

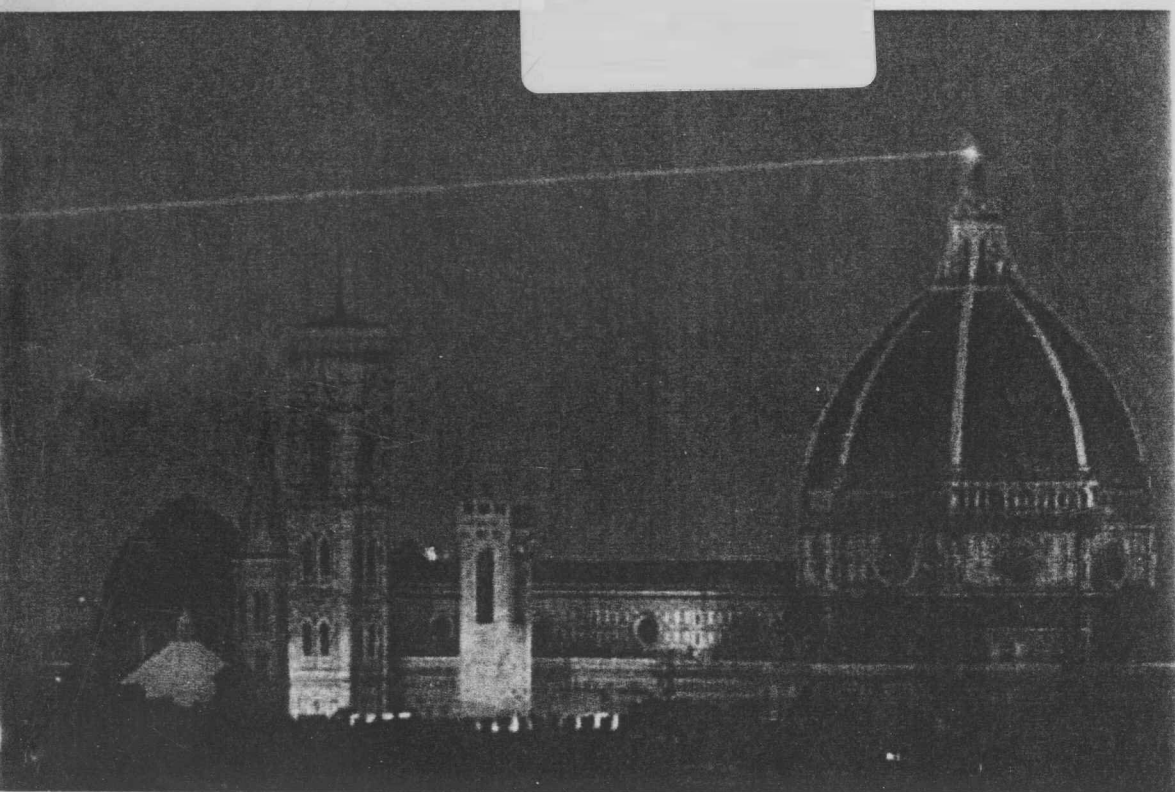
PROBLEMS IN LASER PHYSICS

**G. Cerullo, S. Longhi, M. Nisoli
S. Stagira, and O. Svelto**

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Preface

There is hardly any book that aims at solving problems typically encountered in the laser field, and this book intends to fill the void. Following some initial exercises related to general aspects in laser physics (Chapt. 1), the subsequent problems are organized along the following topics: (i) Interaction of radiation with matter either made of atoms or ions, weakly interacting with surrounding species, or made of more complicated elements such as molecules or semiconductors (Chapters 2 and 3). (ii) Wave propagation in optical media and optical resonators (Chapters 4 and 5). (iii) Optical and electrical pumping processes and systems (Chapter 6). (iv) Continuous wave and transient laser behaviors (Chapters 7 and 8). (v) Solid-state, dye, semiconductor, gas and X-ray lasers (Chapters 9 and 10). (vi) Properties of the output beam and beam transformation by amplification, frequency conversion and pulse compression or expansion (Chapters 11 and 12).

Problems are proposed here and solved following the contents of Orazio Svelto's *Principles of Lasers* (fourth edition; Plenum Press, New York, 1998). Whenever needed, equations and figures of the book mentioned above are currently used with an appropriate reference [e.g., Eq. (1.1.1) of the book is referred to as Eq. (1.1.1) of PL]. One can observe, however, that the types of problems proposed and discussed are of general validity and many of these problems have actually been suggested by our own long-time experience in performing theoretical and experimental researches in the field. Some of these problems are also directly related to real-world lasers (i.e., lasers, laser components and laser systems commonly found in research laboratories or commercially available). Therefore, the reader should be able to solve most of these problems even if his knowledge in laser physics has been acquired through studying other textbooks.

In each chapter, problems are first proposed all together and then solved at the end. This should encourage the reader to solve the problem by himself without immediately looking at the solution. Three kinds of problems are considered with attention being paid to a good balance between them:

1. Problems where one just needs to insert appropriate numbers into some important equation already provided in the previously mentioned book (*applicative problems*): they should help students to become more acquainted with important equations in laser physics and with the typical values of the corresponding parameters that are involved.
2. Problems where students are asked to prove some relevant equation left unproven

in the textbook (*demonstrative problems*): their purpose is to test the maturity acquired by demonstrating some, generally simple, passages.

3. Problems where students are asked to develop topics which go beyond those covered in the above book as well as in many other textbooks in the field (*evolutional problems*): their purpose is to increase the depth of knowledge in the laser field. Whenever appropriate, some hints for the solutions are also added, particularly for some more advanced demonstrative or evolutional problems. However, when the level of difficulty is deemed to be particularly high, a warning in the form "level of difficulty higher than average" is added at the end of the corresponding problem. This should help the reader, on the one hand, know when to apply himself harder and, on the other hand, not to get discouraged at a possible failure. Reading the solution should allow students to considerably enrich their basic knowledge in the field.

Lastly, care has been taken not to have the solution of one problem be dependent on the solution of a preceding problem in the chapter. This should allow more freedom for tackling problems not necessarily in sequential order.

Given the number of problems proposed and their wide variety, it is believed that a proficient student, upon solving these problems, should become more than well prepared to begin a research activity in laser physics and engineering as well as in the general field of photonics.

Giulio Cerullo
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Mauro Nisoli
Salvatore Stagira
Orazio Svelto

Milan, February 2001

PROBLEMS IN LASER PHYSICS

PROBLEMS

1.1P Spectrum of laser emission.

The part of the em spectrum of interest in the laser field starts from the millimeter wave region and decreases in wavelength to the x-ray region. This covers the following regions in succession: far infrared, near infrared, visible, UV, vacuum ultraviolet (VUV), soft X-ray, X-ray. From standard textbooks find the wavelength intervals of these regions.

1.2P Spectrum of visible light.

From standard textbooks find the wavelength intervals corresponding to the different colors of the visible spectrum, and calculate the corresponding frequency intervals.

1.3P Energy of a photon.

Calculate the frequency in hertz and wavenumbers (cm^{-1}) and the energy in electronvolts of a photon of wavelength $4.71 \mu\text{m}$ in vacuum.

1.4P Thermal energy.

Calculate the wavenumbers corresponding to an energy spacing of kT , where k is the Boltzmann constant and T is the absolute temperature. Assume $T = 300 \text{ K}$.

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CHAPTER 1

Introductory Concepts

PROBLEMS

1.1P Spectrum of laser emission.

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