C. Thornton, editor

powders & grains

93

PROCEEDINGS OF THE SECOND INTERNATIONAL CONFERENCE ON MICROMECHANICS OF GRANULAR MEDIA / BIRMINGHAM / UK / 12-16 JULY 1993

Powders & Grains 93

Edited by
C.THORNTON
Aston University, United Kingdom

A.A.BALKEMA / ROTTERDAM / BROOKFIELD / 1993

The texts of the various papers in this volume were set individually by, or under the supervision of, each of the authors concerned.

Published by
A.A. Balkema, P.O. Box 1675, 3000 BR Rotterdam, Netherlands
A.A. Balkema Publishers, Old Post Road, Brookfield, VT 05036, USA

ISBN 90 5410 323 X

Printed in the Netherlands

POWDERS & GRAINS 93

Association pour l'Etude de la Micromécanique des Milieux Granulaires

Association for the Study of Micromechanics of Granular Media

Powders & Grains 93, Thornton (ed.) © 1993 Balkema, Rotterdam, ISBN 90 5410 323 X

Preface

Following the success of the first conference held at Clermont-Ferrand in 1989, the aim of this second conference was to introduce new developments and establish the state of knowledge in research on the micromechanical behaviour of granular media.

An improved understanding of the micromechanics of granular media requires an interdisciplinary approach involving statistical physicists, applied mathematicians and chemical/civil/mechanical/agricultural engineers and is relevant to contemporary technological problems in the process/manufacturing/pharmaceutical/geotechnical/food and agricultural industries. The papers presented in this volume reflect the multidisciplinary activity in a research area which has grown significantly since the conference in Clermont-Ferrand. Contemporary international research on micromechanics of granular media is well represented by the many contributions from the USA, UK, France and Japan.

The focus of this proceedings is the scientific understanding of the nature and description of particle interactions and assembly geometry and how such information obtained at the grain scale may be used to provide prescriptions of the macroscopic behaviour of granular media that will lead to improved end-product/process performance. Many papers are concerned with computer simulation which has now established itself as an essential tool for the investigation of the micromechanics of granular media. However, theoretical and experimental approaches are also well represented.

The work is subdivided into nine topics: packing geometry, particle interactions, quasi-static deformation, powder compacts, vibrating beds, rapid granular flow, avalanches, hopper flow and general flow problems. The reader may have been attracted to a specific paper or a particular area of personal interest. However, we hope (and we believe) that the reader will benefit from browsing through the volume and, as a consequence, gain a wider appreciation of the micromechanics of granular media in its different states. If this increases the reader's appreciation of the complex behaviour of granular media or suggests new avenues of approach to a particular problem then the efforts of the many contributors to this volume will have been worthwhile.

The meeting was organised under the auspices of AEMMG (Association pour l'Etude de la Micromécanique des Milieux Granulaires). I would like to thank the members of the executive committee who persevered in persuading me to organise this conference. It has been a very rewarding personal experience.

Colin Thornton

Powders & Grains 93, Thornton (ed.) © 1993 Balkema, Rotterdam, ISBN 90 5410 323 X

Opening address: The flow of powders

Professor Sir Sam Edwards FRS
Department of Physics, Cavendish Laboratory, University of Cambridge, UK

ABSTRACT: The flow of powders resembles that of liquids but has a very different constitutive structure, i.e. a very different relationship between stress and strain. In spite of this the general structure of equations of flow must be very similar. For liquids and gases the transport equations rely on the conservation laws and are: the equation of continuity governing the density, the equation for velocity which deals with the conservation of momentum and the equation which deals with the conservation of energy which is expressed as an equation for the entropy, or more conveniently, the temperature.

For powders one must have an equation of continuity, and an equation for the velocity of the powder which will contain the gradient of the stress as driving force which is related back to the strain rate via the constitutive equation. There has to be a third equation for the entropy, and just as the temperature $\partial E/\partial S$ is more convenient to use, for powders the compactivity $\partial V/\partial S$ seems to fulfil the role of temperature. Note that the definition of entropy is the usual logarithm of the number of ways the grains of powder can be put into the volume. It is clearly related to the degree of packing but it is a universal definition whereas the packing fraction is specific to a particular powder. It is the compactivity which compares two powders just as the temperature does in conventional thermodynamic problems. For example, simple calculations using this concept can decide whether two powders are miscible. A review will be given of 'static' problems of this kind, but once the concept of a powder entropy is established the universal structure of liquid visco-elastic equations can be initiated in powders. The key quantity is the constitutive equation and this is much more complex than in liquids for a fractal structure can be expected.

This is just a modern phrase for the arching of the powder and the sustaining of stress by any boundary walls of the flow. This situation is well-known in soil mechanics, but what is now aimed for is the universal structure which obtains in the equations of fluid flow. Some simple models will be given of flows which are simple, but examples will also be developed which are much more complex and an attempt will be offered of what constitutive equation can be expected when there is a fractal, i.e. fern-like or multiply arched, transmission of stress. This work is recent and speculative in its detailed form, but it is hoped that it constructs its forms for the deeper issue of a universal structure for powder flow equations.

Powders & Grains 93, Thornton (ed.) © 1993 Balkema, Rotterdam, ISBN 90 5410 323 X

Table of contents

3 *Quasi-static deformation*

4 *Powder compacts*

5 *Vibrating beds*

6 *Rapid granular flow*

10 *Miscellaneous (Posters)*

1 Packing geometry

Powders & Grains 93, Thornton (ed.) © 1993 Balkema, Rotterdam, ISBN 90 5410 323 X

Discrete-mechanical approach to granular media

Masao Satake
Tohoku Gakuin University, Tagajo, Japan

ABSTRACT: Discrete-mechanical approach to granular media is summarized in this paper. First the graph representation of a granular assembly is explained and the importance of duality between particle-graph and void-graph is emphasized. The discrete-mechanical quantities for granular media are introduced and their relationship and the meaning of void quantities are explained. It is shown that the fundamental matrices which connect discrete-mechanical quantities play an important role and they are discretized form of Schaefer's operator. In the last part, the formulation of constitutive relation and its future direction are discussed.

1 INTRODUCTION

Recently it is widely recognized that the mechanical properties of granular media depend mostly on the microstructure of the material. So the micromechanical studies of granular media have been developed increasingly and many useful results have been obtained (Satake & Jenkins 1988, Biarez & Gourves 1989, Shen et al 1992). However, for the micromechanics of granular media, the establishment of discrete-mechanics and the finding of relationship between corresponding continuum mechanics are fundamentally important and desirable. This paper intends to summarize such discrete-mechanical approaches (Satake 1976, 1978, 1987, 1992,1992), and to indicate the future direction of the research.

The granular media are idealized to assemblies of granular particles. The size of particles is assumed to be almost homogeneous and shape of particle is not necessarily circular (in 2D) or sphrical (in 3D) but is assumed to be almost round. In the first part of this paper, we replace such an assembly to a graph called *particle-graph* and also introduce *void-graph*, which is dual to the particle graph. Some graph-theoratical analysis is given and the *redundancy number* is explained.

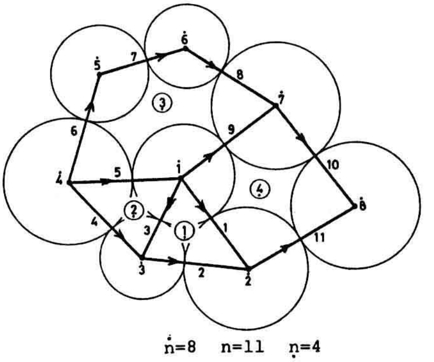
Next mechanical quantities used in discrete-mechanics are defined. The relationships between particle, contact and void quantities are expressed by using two fundamental matrices, i.e. *extended incidence matrix* and and *extended loop matrix*. From the above expression, the meaning of void quantities, which are newly introduced to the present theory, are explained. It is also shown that Schaefer's operator in generalized continuum mechanics correspond to the fundamental matrices.

In the last part, we consider the *particle-stiffnes* for particle displacement and *void-compliance* for void force, taking into account some elasticity at contacts. The particle-stiffness and void-compliance as well are considered to be fundamentally important in the formulation of costitutive relation of granular media, and some discussion is given for the future development.

In this paper, analysis is limitted to 2D, however the extention to 3D is discussed.

2 GRAPH REPRESENTATION

An assembly of granular particles is replaced to a graph (Fig.1), which represents the geometrical structure of the assembly. The graph is called *particle graph*, and it is seen that particle, contact point and void in the assembly correspond to point (centroid of particle), branch and loop in the graph, respectively. For convenience of analysis, we introduce an orientation to each branch, so the graph is an oriented graph. It is noted that the decision of orientation is made arbitrarily except some special cases.



$\dot{n}=8 \quad n=11 \quad \underset{\cdot}{n}=4$

Fig.1 Particle graph

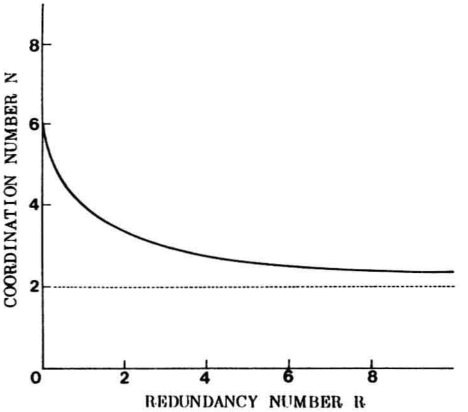


Fig.2 Relationship between redundancy number R and coordination number N

Let $\dot{n}$, n and $\underset{\bullet}{n}$ be numbers of particles, contact points and voids, respectively, we have the following relationship called Euler's formula:

$$\dot{n} - n + \underset{\bullet}{n} = 1 \tag{2.1}$$

As is seen in the Fig.1, loops are not only triangle but also quadrangle, pentagon etc. We call a number $r = s - 3$, where s is number of sides of a loop, the ***redundancy number*** of the loop. We can also introduce the ***(mean) redundancy number*** of an assembly, such that

$$R = \frac{r}{\underset{\bullet}{n}} \tag{2.2}$$

and it is easy to show that

$$R = \frac{6 - N}{N - 2} \tag{2.3}$$

where N is the (mean) coordination number (Satake 1978, 1987). Eq.(2.3) is illustrated in Fig.2. It is seen that R is a measure of density of packing and R increases as the density becomes looser.

A granular assembly consists of particles and voids. In 2D voids contact also at contact points of perticles. It may be clearer if we imagine a ***dual particle*** (dotted line) for each void, as is shown in Fig.3 (a).

Quite similarly as in particles, we can obtain the ***void-graph*** for voids (dual particles) in an assembly of granular particles (Fig.3 (b)). Points in the graph are centroids of dual particles. The void-graph is ***dual*** to the particle-graph and an oriented graph. Branches in void-graph are called ***dual branch***es and each dual branch corresponds to a branch in the particle-graph. The orientation of a dual branch is determined in certain manners, and here it is done so as to make an angle of $0 \sim \pi$ in counter-clockwise to the corresponding branch vector, as is shown in Fig.4. It is noted that incidence matrix and loop matrix (see APPENDIX 1) in void-graph are same to loop matrix and incidence matrix in particle-graph, respectively.

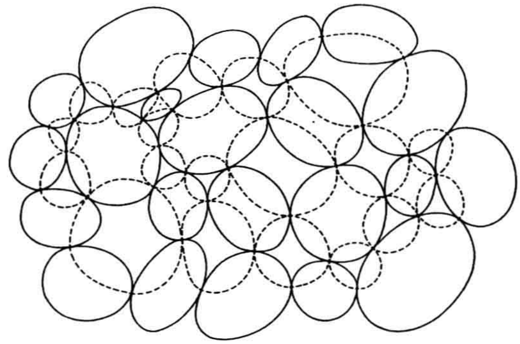


Fig.3(a) Particles and dual particles (voids)

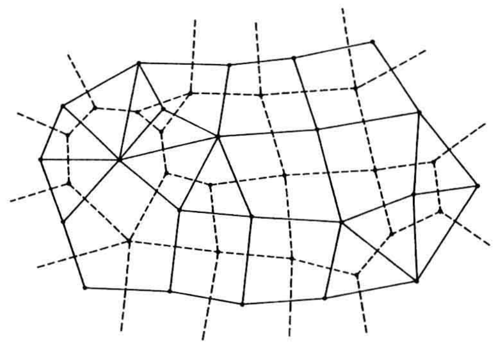


Fig.3(b) Particle-graph and void-graph (orientation of branches is omitted)

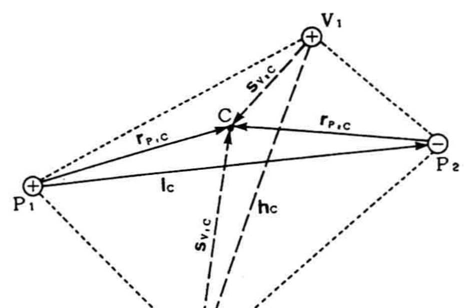


Fig.4 Branch and dual branch

3 MECHANICAL QUANTITIES AND THEIR RELATIONSHIPS

Regarding the most generality, we introduce discrete-mechanical quantities listed in Table 1. Note that subscripts P, C and V specify particles, contact points and voids, respectively. Although couple and rotation