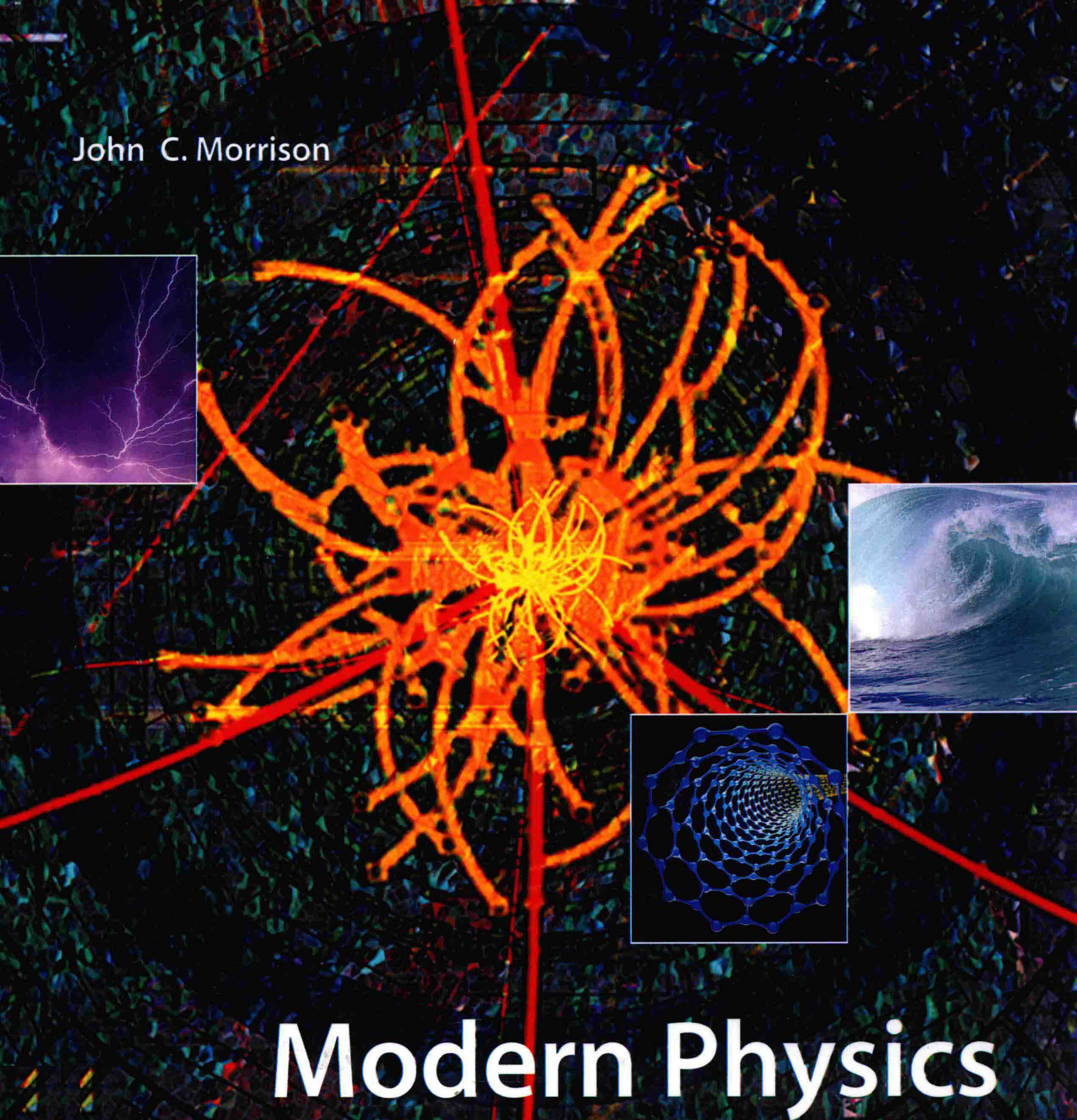
John C. Morrison



Modern Physics

for Scientists and Engineers

Second Edition

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for Scientists and Engineers

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John C. Morrison
Physics Department, University of Louisville, Louisville, KY, USA

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Dedication

This book is dedicated to the scientists and mathematicians in the Holy Lands who are striving for peace in a spiritually and culturally rich part of the world.

Online applets are available to solve realistic problems in atomic and condensed matter physics.

You can find the applets at:
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Preface

Modern Physics for Scientists and Engineers presents the ideas that have shaped modern physics and provides an introduction to current research in the different fields of physics. Intended as the text for a first course in modern physics following an introductory course in physics with calculus, the book begins with a brief and focused account of historical events leading to the formulation of modern quantum theory, while ensuing chapters go deeper into the underlying physics.

This book helps prepare engineering students for the upper division courses on devices they will later take and provides engineering and physics majors an overview of contemporary physics. The course in modern physics is the last course in physics most engineering students will ever take. For this reason, this book covers a few topics that are ordinarily taught at the junior/senior level. I include these advanced topics because they are relevant and interesting to engineering students and because these topics would ordinarily be unavailable to them. Topics such as Bloch's theorem, heterostructures, quantum wells and barriers, and a phenomenological description of semiconductor lasers help to give engineering students the physics background they need for the courses they will later take on semiconductor devices, while subjects like the Hartree-Fock theory, Bose-Einstein condensation, the relativistic Dirac equation, and particle physics help students appreciate the range and scope of contemporary physics. This course helps physics majors by giving them a substantial introduction to quantum theory and to the various fields of modern physics. The books I have used to prepare later chapters of this book are just the books used in upper-division courses in the various fields of contemporary physics.

THIS NEW EDITION

The challenge in preparing this new edition has been to describe the developments that have occurred in physics since the first edition of this book appeared in January 2010. I would like to thank Keith Ellis of the Theory Group at Fermilab for discussing recent developments in particle physics with me and correcting the two new sections I have written on local gauge invariance and the discovery of the Higgs Boson. Thanks are also due to Chris Quigg at Fermilab, Ken Hicks at Ohio University, and Wafaa Khater at Birzeit University. My writing of the two new sections on graphene and carbon nanotubes was also greatly helped by Fendinand Evers at Karlsruher Institute of Technology and by Gamini Sumanesekera and Shi-Yu Wu at University of Louisville.

NEW FEATURES

In this new edition of Modern Physics for Scientists and Engineers, I have included a description of simulations from the educational software package PhET developed at the University of Colorado. These simulations, which can be accessed online, enable students to gain an intuitive understanding of how waves interfere with each other and how waves can be combined to form wave packets. The new edition also contains many exercises using the software package MATLAB. A new appendix on MATLAB has been added. Students are shown how to use MATLAB to plot functions, solve differential equations, and evaluate integrals. To make these techniques available to as large a group of students as possible, I also show how the free software package Octave can be used. The MATLAB programs in the first six chapters of this book run unchanged in either MATLAB or Octave. As I shall show, however, the MATLAB programs in later chapters of the book must be modified slightly to run in Octave.

Many of the electrical devices that have been developed within recent years are quantum devices. The finite potential well provides a fairly realistic description of the active region of a semiconductor laser. This book includes MATLAB programs that can be used to find the energy levels and wave functions for electrons confined to finite wells. Another MATLAB program enables one to calculate the transmission and reflections coefficients for electrons incident upon a potential step where the potential energy changes discontinuously. Potential steps of this kind occur naturally at the interface between two different materials. By expressing the relation between the incoming and outgoing amplitudes of electrons incident upon an interface in matrix form, one can calculate the transmission and reflection coefficients for complex systems by multiplying the matrices for the individual parts. MATLAB and Octave programs described in Chapter 10 enable one to calculate transmission coefficients for barriers where the potential energy assumes a different value for a short interval and for more complex structures with two or three barriers. Interesting interference effects occur for more than a single interface.

This new edition also has new exercises using MATLAB and many more problems at the end of each chapter. In response to the request of several teachers of modern physics, all of the figures in the book will be placed at the website of the book and a digital copy of the book will be made available to teachers of modern physics upon request. Having the figures and a digital copy available makes it easier for teachers to prepare PowerPoint lectures.

THE NATURE OF THE BOOK

As can be seen from the table of contents, Modern Physics for Scientists and Engineers covers atomic and solid-state physics before covering relativity theory. When I was beginning to teach modern physics, I led off with the special theory of relativity as do most books, but I found that this approach had a number of disadvantages. Following the short treatment of relativity, there was invariably an uncertain juncture when I made the transition back to a nonrelativistic framework in order to introduce the ideas of wave mechanics. The students were asked to make this transition when they were just getting started in the course. Then, the important applications of relativity theory to particle and nuclear physics came at the end of the course when we had not used the relativistic formalism for some time. I found it to be better to develop nonrelativistic wave mechanics at the beginning of the course and "go relativistic" in the last 3 or 4 weeks. The course flows better that way.

The first three chapters of this book give an introduction to quantum mechanics at an elementary level. Chapters 4-6 are devoted to atomic physics and the development of lasers. Chapter 7 is devoted to statistical physics and Chapters 8-10 are devoted entirely to condensed matter physics. Each of these chapters has special features that cannot be found in any other book at this level. The new version of the Hartree-Fock applet described in Chapter 5 enables students to do Hartree-Fock calculations on any atom in the periodic table using the Hartree-Fock applet at the website of the book. With the Hartree-Fock program of Charlottte Fischer in the background and a Java interface, the applet comes up showing the periodic table. A student can initiate a Hartree-Fock calculation by choosing a particular atom in the periodic table and clicking on the red arrow in the lower right-hand corner of the web page. The wave functions of the atom immediately appear on the screen and tabs along the upper edge of the web page enable students to gain additional information about the properties of the atom. One can find the average distance of each electron from the nucleus and evaluate the two-electron Slater integrals and the spin-orbit constant of the outer electrons. When I cover the chapters on atomic physics in my course, I keep the focus on the underlying physics. As one moves from one atom to the next along a row of the periodic table, the nuclear charge increases. As a result, the electrons are drawn in toward the nucleus, and the distance between the electrons decreases. The Coulomb interaction between the electrons increases and the "LS" term structure expands. All of this can be understood in simple physical terms.

With the addition of MATLAB to Chapter 7, students can evaluate the probability that the values of the variables of particles lie within a particular range. This enables one to calculate the probability that the velocities of molecules in the upper atmosphere of a planet are greater than the escape velocity with the planet losing its atmosphere, and it enables one to calculate the fractional number of electrons in a semiconductor with an energy above the Fermi energy. In this new edition, Chapter 8 has a detailed description of graphene and carbon nanotubes. One of my surprises in preparing the new edition was to find that the charge carriers of graphene are Fermions with zero mass that are accurately described by the Dirac equation. Physics is a whole with all of the individual pieces fitting together.

Chapters 11 and 12, which are devoted to relativity theory, include a careful treatment of the Dirac equation and a qualitative description of quantum electrodynamics. Chapter 13 on particle physics includes a description of the conservation laws of lepton number, baryon number, and strangeness. Also included is a treatment of the parity and charge conjugation symmetries, isospin, and the flavor and color $SU(3)$ symmetries. The chapter on particle physics concludes with two new sections on local gauge invariance and the recent discovery of the Higgs boson.

Most chapters of this book are fewer than 40 pages long, making it possible for an instructor to cover the main topics in each chapter in 1 week. To give myself some flexibility in presenting the material, I usually choose two or three chapters that I will not cover apart from a few qualitative remarks and then choose another three chapters that I will only expect my students to know in a qualitative way. My selection of the subjects I cover more extensively depends upon the interests of the particular class. Typically, the students might be expected to be able to work problems for the first three chapters and the first section of Chapter 4, for Chapter 7 on Boltzman and Fermi-Dirac statistics, for Chapter 8 on condensed matter physics, for Chapter 11 and the first two sections of Chapter 12 on relativity theory, and for the first two sections of Chapter 13 on particle physics. The students might then be asked qualitative quiz questions for Chapters 5, 6, 9, and 10, and the concluding sections of Chapters 12 and 13. Suitable quiz questions and test problems can be found at the end of each chapter. In my own classes, I typically give six quizzes and two tests. The practice of giving frequent quizzes keeps students up on the reading and better prepared for discussion in class. Also, as a practical matter, our physics courses are always competing with the engineering program for the study time of our students. Only by requiring in some concrete way that students keep up with our courses can we expect a continuous investment of effort on their part.

I feel strongly that any class in physics should reach out to the broad majority of students, but that the class should also allow students the opportunity to follow their interests beyond the level of the general course. Each chapter of this book begins with a sound, rudimentary treatment of the fundamental subject matter, but then treats subjects such as the Dirac theory that challenges the abilities of my better students. I always encourage my students to do extra-credit projects in which they have a special interest and to work additional problems in areas that have been reserved for the quizzes. The few physics majors I have had in my class often choose more advanced topics in which they have a special interest. For the physics majors, my course gives them a valuable overview of the fields of contemporary physics that helps them with the specialty course they later take as juniors and seniors.

Acknowledgments

Many people have helped me to produce this book. I would like to thank Leslie Friesen who drew all of the figures for the two editions of this book and responded to numerous suggestions that I have made of how the figures could be improved. Special thanks is also due Ken Hicks at Ohio University who suggested that I use MATLAB to solve the problems that arise in modern physics and provided many of the MATLAB exercises and problems in the text. Ken wrote the first draft of the appendix on MATLAB. I would also like to thank Thomas Ericsson of the Mathematics Department of Göteborg University for bringing our MATLAB exercises and problems up to the level of modern books on mathematics and Geoffrey Lentner of the Department of Physics and Astronomy at University of Louisville for helping me with Octave. I appreciate the kind help Charlotte Fischer provided me so that our applet could take advantage of all of the special features of her atomic Hartree-Fock program and the work of Simon Rochester who wrote the current version of our Hatree-Fock applet.

I would like to express my appreciation for the help I have received from many physicists who are at the forefront of their research areas and have helped me during the course of producing this book. In the area of condensed matter physics, I would like to thank Jim Davenport, Dick Watson, and Vic Emory for their hospitality in the Condensed Matter Theory Group of Brookhaven National Laboratory during the summer when I wrote my first draft of the solid-state chapter. I appreciate the guidance of John Wilkins of Ohio State University, who has served as the Chair of the condensed matter section of the American Physical Society. In the area of particle physics, I would like to thank Keith Ellis and Chris Quigg at Fermilab, William Palmer at Ohio State University, and Howard Georgi, who allowed me to attend his class on group theory and particle physics at Harvard University.

Several well-known physicists have distinguished themselves not only for their research but also for their teaching and writing. I would like to thank Dirk Walecka at College of William and Mary, Dick Furnstahl at Ohio State University, and I would like to thank Thomas Moore at Pomona College whose writings on elementary physics have been a source of inspiration for me.

This book evolved over a number of years and several of the early reviewers of the manuscript played an important role in its development. For their ideas and guidance, I would like to thank Massimilliano Galeazzi at University of Miami, Amitabh Lath at SUNY Rutgers, and Mike Santos and Michael Morrison at University of Oklahoma.

Finally, I would like to thank the teachers of modern physics, who have sent me valuable suggestions and extended to me their hospitality when I have visited their university. Thanks are due to Jay Tang at Brown University; W. Andreas Schroeder at University of Illinois, Chicago; Roger Bengtson at University of Texas; Michael Jura at UCLA; Dmitry Budker at University of California Berkeley; Paul Dixon at California State San Bernadino; Murtadha Khakoo at California State Fullerton; Charlotte Elster at Ohio University; Ronald Reifenberger at Purdue University; Michael Schulz at University of Missouri, Rolla; Bill Skocpol at Boston University; David Jasnow at University of Pittsburgh; Sabine Lammers, Lisa Kaufman, and Jon Urheim at Indiana University; Connie Roth and Fereydoon Family at Emery University; David Maurer at Auburn University; Xuan Gao and Peter Kerman at Case Western Reserve; and Cheng Cen and Earl Scime at University of West Virginia. I would also like to thank Lee Larson, Dave Brown, Chris Davis, Humberto Gutierrez, Christian Tate, Kyle Stephen, and Joseph Brock at University of Louisville; Matania Ben-Atrzi at Hebrew University in Jerusalem; Ramzi Rihan, Aziz Shawabka, Henry Jagaman, Wael Qaran, and Wafaa Khater at Birzeit University in Ramallah; and Jacob Katriel at Technion University in Haifa.

I welcome the suggestions and the questions of any teacher who takes to the phone or keyboard and wants to talk about a particular topic.

Louisville, Kentucky
John C. Morrison
October 21, 2014
johnc@erdos.math.louisville.edu

Introduction

Every physical system can be characterized by its size and the length of time it takes for processes occurring within it to evolve. This is as true of the distribution of electrons circulating about the nucleus of an atom as it is of a chain of mountains rising up over the ages.

Modern physics is a rich field including decisive experiments conducted in the early part of the twentieth century and more recent research that has given us a deeper understanding of fundamental processes in nature. In conjunction with our growing understanding of the physical world, a burgeoning technology has led to the development of lasers, solid-state devices, and many other innovations. This book provides an introduction to the fundamental ideas of modern physics and to the various fields of contemporary physics in which discoveries and innovation are going on continuously.

I.1 THE CONCEPTS OF PARTICLES AND WAVES

While some of the ideas currently used to describe microscopic systems differ considerably from the ideas of classical physics, other important ideas are classical in origin. We begin this chapter by discussing the important concepts of a particle and a wave which have the same meaning in classical and modern physics. A *particle* is an object with a definite mass concentrated at a single location in space, while a *wave* is a disturbance that propagates through space. The first section of this chapter, which discusses the elementary properties of particles and waves, provides a review of some of the fundamental ideas of classical physics. Other elements of classical physics will be reviewed later in the context for which they are important. The second section of this chapter describes some of the central ideas of modern quantum physics and also discusses the size and time scales of the physical systems considered in this book.

I.1.1 The Variables of a Moving Particle

The position and velocity vectors of a particle are illustrated in Fig. I.1. The position vector $\mathbf{r}$ extends from the origin to the particle, while the velocity vector $\mathbf{v}$ points in the direction of the particle's motion. Other variables, which are appropriate for describing a moving particle, can be defined in terms of these elementary variables.

The *momentum* $\mathbf{p}$ of the particle is equal to the product of the mass and velocity $\mathbf{v}$ of the particle

$$\mathbf{p} = m\mathbf{v}.$$

We shall find that the momentum is useful for describing the motion of electrons in an extended system such as a crystal.

The motion of a particle moving about a center of force can be described using the *angular momentum*, which is defined to be the cross product of the position and momentum vectors

$$\ell = \mathbf{r} \times \mathbf{p}.$$

The cross product of two vectors is a vector having a magnitude equal to the product of the magnitudes of the two vectors times the sine of the angle between them. Denoting the angle between the momentum and position vectors by θ as in Fig. I.1, the magnitude of the angular momentum vector momentum can be written

$$|\ell| = |\mathbf{r}|\,|\mathbf{p}|\sin\theta.$$

This expression for the angular momentum may be written more simply in terms of the distance between the line of motion of the particle and the origin, which is denoted by r_0 in Fig. I.1. We have

$$|\ell| = r_0|\mathbf{p}|.$$

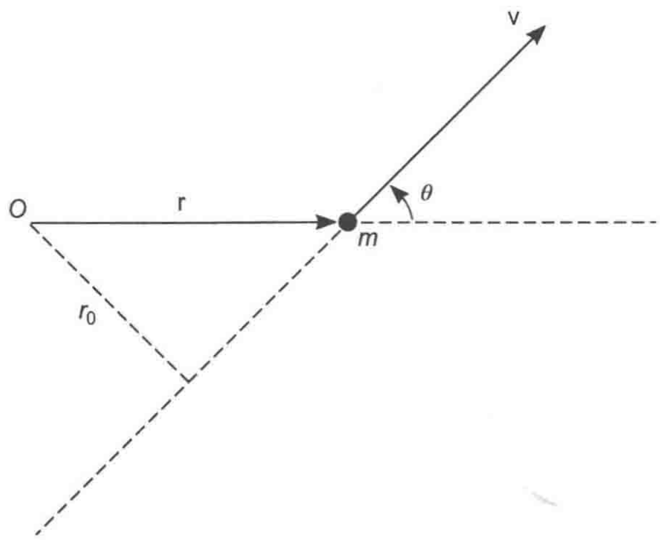

FIGURE I.1 The position **r** and the velocity **v** of a moving particle of mass *m*. The point *O* denotes the origin, and r_0 denotes the distance between the line of motion and the origin.

The angular momentum is thus equal to the distance between the line of motion of the particle and the origin times the momentum of the particle. The direction of the angular momentum vector is generally taken to be normal to the plane of the particle's motion. For a classical particle moving under the influence of a central force, the angular momentum is conserved. The angular momentum will be used in later chapters to describe the motion of electrons about the nucleus of an atom.

The *kinetic energy* of a particle with mass m and velocity $\mathbf{v}$ is defined by the equation

$$KE = \frac{1}{2}mv^2,$$

where v is the magnitude of the velocity or the speed of the particle. The concept of *potential energy* is useful for describing the motion of particles under the influence of conservative forces. In order to define the *potential energy* of a particle, we choose a point of reference denoted by R. The potential energy of a particle at a point P is defined as the negative of the work carried out on the particle by the force field as the particle moves from R to P. For a one-dimensional problem described by a variable x, the definition of the potential energy can be written

$$V_P = -\int_R^P F(x)\mathrm{d}x. \tag{I.1}$$

As a first example of how the potential energy is defined we consider the harmonic oscillator illustrated in Fig. I.2(a). The harmonic oscillator consists of a body of mass m moving under the influence of a linear restoring force

$$F = -kx, \tag{I.2}$$

where x denotes the distance of the body from its equilibrium position. The constant k, which occurs in Eq. (I.2), is called the *force constant*. The restoring force is proportional to the displacement of the body and points in the direction opposite to the displacement. If the body is displaced to the right, for instance, the restoring force points to the left. It is natural to take the reference position R in the definition of the potential energy of the oscillator to be the equilibrium position for which $x = 0$. The definition of the potential energy (I.1) then becomes

$$V(x) = -\int_0^x (-kx')\mathrm{d}x' = \frac{1}{2}kx^2. \tag{I.3}$$

Here x' is used within the integration in place of x to distinguish the variable of integration from the limit of integration.

If one were to pull the mass shown in Fig. I.2(a) from its equilibrium position and release it, the mass would oscillate with a frequency independent of the initial displacement. The angular frequency of the oscillator is related to the force constant of the oscillator and the mass of the particle by the equation

$$\omega = \sqrt{k/m}.$$

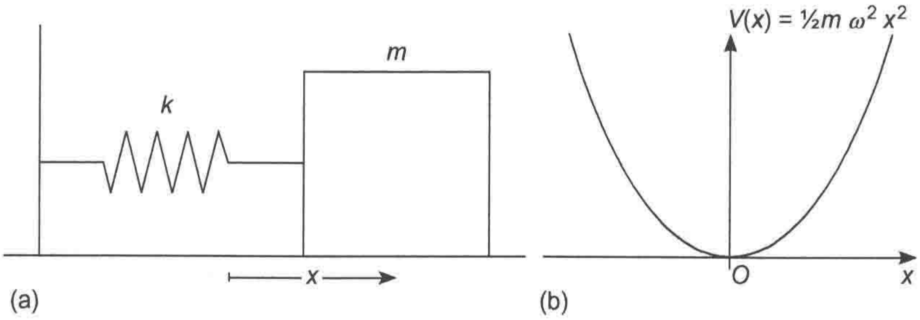

FIGURE I.2 (a) A simple harmonic oscillator in which a mass m is displaced a distance x from its equilibrium position. The mass is attracted toward its equilibrium position by a linear restoring force with force constant k. (b) The potential energy function for a simple harmonic oscillator.

or

$$k = m\omega^2.$$

Substituting this expression for k into Eq. (I.3), we obtain the following expression for the potential energy of the oscillator

$$V(x) = \frac{1}{2}m\omega^2 x^2. \tag{I.4}$$

The oscillator potential is illustrated in Fig. I.2(b). The harmonic oscillator provides a useful model for a number of important problems in physics. It may be used, for instance, to describe the vibration of the atoms in a crystal about their equilibrium positions.

As a further example of potential energy, we consider the potential energy of a particle with electric charge q moving under the influence of a charge Q. According to Coulomb's law, the electromagnetic force between the two charges is equal to

$$F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2},$$

where r is the distance between the two charges and ϵ_0 is the permittivity of free space. The reference point for the potential energy for this problem can be conveniently chosen to be at infinity where $r = \infty$ and the force is equal to zero. Using Eq. (I.1), the potential energy of the particle with charge q at a distance r from the charge Q can be written

$$V(r) = -\frac{Qq}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{r'^2}\mathrm{d}r'.$$

Evaluating the above integral, one finds that the potential energy of the particle is

$$V(r) = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r}.$$

An application of this last formula will arise when we consider the motion of electrons in an atom. For an electron with charge $-\mathrm{e}$ moving in the field of an atomic nucleus having Z protons and hence a nuclear charge of Ze, the formula for the potential energy becomes

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0}\frac{1}{r}. \tag{I.5}$$

The energy of a body is defined to be the sum of its kinetic and potential energies

$$E = KE + V.$$

For an object moving under the influence of a conservative force, the energy is a constant of the motion.

I.1.2 Elementary Properties of Waves

We consider now some of the elementary properties of waves. Various kinds of waves arise in classical physics, and we shall encounter other examples of wave motion when we apply the new quantum theory to microscopic systems.

Traveling Waves

If one end of a stretched string is moved abruptly up and down, a pulse will move along the string as shown in Fig. I.3(a). A typical element of the string will move up and then down as the pulse passes. If instead the end of the string moves up and down with the time dependence,

$$y = \sin \omega t,$$

an extended sinusoidal wave will travel along the string as shown in Fig. I.3(b). A wave of this kind which moves up and down with the dependence of a sine or cosine is called a *harmonic wave*.

The wavelength of a harmonic wave will be denoted by λ and the speed of the wave by v. The wavelength is the distance from one wave crest to the next. As the wave moves, a particular element of the string which is at the top of a crest will move down as the trough approaches and then move back up again with the next crest. Each element of the string oscillates up and down with a period, T. The frequency of oscillation f is equal to $1/T$. The period can also be thought of as the time for a crest to move a distance of one wavelength. Thus, the wavelength, wave speed, and period are related in the following way:

$$\lambda = vT.$$

Using the relation, $T = 1/f$, this equation can be written

$$\lambda f = v. \tag{I.6}$$

The dependence of a harmonic wave upon the space and time coordinates can be represented mathematically using the trigonometric sine or cosine functions. We consider first a harmonic wave moving along the x-axis for which the displacement is

$$y(x, t) = A \sin[2\pi(x/\lambda - t/T)], \tag{I.7}$$

where A is the amplitude of the oscillation. One can see immediately that as the variable x in the sine function increases by an amount λ or the time increases by an amount T, the argument of the sine will change by an amount 2π, and the function $y(x, t)$ will go through a full oscillation. It is convenient to describe the wave by the *angular wave number*,

$$k = \frac{2\pi}{\lambda}, \tag{I.8}$$

and the *angular frequency*,

$$\omega = \frac{2\pi}{T}. \tag{I.9}$$

Using the relation, $T = 1/f$, the second of these two equations can also be written

$$\omega = 2\pi f. \tag{I.10}$$

The angular wave number k, which is defined by Eq. (I.8), has SI units of radians per meter, while ω, which is defined by Eq. (I.9), has SI units of radians per second. Using Eqs. (I.8) and (I.9), the wave function (I.7) can be written simply

$$y(x, t) = A \sin(kx - \omega t). \tag{I.11}$$

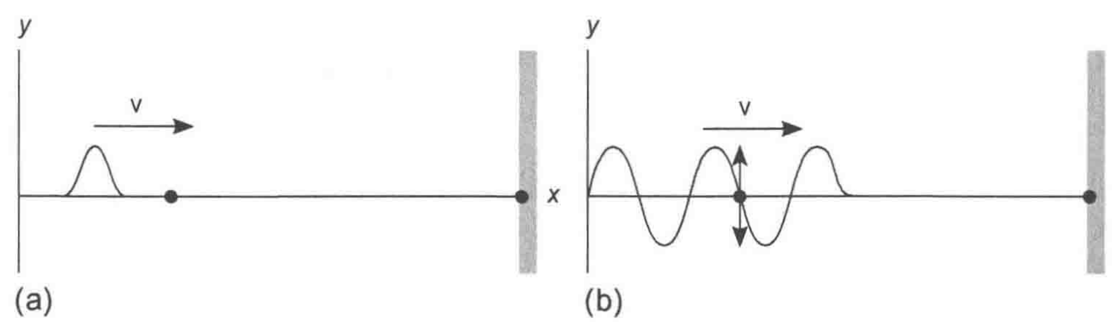

FIGURE I.3 (a) A pulse moving with velocity v along a stretched string. (b) An extended sinusoidal wave moving along a string.

Equation (I.11) describes a traveling wave. We can see this by considering the crest of the wave where the value of the phase of the sine function in Eq. (I.11) is equal to $\pi/2$. The location of the crest is given by the equation

$$kx_{\text{crest}} - \omega t = \frac{\pi}{2}.$$

Solving this last equation for x_{crest}, we get

$$x_{\text{crest}} - = \frac{\omega t}{k} + \frac{\pi}{2k}.$$

An expression for the velocity of the wave crest can be obtained by taking the derivative of x_{crest} with respect to time to obtain

$$v = \frac{dx_{\text{crest}}}{dt} = \frac{\omega}{k}. \quad (I.12)$$

Equation (I.11) thus describes a sinusoidal wave moving in the positive x-direction with a velocity of ω/k. Equation (I.12) relating the velocity of the wave to the angular wave number k and angular frequency ω can also be obtained by solving Eq. (I.8) for λ and solving Eq. (I.10) for f. Equation (I.12) is then obtained by substituting these expressions for λ and f into Eq. (I.6).

Using the same approach as that used to understand the significance of Eq. (I.11), one can show that

$$y(x,t) = A\sin(kx + \omega t) \quad (I.13)$$

describes a sinusoidal wave moving in the negative x-direction with a velocity of ω/k.

Figure I.4(a) illustrates how the harmonic function (I.11) varies with position at a fixed time chosen to be $t = 0$. Setting t equal to zero, Eq. (I.11) becomes

$$y(x,0) = A\sin kx. \quad (I.14)$$

The wave described by the function $A\sin x$ and illustrated in Fig. I.4(a) does not depend upon the time. Such a wave, which is described by its dependence upon a spatial coordinate, is called a *stationary wave*. As for the traveling wave (I.11), the angular wave number k is related to the wavelength by Eq. (I.8). Similarly, Fig. I.4(b) shows how the function (I.11 varies with time at a fixed position chosen to be $x = 0$. Setting x equal to zero in Eq. (I.11) and using the fact that the sine is an odd function, we obtain

$$y(0,t) = -A\sin\omega t. \quad (I.15)$$

The wave function (I.15) oscillates as the time increases with an angular frequency ω given by Eq. (I.10).

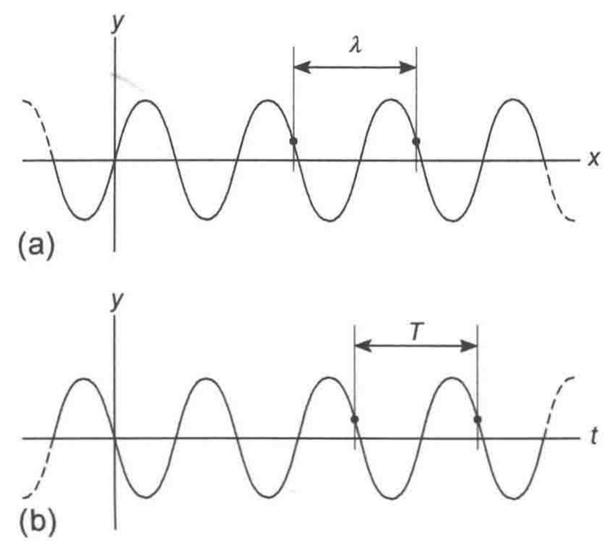

FIGURE I.4 (a) The x-dependence of the sinusoidal function for a fixed time $t = 0$. (b) The time dependence of the sinusoidal function at the fixed point $x = 0$.

Standing Waves

Suppose two waves travel simultaneously along the same stretched string. Let $y_1(x,t)$ and $y_2(x,t)$ be the displacements of the string due to the two waves individually. The total displacement of the string is then

$$y(x,t) = y_1(x,t) + y_2(x,t).$$

This is called the *principle of superposition*. The displacement due to two waves is generally the algebraic sum of the displacements due to the two waves separately. Waves that obey the superposition principle are called *linear* waves and waves that do not are called *nonlinear* waves. It is found experimentally that *most* of the waves encountered in nature obey the superposition principle. Shock waves produced by an explosion or a jet moving at supersonic speeds are uncommon examples of waves that do not obey the superposition principle. In this text, only linear waves will be considered. Two harmonic waves reinforce each other or cancel depending upon whether or not they are in phase (in step) with each other. This phenomena of reinforcement or cancelation is called *interference*.

We consider now two harmonic waves with the same wavelength and frequency moving in opposite directions along the string. The two waves having equal amplitudes are described by the wave functions

$$y_1(x) = A\sin(kx - \omega t)$$

and

$$y_2(x) = A\sin(kx + \omega t).$$

According to the principle of superposition, the combined wave is described by the wave function

$$y(x,t) = y_1(x) + y_2(x) = A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]. \tag{I.16}$$

Using the trigonometric identity,

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B, \tag{I.17}$$

Eq. (I.16) may be written

$$u(x,t) = [2A\cos\omega t]\sin kx. \tag{I.18}$$

This function describes a *standing wave*.

At a particular time, the quantity within square brackets in Eq. (I.18) has a constant value and may be thought of as the amplitude of the wave. The amplitude function $2A\cos\omega t$ varies with time having both positive and negative values. The function $\sin kx$ has the spatial form illustrated in Fig. I.5 being zero at the points satisfying the equation

$$kx = n\pi, \quad \text{for } n = 0, 1, 2, \ldots.$$

Substituting $k = 2\pi/\lambda$ into this equation, we get

$$x = n\frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \ldots.$$

The function $\sin kx$ is thus equal to zero at points separated by half a wavelength. At these points, which are called *nodes*, the lateral displacement is *always* equal to zero. An example of a standing wave is provided by the vibrating strings of a guitar. The ends of the guitar strings are fixed and cannot move. In addition to the ends of the strings, other points along the strings separated by half a wavelength have zero displacements. We shall find many examples of traveling and standing waves later in the book when we consider microscopic systems.

One can gain an intuitive understanding of the properties of waves by using the PhET simulation package developed at the University of Colorado. The simulations can be found at the Web site: phet.colorado.edu/en/simulations. Choosing the

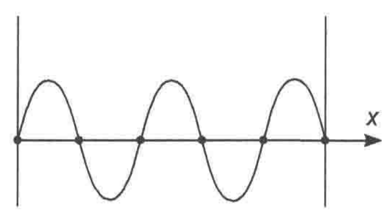

FIGURE I.5 Function describing the spatial form of a standing wave. The nodes, which have zero displacement, are represented by dots.

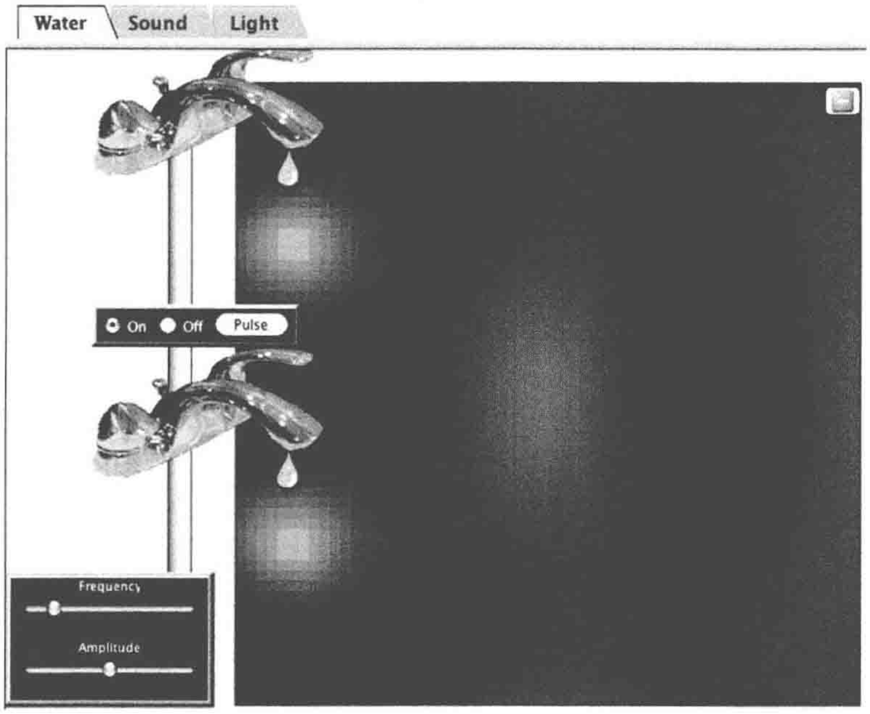

FIGURE I.6 A simulation of wave interference in the PhET simulation package developed at the University of Colorado.

categories "physics" and "sound and waves," one can initiate the simulation called "wave interference." Choosing the tab "water" and the option "one drip," one sees the waves spreading across a body of water when drips from a single faucet strike the water surface. Choosing then the option "two drips," one sees the waves, produced by the drips of two faucets striking the water surface. This figure is shown in Fig. I.6. As we have just described the waves from the two disturbances add together and destructively interfere to produce a complex disturbance on the surface of the water. One can observe similar effects with sound and light waves by choosing the tabs "sound" and "light."

The Fourier Theorem

We have thus far considered sinusoidal waves on a string and would now like to consider wave phenomenon when the shape of the initial disturbance is *not* sinusoidal. In the decade of the 1920s, Jean Baptiste Fourier showed that *any* reasonably continuous function $f(x)$, which is defined in the interval $0 \leq x < L$, can be represented by a series of sinusoidal waves

$$f(x) = \sum_{n=1,2,\ldots} S_n \sin nkx, \quad \text{for } 0 \leq x \leq L, \tag{I.19}$$

where $k = 2\pi/L$ and

$$S_n = \frac{2}{L}\int_0^L \sin nkx f(x)\mathrm{d}x. \tag{I.20}$$

A sketch of the derivation of Eq. (I.20) is given in Problem 4.

As an example, we consider a square wave

$$f(x) = \begin{cases} -A & \text{if } 0 \leq x < \frac{1}{2}L \\ +A & \text{if } \frac{1}{2}L < x \leq L \end{cases} \tag{I.21}$$

Using Eqs. (I.19) and (I.20), the square wave (I.21) can be shown to be equal to the following infinite sum of sinusoidal waves

$$f(x) = -A\frac{4}{\pi}\left(\sin kx + \frac{1}{3}\sin 3kx + \frac{1}{5}\sin 5kx + \cdots\right), \tag{I.22}$$

where k is the angular wave number of the fundamental mode of vibration.

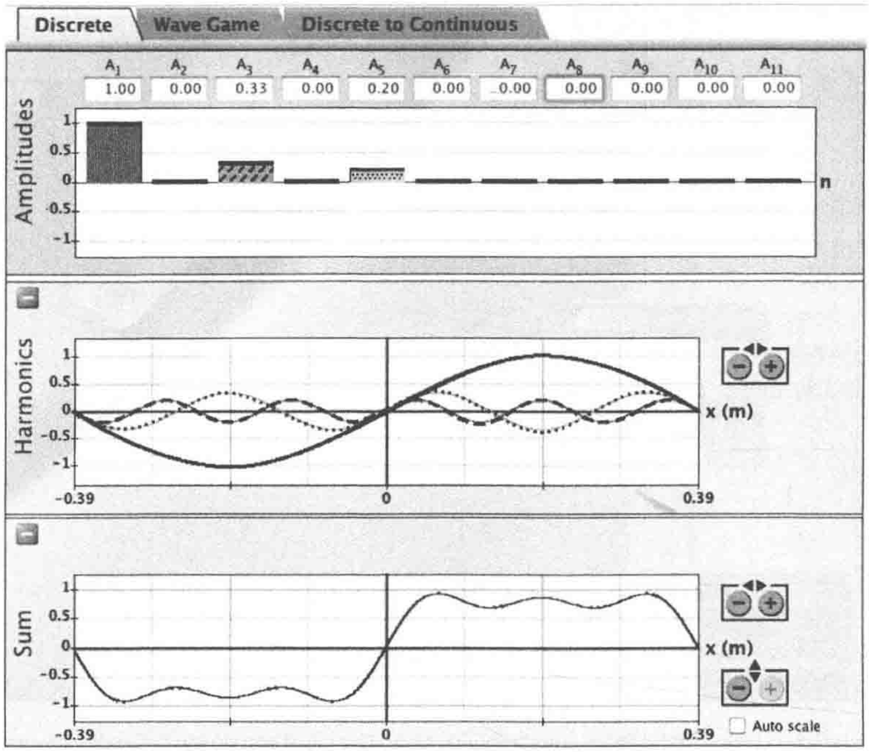

FIGURE I.7 A representation of a square wave function formed by adding harmonic waves together using the PhET simulation package developed at the University of Colorado.

We can gain some insight into how harmonic waves combine to form the square wave function (I.21) by using the simulation package "Fourier: Making Waves" at the Web site: http://phet.colorado.edu/en/simulation/fourier. A reproduction of the window that comes up is shown in Fig. I.7. With "Preset Function" set to "sine/cosine" and "Graph controls" set at "Function of: space (x)" and "sin," one can begin by setting $A_1 = 1$ and $A_3 = 0.33$ and then gradually adding $A_5 = 0.20, A_7 = 0.14, A_9 = 0.11$, and $A_{11} = 0.09$. As one adds more and more sine functions of higher frequency, the sum of the waves shown in the lower screen becomes more and more like a square wave. One can understand in qualitative terms how the harmonic waves add up to produce the square wave. Using the window reproduced by Fig. I.7, one can view each sinusoidal wave by setting the amplitude of the wave equal to one and all other amplitudes equal to zero. The amplitude A_1 corresponds to the fundamental wave for which a single wavelength stretches over the whole region. This sinusoidal wave—like the square wave—is zero at the center of the region and assumes negative values to the left of center and positive values to the right of center. The sinusoidal waves with amplitudes A_3, A_5, A_7, A_9, and A_{11} all have these same properties but being sinusoidal waves of higher frequencies they rise more rapidly from zero as one moves to the right from the center of the region. By adding waves with higher frequencies to the fundamental wave, one produces a wave which rises more rapidly as one moves to the right from center and declines more rapidly as one moves to the left from center; however, the sum of the waves oscillate with a higher frequency than the fundamental frequency in the region to the right and left of center. As one adds more and more waves, the oscillations due to the various waves of high frequency destructively interfere and one obtains the square wave.

The above result can also be obtained using the MATLAB software package. A short introduction to MATLAB can be found in Appendix C and a more extensive presentation in Appendix CC. MATLAB Program I.1 given below adds sinusoidal waves up to the fifth harmonic. The first three lines of the program define the values of A, L, and k, and the next line defines a vector x with elements between −L/2, and +L/2 with equal steps of L/100. The plot of x versus y produced by this MATLAB program is shown in Fig. I.8. This figure is very similar to Fig. I.7 produced by the PhET simulation package.
MATLAB Program I.1
This program adds the Fourier components up to the fifth harmonic to produce a square wave of amplitude 1.0 and width 1.0.

```
A=1;
L=1;
k=2*pi/L;
x = -L/2 : L/100 : L/2;
y = (A*4/pi)*( sin(k*x)+(1/3)*sin(3*k*x)+(1/5)*sin(5*k*x) );
plot(x,y)
```