{x|y\}
                   the expectation of the stochastic variable x conditional on y
(\Omega, \mathcal{F}, Prob)
                   the complete probability space
\lambda_{\min}(A)
                   the smallest eigenvalue of a square matrix A
\lambda_{\max}(A)
                   the largest eigenvalue of a square matrix A
                   the ellipsis for terms induced by symmetry, in symmetric block matrices
diag\{\cdots\}
                   the block-diagonal matrix
l_2[0,\infty)
                   the space of square summable sequences
|| . || 2
                   the usual l_2 norm
                   the trace of a matrix A
tr(A)
\min \operatorname{tr}(A)
                   the minimization of tr(A)
Var\{x_i\}
                   the variance of x_i
                   the Kronecker product
0
                   \mathbf{1}_n = [1, 1, \dots, 1]^{\mathrm{T}} \in \mathbb{R}^n
\mathbf{1}_n
                   e_i = [0, \dots, 0, 1, 0, \dots, 0]^T
e_i
X > Y
                   the X - Y is positive definite, where X and Y are real symmetric matrices
X > Y
                   the X - Y is positive semi-definite, where X and Y are real symmetric matrices
```

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# Introduction

In the past decade, networked control systems (NCSs) have attracted much attention owing to their successful applications in a wide range of areas for the advantage of decreasing the hardwiring, the installation cost, and implementation difficulties. Nevertheless, network-related challenging problems inevitably arise due to the physical equipment constraints, the complexity, and uncertainty of the external environment in the process of modeling or information transmission, which would drastically degrade the system performance. Such network-induced problems include, but are not limited to, missing measurements, communication delays, sensor and actuator saturations, signal quantization, and randomly varying nonlinearities. These phenomena may occur in a probabilistic way that is customarily referred to as randomly occurring incomplete information.

For several decades, nonlinear analysis and stochastic analysis have arguably been two of the most active research areas in systems and control. This is simply because (1) nonlinear control problems are of interest to engineers, physicists, and mathematicians as most physical systems are inherently nonlinear in nature, and (2) stochastic modeling has come to play an important role in many branches of science and industry as many real-world system and natural processes may be subject to stochastic disturbances. There has been a rich literature on the general nonlinear stochastic control problems. A great number of techniques have been developed on filtering, control, and fault detection problems for nonlinear stochastic systems in order to meet the needs of practical engineering. Recently, with the development of NCSs, the analysis and synthesis problems for nonlinear stochastic systems with the aforementioned network-induced phenomena have become interesting and imperative, yet challenging, topics. Therefore, the aim of this book is to deal with the filtering, control, and fault detection problems for nonlinear stochastic systems with randomly occurring incomplete information.

The focus of this chapter is to provide a timely review on the recent advances of the analysis and synthesis issues for complex systems with randomly occurring incomplete information. Most commonly used methods for modeling randomly occurring incomplete information are summarized. Based on the models established, various filtering, control, and fault detection problems with randomly occurring incomplete information are discussed in great detail. Subsequently, some challenging issues for future research are pointed out. Finally, we give the outline of this book.

### 1.1 Background, Motivations, and Research Problems

### 1.1.1 Randomly Occurring Incomplete Information

Accompanied by the rapid development of communication and computer technology, NCSs have become more and more popular for their successful applications in modern complicated industry processes, e.g., aircraft and space shuttle, nuclear power stations, high-performance automobiles, etc. However, the insertion of network makes the analysis and synthesis problems much more complex due to the randomly occurring incomplete information that is mainly caused by the limited bandwidth of the digital communication channel. The randomly occurring incomplete information under consideration mainly includes randomly missing measurements (RMMs), randomly occurring communication delays, sensor and actuator saturations (SASs), randomly occurring quantization and randomly varying nonlinearities (RVNs).

### **Missing Measurements**

In practical systems within a networked environment, measurement signals are usually subject to missing probabilistic information (data dropouts or packet losses). This may be caused for a variety of reasons, such as the high maneuverability of the tracked target, a fault in the measurement, intermittent sensor failures, network congestion, accidental loss of some collected data, or some of the data may be jammed or coming from a very noisy environment, and so on. Such a missing measurement phenomenon that typically occurs in NCSs has attracted considerable attention during the past few years; see Refs [1-24] and the references cited therein. Various approaches have been presented in the literature to model the packet dropout phenomenon. For example, the data packet dropout phenomenon has been described as a binary switching sequence that is specified by a conditional probability distribution taking on values of 0 or 1 [25, 26]. A discrete-time linear system with Markovian jumping parameters was employed by Shu et al. [27] and Xiong and Lam [28] to construct the random packet dropout model. A model that comprises former measurement information of the process output was introduced by Sahebsara et al. [29] to account for the successive packet dropout phenomenon. A model of multiple missing measurements was proposed by Wei et al. [18] using a diagonal matrix to describe the different missing probability for individual sensors.

### **Communication Delays**

Owing to the fact that time delays commonly reside in practical systems and constitute a main source for system performance degradation or even instability, the past decade has witnessed significant progress on analysis and synthesis for systems with various types of delays, and a large amount of literature has appeared on the general topic of time-delay systems. For example, the stability of NCSs under a network-induced delay was studied by Zhao *et al.* [30] using a hybrid system technique. The optimal stochastic control method was proposed by Nilsson [31] to control the communication delays in NCSs. A networked controller was designed in the frequency domain using robust control theory by Gokas [32] in which the network delays were considered as an uncertainty. However, most of the relevant literature mentioned above has focused on the *constant time-delays*. Delays resulting from network transmissions are inherently *random* and *time varying* [33–41]. This is particularly true when signals are transmitted over the internet and, therefore, existing control methods for constant time-delay

cannot be utilized directly [42]. Recently, some researchers have started to model the network-induced time delays in multi-form probabilistic ways and, accordingly, some initial results have been reported. For example, the random communication delays have been modeled as Markov chains and the resulting closed-loop systems have been represented as Markovian jump linear systems with two jumping parameters [43, 44]. Two kinds of random delays, which happen in the channels from the controller to the plant and from the sensor to the controller, were simultaneously considered by Yang *et al.* [45]. The random delays were modeled by Yang *et al.* [45] as a linear function of the stochastic variable satisfying a Bernoulli random binary distribution. Different from Yang *et al.* [45], the problem of stability analysis and stabilization control design was studied by Yue *et al.* [46] for Takagi–Sugeno (T–S) fuzzy systems with probabilistic interval delay, and the Bernoulli distributed sequence was utilized to describe the probability distribution of the time-varying delay taking values in an interval. It should be mentioned that, among others, the binary representation of the random delays has been fairly popular because of its practicality and simplicity in describing communication delays.

However, most research attention has been centered on the *single* random delay having a *fixed* value if it occurs. This would lead to conservative results or even degradation of the system performance since, at a certain time, the NCSs could give rise to multiple time-varying delays but with different occurrence probabilities. Therefore, a more advanced methodology is needed to handle time-varying network-induced time delays in a closed-loop control system.

### **Signal Quantization**

As is well known, quantization always exists in computer-based control systems employing finite-precision arithmetic. Moreover, the performance of NCSs will be inevitably subject to the effect of quantization error owing to the limited network bandwidth caused possibly by strong signal attenuation and perturbation in the operational environment. Hence, the quantization problem of NCSs has long been studied and many important results have been reported; see Refs [47-64] and references cited therein. For example, in Brockett and Liberzon [65], the time-varying quantization strategy was first proposed where the number of quantization levels is fixed and finite while at the same time the quantization resolution can be manipulated over time. The problem of input-to-state stable with respect to the quantization error for nonlinear continuous-time systems has been studied by Liberzon [66]. In this framework, the effect of quantization is treated as an additional disturbance whose effect is overcome by a Lyapunov redesign of the control law. A switching control strategy with dwell time was proposed by Ishii and Francis [67] to use as a quantizer for single-input systems. The quantizer employed in this framework is in fact an extension of the static logarithmic quantizer in [68] to the continuous case. So far, there have been mainly two different types of quantized communication models adopted in the literature: uniform quantization [62-64] and logarithmic quantization [56-59, 61]. It has been proved that, compared with a uniform quantizer, logarithmic quantization is more preferable since fewer bits need to be communicated. A sector bound scheme to handle the logarithmic quantization effects in feedback control systems was proposed by Fu and Xie [69], and such an elegant scheme was then extensively employed later on; for example, see Refs [58, 70, 71] and references cited therein. However, we note that the methods in most of the references cited above could not be directly applied to NCSs, because in NCSs the effects of network-included delay and packet dropout should also be considered.

### Sensor and Actuator Saturations

In practical control systems, sensors and actuators cannot provide unlimited amplitude signal due primarily to the physical, safety, or technological constraints. In fact, actuator/sensor saturation is probably the most common nonlinearity encountered in practical control systems, which can degrade the system performance or even cause instability if such a nonlinearity is ignored in the controller/filter design. Because of their theoretical significance and practical importance, considerable attention has been focused on the filtering and control problems for systems with actuator saturation [72-82]. As for sensor saturation, the associated results have been relatively few due probably to the technical difficulty [83–88]. Nevertheless, in the scattered literature regarding sensor saturation, it has been implicitly assumed that the occurrence of sensor saturations is deterministic; that is, the sensor always undergoes saturation. Such an assumption, however, does have its limitations, especially in a sensor network. The sensor saturations may occur in a probabilistic way and are randomly changeable in terms of their types and/or levels due to the random occurrence of networked-induced phenomena such as random sensor failures, sensor aging, or sudden environment changes. To reflect the reality in networked sensors, it would make practical sense to consider the randomly occurring sensor saturations (ROSSs) where the occurrence probability can be estimated via statistical tests. Also, it should be mentioned that very few results have dealt with the systems with simultaneous presence of actuator and sensor saturations [89], although such a presence is quite typical in engineering practice.

### **Randomly Varying Nonlinearities**

It is well known that nonlinearities exist universally in practice, and it is quite common to describe them as additive nonlinear disturbances that are caused by environmental circumstances. In a networked system such as the internet-based three-tank system for leakage fault diagnosis, such nonlinear disturbances may occur in a probabilistic way due to the random occurrence of a networked-induced phenomenon. For example, in a particular moment, the transmission channel for a large amount of packets may encounter severe network-induced congestions due to the bandwidth limitations, and the resulting phenomenon could be reflected by certain randomly occurring nonlinearities where the occurrence probability can be estimated via statistical tests. As discussed in Refs [90–93], in the NCSs that are prevalent nowadays, the nonlinear disturbances themselves may experience random abrupt changes due to random changes and failures arising from networked-induced phenomena, which give rise to the so-called RVNs. In other words, the type and intensity of the so-called RVNs could be changeable in a probabilistic way.

### 1.1.2 The Analysis and Synthesis of Nonlinear Stochastic Systems

For several decades now, stochastic systems have received considerable research attention in which stochastic differential equations are the most useful stochastic models with broad applications in aircraft, chemical, or process control systems and distributed networks. Generally speaking, stochastic systems can be categorized into two types, namely internal stochastic systems and external stochastic systems [94].