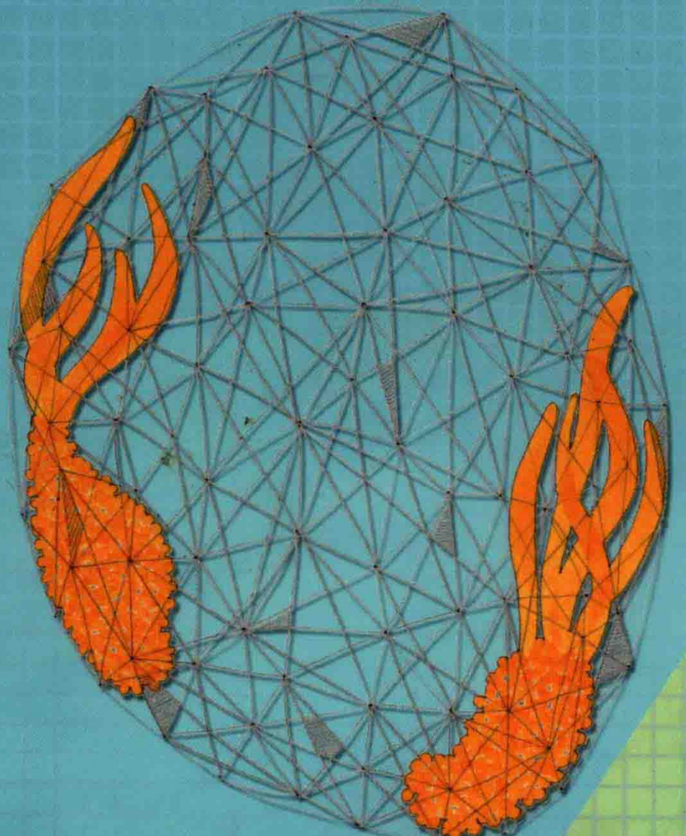
ALGEBRAIC AND GEOMETRIC IDEAS IN THE THEORY OF DISCRETE OPTIMIZATION



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MOS-SIAM Series on Optimization

Algebraic and Geometric Ideas in the Theory of Discrete Optimization

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Algebraic and Geometric Ideas in the Theory of Discrete Optimization

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Preface

It is undeniable that geometric ideas have been very important to the foundations of modern discrete optimization. The influence that geometric algorithms have in optimization was elegantly demonstrated in the, now classic, book *Geometric Algorithms and Combinatorial Optimization* [145] written more than 25 years ago by M. Grötschel, L. Lovász, and A. Schrijver. There, in a masterful way, we were introduced to the power that the geometry of ellipsoids, hyperplanes, convex bodies, and lattices can wield in optimization. After many years, students of integer programming today are exposed to notions such as the equivalence of separation and optimization, convex hulls, and membership, and to the many examples of successful application of these ideas such as efficient algorithms for matchings on graphs and other problems with good polyhedral characterizations [298], [299], [300] and the solution of large-scale traveling salesman problems [14]. These results were a landmark success in the theory of integer optimization.

But in just the past 15 years, there have been new developments in the understanding of the structure of polyhedra, convex sets, and their lattice points that have produced new algorithmic ideas for solving integer programs. These techniques add a new set of powerful tools for discrete optimizers and have already proved very suitable for the solution of a number of hard problems, including attempts to deal with nonlinear objective functions and constraints in discrete optimization. Unfortunately, many of these powerful tools are not yet widely known or applied. Perhaps this is because many of the developments have roots in areas of mathematics that are not normally part of the standard curriculum of students in optimization and have a much more algebraic flavor. Examples of these new tools include algebraic geometry, commutative algebra, representation theory, and number theory.

We feel that the unfamiliar technical nature of these new ideas and the lack of expository accounts have unnecessarily delayed the popularity of these techniques among those working in optimization. We decided to write a text that would not demand any background beyond what we already assume from people in mathematical programming courses. This monograph is then intended as a short, self-contained, introductory course in these new ideas and algorithms with the hope of popularizing them and inviting new applications. We were deeply inspired by the influential book [145] and we humbly try to follow in its footsteps in the hope that future generations continue to see the interdependence between beautiful mathematics and the creation of efficient optimization algorithms.

This book is meant to be used in a quick, intense course, no longer than 15 weeks. This is not a complete treatise on the subject, but rather an invitation to a set of new ideas and tools. Our aim is to popularize these new ideas among workers in optimization.

- We want to make it possible to read this book even if you are a novice of integer and linear programming (and we have taught courses with some students in that category). For this reason, we open in Part I with some of the now well-established techniques that originated before the beginning of the 1990s, a time when linear and convex programming and integer programming underwent major changes thanks to the ellipsoid method, semidefinite programming, lattice basis reduction, etc. Most of what is contained in Part I is a short summary of tools that students in optimization normally encounter in a course based on the excellent books [50, 145, 206, 259, 296] and probably should be skipped by such readers. They should go directly to the new exciting techniques in Parts II, III, IV, and V. Readers that start with Part I will add an extra three or four weeks to the course.

- Parts II, III, IV, and V form the core of this book. Roughly speaking, when the reader works in any of these sections, nonlinear, nonconvex conditions are central, making the tools of algebra necessary. When studying only these parts, the course is planned to take about 12 weeks. In fact, all the parts are quite independent from each other and each can be the focus of independent student seminars.
  - We begin in Part II with the idea of test sets and Graver bases. We show how they can be used to prove results about integer programs with linear constraints and convex objective functions.
  - Part III discusses the use of generating functions to deal with integer programs with linear constraints but with nonlinear polynomial objectives and/or with multiobjectives.
  - Part IV discusses the notion of Gröbner bases and their connection with integer programming.
  - Part V discusses the solution of global optimization problems with polynomial constraints via a sequence of linear algebra or semidefinite programming systems. These are generated based on Hilbert's Nullstellensatz and its variations.

The book contains several exercises to help students learn the material. A course based on these lectures should be suitable for advanced undergraduates with a solid mathematical background (i.e., very comfortable with proofs in linear algebra and real analysis) or for graduate students who have already taken an introductory linear programming class.

Acknowledgments. We are truly grateful to many people who helped us both on producing the research presented here and later on presenting it to a larger audience of students and colleagues.

First and foremost our collaborators and coauthors in many parts of this book were fundamental for arriving at this point; their energy and ideas show on every page. In fact several portions of the book are taken partially from our joint work. So many, many thanks for everything to Robert Hildebrand, Chris Hillar, Jon Lee, Peter N. Malkin, Susan Margulies, Mohamed Omar, Shmuel Onn, Pablo Parrilo, Uriel Rothblum, Maurice Queyranne, Christopher T. Ryan, Rüdiger Schultz, Sven Verdoolaege, Robert Weismantel, and Kevin Woods. Thanks!

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Finally, our families are very special in our lives and this project is partly theirs too, built with their love and patience in our long crazy hours and very distracted minds.

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Jesús A. De Loera, Raymond Hemmecke, Matthias Köppe

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