Kazimierz Kuratowski

Introduction to set theory and topology

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INTRODUCTION TO SET THEORY AND TOPOLOGY

COMPLETELY REVISED SECOND ENGLISH EDITION

INTRODUCTION TO SET THEORY AND TOPOLOGY

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Member of the Polish Academy of Sciences

Containing a Supplement
on
ELEMENTS OF ALGEBRAIC TOPOLOGY
by
Professor RYSZARD ENGELKING

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Elements of Algebraic Topology
by R. Engelking

FOREWORD TO THE FIRST ENGLISH EDITION

The ideas and methods of set theory and topology penetrate modern mathematics. It is no wonder then that the elements of these two mathematical disciplines are now an indispensable part of basic mathematical training. Concepts such as the union and intersection of sets, countability, closed set, metric space, and homeomorphic mapping are now classical notions in the whole framework of mathematics.

The purpose of the present volume is to give an accessible presentation of the fundamental concepts of set theory and topology; special emphasis being placed on presenting the material from the viewpoint of its applicability to analysis, geometry, and other branches of mathematics such as probability theory and algebra. Consequently, results important for set theory and topology but not having close connections with other branches of mathematics, are given a minor role or are omitted entirely. Such topics are, for instance, investigations on foundations, the theory of alephs, and the theory of curves.

The main body of the book is an introduction to set theory and topology, intended for the beginner. Sections marked with an asterisk cover either more complicated topics or points which are frequently omitted in a first course; this holds also for some exercises which allow the reader to get acquainted with many applications and some important results which could not be included in the text without unduly expanding it. Many new exercises not contained in the Polish edition have been included here.

I take great pleasure in thanking Professor J. Jaworowski and Dr. A. Granas for their cooperation in preparing the Polish edition and to thank also Professors A. Mostowski and R. Sikorski, Dr. S. Mrówka, Mr. R. Engelking and Dr. A. Schinzel for numerous comments which helped me to improve the original manu-

script. Also, my thanks go to Mr. Leo F. Boroń and to Mr. A.H. Robinson for preparing the present text for English-speaking students of mathematics.

KAZIMIERZ KURATOWSKI

Warsaw
September 1960

FOREWORD TO THE SECOND ENGLISH EDITION

Since the first English edition appeared, set theory and point-set topology have developed to such an extent that the author found it necessary to modify in many points the previous edition. This was done partially in the Polish edition (1965) and in the French edition (1966).

The most essential changes concern the second part of this book (devoted to topology). However, there are also changes worth noticing in the first part (on set theory).

The concepts of inverse limit, of lattice, of ideal, of filter, of a commutative diagram, and of a cartesian product of an arbitrary number of factors are considered. A slightly deeper insight into the axioms of set theory was needed; in particular, the notion of a class (in the sense of Bernays) is mentioned (and later applied, mainly in connection with the concept of category used in the Supplement).

In the theory of ordering relations, more emphasis was put on what was previously called partial ordering. This is now called, more concisely, ordering, and this change of terminology seems to be more appropriate to common use.

For the same reason, some notations have been changed. In particular, the Lebesgue notation $E_x\varphi(x)$ has been replaced by $\{x\colon \varphi(x)\}$; the union of members of a family A of sets is denoted by $\bigcup A$, and the intersection by $\bigcap A$.

The changes in the second part of the book are more essential. In the first edition, this part of the book was chiefly devoted to the study of metric spaces. In this second edition, the general topological spaces form its main subject. Consequently, more than a half of the second part had to be written anew. It contains new topics which were not considered in the first edition, such as cartesian products of topological spaces, the Čech–Stone compactification, quotient-spaces, completely regular spaces, quasi-components, and a large number of exercises have been added.

In Chapter XX, on simplexes, more material will be found on simplicial mappings, on the nerve of a cover and related problems.

Finally, the rather short Chapter XXI, on complexes, chains and homologies, has been replaced by a much more extensive Supplement on the elements of algebraic topology. This Supplement, written by Professor Engelking, will certainly be a very valuable complement to my book.

I have received considerable help from the persons mentioned in the Foreword to the first edition and also from the young ladies Dr. Karłowicz and Dr. Vuilleumier. To all of them go my heartiest thanks.

KAZIMIERZ KURATOWSKI

Warsaw
October 1968

Part I

SET THEORY