Laser Fundamentals

SECOND EDITION

William T. Silfvast

CAMBRIDGE

LASER FUNDAMENTALS

SECOND EDITION

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OPTICAL CONSTANTS

Complex index of refraction

$$\mathcal{N} = \eta + i\kappa \quad (2.94)$$

Absorption coefficient

$$\alpha = 2\alpha_E = \frac{2\omega\kappa}{c} \quad (2.98)$$

Frequency-dependent relationships for optical constants

$$\eta^2 - \kappa^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right) \quad (2.102)$$

$$2\eta\kappa = \frac{Ne^2}{m\varepsilon_0}\left(\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right) \quad (2.103)$$

Sellmeier's formula

$$\mathcal{N}^2 \cong \eta^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_j\left(\frac{f_j}{\omega_j^2 - \omega^2}\right) \quad (2.105)$$

COHERENCE

Temporal coherence (longitudinal coherence)

$$l_c = \lambda\left(\frac{\lambda}{\Delta\lambda}\right) = \frac{\lambda^2}{\Delta\lambda} \quad (2.116)$$

Spatial coherence (transverse coherence)

$$l_t = \frac{r\lambda}{s} = \frac{\lambda}{\theta_s} \quad (2.117)$$

RELATION BETWEEN OSCILLATOR STRENGTH AND TRANSITION PROBABILITY

$$A_{ul} = \frac{e^2\omega_{ul}^2}{2\pi\varepsilon_0 m_e c^3}\left(\frac{g_l}{g_u}\right)f_{lu} = \frac{2\pi e^2\nu_{ul}^2}{\varepsilon_0 m_e c^3}\left(\frac{g_l}{g_u}\right)f_{lu}$$
$$= \frac{2\pi e^2}{\varepsilon_0 m_e c\lambda_{ul}^2}\left(\frac{g_l}{g_u}\right)f_{lu} \quad (4.78)$$

Relation between absorption and emission oscillator strengths

$$f_{ul} = -\frac{g_l}{g_u}f_{lu} \quad (4.79)$$

Empirical expression for relationship between A_{ul} and f_{ul}

$$A_{ul} = \frac{10^{-4}(f_{lu})}{1.5(g_u/g_l)\lambda_{ul}^2}\ \mathrm{s}^{-1}\ [\lambda \text{ in m}] \quad (4.7)$$

HOMOGENEOUS BROADENING

Homogeneous linewidth

$$\Delta\nu_{ul}^H = \frac{\gamma_{ul}^T}{2\pi} = \frac{1}{2\pi}\left[\left(\sum_i A_{ui} + \sum_j A_{lj}\right) + \frac{1}{T_1^u} + \frac{1}{T_1^l} + \frac{2}{T_2}\right] \quad (4.44)$$

Homogeneous lineshape

$$I(\nu) = I_0\frac{\gamma_{ul}^T/4\pi^2}{(\nu - \nu_0)^2 + (\gamma_{ul}^T/4\pi)^2} \quad (4.37)$$

DOPPLER (INHOMOGENEOUS) BROADENING

Average velocity

$$\bar{v} = \sqrt{\frac{8kT}{M\pi}} \quad (4.49)$$

Doppler width

$$\Delta\nu_D = 2\nu_0\sqrt{\frac{2(\ln 2)kT}{Mc^2}}$$
$$= (7.16 \times 10^{-7})\nu_0\sqrt{\frac{T}{M_N}} \quad (4.59)$$

[T in K, M_N is mass number]

Doppler lineshape

$$I(\nu) = \frac{2(\ln 2)^{1/2}}{\pi^{1/2}\Delta\nu_D}I_0\exp\left\{-\left[\frac{4(\ln 2)(\nu - \nu_0)^2}{(\Delta\nu_D)^2}\right]\right\} \quad (4.60)$$

SELECTION RULES FOR ALLOWED ELECTRIC DIPOLE TRANSITIONS

For atoms

$\Delta l = \pm 1$ for the changing electron (change in parity)

$\Delta S = 0, \quad \Delta L = 0, \pm 1$ (4.104)

$\Delta J = 0, \pm 1$ but $J = 0 \nrightarrow J = 0$

$\Delta M_J = 0, \pm 1$ but $M_J = 0 \nrightarrow M_J = 0$ if $\Delta J = 0$

Also parity must change

For molecules

Rotational transitions

$\Delta J = 0, \pm 1, \quad \Delta K = \pm 1$ (5.15)

Rotational–vibrational transitions

$\Delta v = \pm 1$ (5.17)

$\Delta J = 0, \pm 1, \quad \Delta J = J_u - J_l$ (5.18)

Branch definitions

$\Delta J = -1$ P branch

$\Delta J = 0$ Q branch (5.19)

$\Delta J = +1$ R branch

Electronic transitions

$\Delta\Lambda = 0, \pm 1$ (5.21)

$\Delta S = 0$ (5.22)

BLACKBODY RADIATION (intensity per unit λ)

$$I_{\rm BB}(\nu) = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \quad (6.39)$$

$$I_{\rm BB}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5(e^{ch/\lambda kT}-1)} \quad (6.42)$$

$$I_{\rm BB}(\lambda, T) = \frac{3.75\times 10^{-22}}{\lambda^5(e^{0.0144/\lambda T}-1)}\ \mathrm{W/m^2\text{-}\mu m} \quad [\lambda \text{ in m}, T \text{ in K}] \quad (6.44)$$

$$I_{\rm BB}(\lambda, T) = \frac{3.75\times 10^{-25}}{\lambda^5(e^{0.0144/\lambda T}-1)}\ \mathrm{W/m^2\text{-}nm} \quad [\lambda \text{ in m}, T \text{ in K}] \quad (6.45)$$

EINSTEIN *A* AND *B* COEFFICIENTS

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1 \quad \text{or} \quad g_l B_{lu} = g_u B_{ul} \quad (6.50)$$

$$B_{ul} = \frac{c^3}{8\pi h\eta^3\nu^3}A_{ul} \quad (6.52)$$

Ratio between stimulated emission rate and spontaneous emission rate

$$\frac{B_{ul}u(\nu)}{A_{ul}} = \frac{1}{e^{h\nu_{ul}/kT}-1} \quad (6.57)$$

GAIN COEFFICIENTS AND STIMULATED EMISSION CROSS SECTION

Homogeneous broadening

$$g^H(\nu) = \left[N_u - \frac{g_u}{g_l}N_l\right]\frac{c^2}{8\pi\eta^2\nu^2} \times \left[\frac{\gamma_{ul}^T/4\pi^2}{(\nu-\nu_0)^2+(\gamma_{ul}^T/4\pi)^2}\right]A_{ul} \quad (7.11)$$

$$\Delta N_{ul} = \left[N_u - \frac{g_u}{g_l}N_l\right] \quad (7.12)$$

$$\sigma_{ul}^H = \frac{c^2}{8\pi\eta^2\nu^2}A_{ul}(\nu) = \frac{c^2}{8\pi\nu^2}\left[\frac{\gamma_{ul}^T/4\pi^2}{(\nu-\nu_0)^2+(\gamma_{ul}^T/4\pi)^2}\right]A_{ul} \quad (7.13)$$

$$g^H(\nu=\nu_0) \equiv g^H(\nu_0) \equiv g_0^H = \frac{c^2}{2\pi\eta^2\nu_0^2\gamma_{ul}}A_{ul}\left[N_u - \frac{g_u}{g_l}N_l\right] \quad (7.15)$$

$$\sigma_{ul}^H(\nu_0) = \frac{c^2 A_{ul}}{2\pi\eta^2\nu_0^2\gamma_{ul}} = \frac{\lambda_{ul}^2 A_{ul}}{4\pi^2\Delta\nu_{ul}^H} \quad (7.16)$$

Exponential growth

$$I = I_0 e^{g^H(\nu)z} = I_0 e^{\sigma_{ul}^H(\nu)[N_u-(g_u/g_l)N_l]z} = I_0 e^{\sigma_{ul}^H(\nu)\Delta N_{ul}z} \quad (7.18)$$

Doppler broadening

$$g^D(\nu) = \sqrt{\frac{\ln 2}{16\pi^3}}\frac{c^2 A_{ul}}{\eta^2\nu_0^2\Delta\nu_D}\left[N_u - \frac{g_u}{g_l}N_l\right] \times \exp\left\{-\left[\frac{4\ln 2(\nu-\nu_0)^2}{\Delta\nu_D^2}\right]\right\} \quad (7.25)$$

$$g^D(\nu=\nu_0) \equiv g^D(\nu_0) \equiv g_0^D = \sqrt{\frac{\ln 2}{16\pi^3}}\frac{\lambda_{ul}^2 A_{ul}}{\Delta\nu_D}\left[N_u - \frac{g_u}{g_l}N_l\right] \quad (7.26)$$

$$g^D(\nu) = \sigma_{ul}^D(\nu)\left[N_u - \frac{g_u}{g_l}N_l\right] = \sigma_{ul}^D(\nu)\Delta N_{ul} \quad (7.27)$$

$$\sigma_{ul}^D(\nu_0) = \sqrt{\frac{\ln 2}{16\pi^3}}\frac{\lambda_{ul}^2 A_{ul}}{\Delta\nu_D} \quad (7.28)$$

$$\sigma_{ul}^D(\nu_0) = (1.74\times 10^{-4})\lambda_{ul}^3 A_{ul}\sqrt{M_N/T} \quad (7.29)$$

[λ in m, A_{ul} in $\mathrm{s^{-1}}$, T in K, M_N is mass number]

Exponential growth

$$I = I_0 e^{g^D(\nu)z} = I_0 e^{\sigma_{ul}^D(\nu)[N_u-(g_u/g_l)N_l]z} = I_0 e^{\sigma_{ul}^D(\nu)\Delta N_{ul}z} \quad (7.30)$$

SATURATION INTENSITY

$$I_{\rm sat} = \frac{h\nu_{ul}}{\sigma_{ul}^H(\nu)\tau_u} \quad (7.42)$$

$$F_{\rm sat} = \frac{h\nu_{ul}}{\sigma_{ul}} \quad (8.92b)$$

GAIN SATURATION

$$g = \frac{g_0}{1+I/I_{\rm sat}} = \frac{\sigma_{ul}\Delta N_{ul}^0}{1+I/I_{\rm sat}} \quad (8.8)$$

THRESHOLD CONDITIONS FOR LASERS

No mirrors

$$\sigma_{ul}\Delta N_{ul}L_{\rm sat} \cong 12 \pm 5 \quad (7.55)$$

One mirror

$$\sigma_{ul}\Delta N_{ul}(2L) \cong 12 \pm 5 \quad [L_{\rm sat} = 2L] \quad (7.56)$$

Two mirrors

$$g_{\rm th} = \frac{1}{2L}\ln\frac{1}{R^2} \quad (7.58)$$

$$g_{\rm th} = \frac{1}{2L}\ln\left[\frac{1}{R_1R_2(1-a_1)(1-a_2)}\right] + \alpha \quad (7.60)$$

$$t_s = m[\eta_C(d-L)+\eta_L L]/c \quad (7.64)$$

LASER FUNDAMENTALS

SECOND EDITION

Laser Fundamentals provides a clear and comprehensive introduction to the physical and engineering principles of laser operation and design. Simple explanations, based throughout on key underlying concepts, lead the reader logically from the basics of laser action to advanced topics in laser physics and engineering.

In addition to many improvements to the text and figures of the first edition, much new material has been added to this second edition – especially in the areas of solid-state lasers, semiconductor lasers, and laser cavities. This edition contains a new chapter on laser operation above threshold, including extensive discussion of laser amplifiers. It also provides details on new types of lasers (e.g., Nd:YLF, Nd:YVO_4, and Yb:YAG) plus a new section on diode pumping of solid-state lasers. The coverage extends to the four basic categories (GaAs, InP, ZnSe, and GaN) of semiconductor lasers. The analysis is applied to electron and hole concentrations for both heterojunction semiconductor lasers and quantum-well lasers, and a thorough discussion of laser cavities features *ABCD* matrix analysis of two-, three-, and four-mirror cavities.

The book first develops the fundamental wave and quantum properties of light, such as coherence, energy levels, emission linewidth, and stimulated emission. It then uses those properties to develop the concepts of population inversion, gain, saturation intensity, laser operation above threshold, excitation or pumping, and cavity properties, which include longitudinal and transverse modes, Gaussian beams, unstable resonators, Q-switching, and mode-locking. The book addresses aspects that are common to all laser amplification. It examines the development of population inversions in such low-density materials as gases and plasmas as well as in the usual three- or four-level systems of such high-density materials as liquids and solids. Included are extensive accounts of both solid-state and semiconductor lasers, and detailed descriptions and data tables of the most common lasers are provided. The book concludes with a chapter on nonlinear frequency conversion as it relates to lasers.

The clear explanations, worked examples, and many homework problems make this book eminently suitable for undergraduate and first-year graduate students in science and engineering who are taking courses on lasers. The summaries of key types of lasers, the use of many unique theoretical descriptions, and the chapter-by-chapter bibliography make this an invaluable reference work for researchers as well.

William Silfvast received a B.S. degree in both physics and mathematics (1961) and a Ph.D. in physics (1965) from the University of Utah. From 1967 to 1989 he worked at AT&T Bell Laboratories, becoming a Distinguished Member of the Technical Staff in 1983. In 1990 he joined the faculty of the University of Central Florida in Orlando, where he was a Professor of Physics and Electrical Engineering as well as a member of the Center for Research and Education in Optics and Lasers (CREOL). In 1999 he also became a Professor of Optics at the School of Optics, and he is presently Emeritus Professor of Optics. He was a NATO Postdoctoral Fellow at Oxford University in 1966–67 and a Guggenheim Fellow at Stanford University in 1982–83. He was Chair of the Department of Physics at the University of Central Florida from 1994 to 1997. Professor Silfvast is a Fellow of the American Physical Society, the Optical Society of America, and the IEEE. He has carried out pioneering work in the fields of metal vapor lasers, recombination lasers, photoionization-pumped lasers, laser plasmas, and EUV lithography. He has authored more than 100 technical papers and holds more than 30 patents.

To my wife, Susan, and my three children, Scott, Robert and Stacey, all of whom are such an important part of my life.

Preface to the Second Edition

I am very pleased to have completed this Second Edition of *Laser Fundamentals.* The encouragement I have received over the past few years from readers as well as from my editors was sufficient to provide me with the enthusiasm to take on this new task. Writing the first edition was essentially a ten-year endeavor from first thoughts to the completed book. I thought I had a better way to explain to senior-level and first-year graduate students how lasers work. Apparently there were others who agreed with me, judging from comments I have received. Writing the second edition was an attempt to fill in some of the gaps, so to speak; not surprisingly, it took much more time than I had anticipated. Some of the areas of the First Edition were not as complete as I would have liked. There were also errors that had to be corrected. In addition, there have been advances – primarily in the areas of solid-state and semiconductor lasers – that needed to be included. I think the new edition addresses those issues pretty well. I suppose it's up to the readers to make that judgment.

Naturally one can't take on a task like this without gleaning information from experts in the various fields of lasers. I offer special thanks to my colleagues at the School of Optics/CREOL at the University of Central Florida: Michael Bass, Glenn Boreman, Peter Delfyett, Dave Hagan, Hans Jenssen, Patrick Li Kam Wa, Alexandra Rapaport, Kathleen Richardson, Martin Richardson, Craig Siders, Eric Van Stryland, Nikolai Vorobiev, and Boris Zeldovich. Others who were very helpful include Norm Hodgson, Jason Eichenholz, Jack Jewell, Shuji Nakamura, Jorge Rocca, Rita Petersen, and Colin Webb (and I'm sure I've inadvertently left out a few).

I am grateful to have Simon Capelin as my editor at Cambridge University Press. He has been most encouraging without pressing me with a specific deadline. It is also a pleasure to work again with Matt Darnell as my production editor. Most importantly, I thank my wonderful wife, Susan, who was always very supportive while putting up with the many long hours that I spent in completing this Second Edition.

Preface to the First Edition

I wrote *Laser Fundamentals* with the idea of simplifying the explanation of how lasers operate. It is designed to be used as a senior-level or first-year graduate student textbook and/or as a reference book. The first draft was written the first time I taught the course "Laser Principles" at the University of Central Florida. Before that, I authored several general laser articles and taught short courses on the subject, giving careful consideration to the sequence in which various topics should be presented. During that period I adjusted the sequence, and I am now convinced that it is the optimal one.

Understanding lasers involves concepts associated with light, viewed either as waves or as photons, and its interaction with matter. I have used the first part of the book to introduce these concepts. Chapters 2 through 6 include fundamental wave properties, such as the solution of the wave equation, polarization, and the interaction of light with dielectric materials, as well as the fundamental quantum properties, including discrete energy levels, emission of radiation, emission broadening (in gases, liquids, and solids), and stimulated emission. The concept of amplification is introduced in Chapter 7, and further properties of laser amplifiers dealing with inversions and pumping are covered in Chapters 8 and 9 [Chapters 8–10 in the Second Edition – Ed.]. Chapter 10 [11] discusses cavity properties associated with both longitudinal and transverse modes, and Chapters 11 and 12 [12 and 13] follow up with Gaussian beams and special laser cavities. Chapters 13 and 14 [14 and 15] provide descriptions of the most common lasers. The book concludes in Chapter 15 [16] with a brief overview of some of the nonlinear optical techniques for laser frequency conversion.

Some of the unique aspects of the book are the treatment of emission linewidth and broadening in Chapter 4, the development of a simple model of a laser amplifier in Chapter 7, the discussion of special laser cavities in Chapter 12 [13], and the laser summaries in Chapters 13 and 14 [14 and 15]. Throughout the book, whenever a particular concept is introduced, I have tried to relate that concept to all the various types of laser amplifiers including gas lasers, liquid (dye) lasers, and solid-state lasers. My intention is to give the reader a good understanding, not just of one specific type of laser but rather of all types of lasers, as each concept is introduced.

The book can be used in either a one- or two-semester course. In one semester the topics of Chapters 2 through 12 [13] would be emphasized. In two semesters, extended coverage of the specific lasers of Chapters 13 and 14 [14 and 15], as well as the frequency multiplication in Chapter 15 [16], could be included. In a one-semester course I have been able to cover a portion of the material in Chapters 13 and 14 [14 and 15] by having each student write a report about one specific laser and then give a ten- or fifteen-minute classroom presentation about that laser. The simple quantum mechanical descriptions in Chapters 3 and 4 were introduced to describe how radiative transitions occur in matter. If the instructor chooses to avoid quantum mechanics in the course, it would be sufficient to stress the important results that are highlighted at the ends of each of those sections.

Writing this book has been a rewarding experience for me. I have been associated with lasers since shortly after their discovery in 1960 when, as an undergraduate student at the University of Utah, I helped build a ruby laser for a research project under Professor Frank Harris. He was the first person to instill in me an enthusiasm for optics and light. I was then very fortunate to be able to do my thesis work with Professor Grant Fowles, who encouraged me to reduce ideas to simple concepts. We discovered many new metal vapor lasers during that period. I also thank Dr. John Sanders for giving me the opportunity to do postdoctoral work at the Clarendon Laboratory at Oxford University in England, and Dr. Kumar Patel for bringing me to Bell Laboratories in Holmdel, New Jersey. Being a part of a stimulating group of researchers at Bell Laboratories during the growth of the field of lasers was an unparalleled opportunity. During that period I was also able to spend an extremely rewarding sabbatical year at Stanford University with Professor Steve Harris. Finally, to round out my career I put on my academic hat at the University of Central Florida as a member of the Center for Research and Education in Optics and Lasers (CREOL) and the Department of Physics and of Electrical and Computer Engineering. Working in the field of lasers at several different institutions has provided me with a broad perspective that I hope has successfully contributed to the manner in which many of the concepts are presented in this book.

Acknowledgments

I first acknowledge the support of my wife, Susan. Without her encouragement and patience, I would never have completed this book.

Second, I am deeply indebted to Mike Langlais, an undergraduate student at the University of Central Florida and a former graphics illustrator, who did most of the figures for the book. I provided Mike with rough sketches, and a few days later he appeared with professional quality figures. These figures add immensely to the completeness of the book.

Colleagues who have helped me resolve particular issues associated with this book include Michael Bass, Peter Delfyett, Luis Elias, David Hagan, James Harvey, Martin Richardson, and Eric Van Stryland of CREOL; Tao Chang, Larry Coldren, Dick Fork, Eric Ippen, Jack Jewell, Wayne Knox, Herwig Kogelnik, Tingye Li, David Miller, Peter Smith, Ben Tell, and Obert Wood of Bell Laboratories; Bob Byer, Steve Harris, and Tony Siegman of Stanford University; Boris Stoicheff of the University of Toronto; Gary Eden of the University of Illinois; Ron Waynant of the FDA; Arto Nurmiko of Brown University; Dennis Matthews of Lawrence Livermore National Laboratories; Syzmon Suckewer of Princeton University; Colin Webb of Oxford University; John Macklin of Stanford University and Bell Labs; Jorgé Rocca of Colorado State University; Frank Tittle of Rice University; Frank Duarte of Kodak; Alan Petersen of Spectra Physics; Norman Goldblatt of Coherent, Inc.; and my editor friend, Irwin Cohen. I also thank the many laser companies who contributed figures, primarily in Chapters 13 and 14 [14 and 15 in 2e]. I'm sure that I have left a few people out; for that, I apologize to them. In spite of all the assistance, I accept full responsibility for the final text.

I thank my editor, Philip Meyler, at Cambridge University Press for convincing me that CUP was the best publishing company and for assisting me in determining the general layout of my book. I also thank editor Matt Darnell for doing such a skillful job in taking my manuscript and making it into a "real" book.

I am indebted to several graduate students at CREOL. Howard Bender, Jason Eichenholz, and Art Hanzo helped with several of the figures. In addition, Jason Eichenholz assisted me in taking the cover photo, Howard Bender and Art Hanzo helped with the laser photo on the back cover, and Marc Klosner did a careful proofreading of one of the later versions of the text. I am also indebted to Al Ducharme for suggesting the title for the book.

Finally, I thank the students who took the "Laser Principles" course the first year I taught it (Fall 1991). At that point I was writing and passing out drafts of my chapters to the students at a frantic pace. Because those students had to suffer through that first draft, I promised all of them a free copy of the book. I stand by that promise and hope those students will get in touch with me to collect.

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