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PREFACE

The Fifth European Conference on Industrial Mathematics (ECMI 90) took place at Lahti, Finland on June 6-9, 1990. The conference was organised by the Rolf Nevanlinna Institute together with the Lahti Research and Training Centre of the University of Helsinki. Like its predecessors the Lahti meeting was devoted to the exchange of experience, ideas and methods from various fields of industrial mathematics. The series of ECMI conferences have clearly established an important forum of interaction between the advancing front of technology and one of its crucial development resources, modern applications-oriented mathematics.

The precise title of the conferences has been the subject of some discussion and it has been argued that there is no such area which can be labelled as "industrial mathematics". This is certainly true if one thinks only in terms of the range of ideas, theorems, methods and algorithms constituting mathematics all of which may be applied. However with another viewpoint industrial mathematics is not a collection of topics but refers to the interactive process in which mathematics, the science, meets the real world of applications. Ideally this interaction involves both good mathematics and technological advance. The computer revolution has created a new era in technology with the increased computational capability making it possible to simulate complex industrial processes, devices, and other technological systems. This simulation depends on mathematical modelling and analysis and these techniques, sometimes ingenious but often quite routine, have provided a powerful tool for industrial scientists and creative research management.

The series of ECMI conferences, which began in Amsterdam in 1985, is intended to offer a state-of-the-art survey on the developments in this vital area of technology transfer. The Lahti conference confirmed both the relevance of the subject matter and the growth of the field of application. Despite the remote geographical location, and hence unduly high air fares, about 175 participants from 24 countries attended the meeting. The programme was composed of 11 invited lectures and 90 contributed papers. The range of the topics was broad reflecting the variety of applications that mathematical methods have in industry.

From the application areas displayed in the programme some examples could be mentioned: the supply and distribution of energy, electromagnetic field computations, fluid- and gas dynamics, welding and casting phenomena, phase and shape transitions, process and device simulation, chemical reactors, physical measurements and signals, picture processing, systems analysis and control, robotics and education in industrial mathematics. The invited lectures covered topics like chemical engineering, shell problems, sedimentation and aggregation phenomena, stochastic systems, radar technology, shape design and nondestructive testing, VLSI industry, random functions in mechanical systems, wavelets and modelling the ocean surface.

These proceedings consist of 7 invited lectures and 64 contributed papers. The first section of this volume contains the invited lectures. The second section consists of the papers presented at the special session on energy production, a minisymposium organised by Hansjörg Wacker. Because of the wide range of topics no specific grouping has been applied to the rest of the papers. Within each section the papers appear in alphabetical order according to the first author.

We would like to thank the authors for their cooperation in editing this volume. Also the valuable work of the referees is gratefully acknowledged. We would also like to thank the conference secretary Mrs. Sinikka Vaskelainen and her staff at the Lahti Research and Training Centre for successful running of the local arrangements. Finally I express my sincere thanks to my secretary Ms. Tarja Nieminen at the Rolf Nevanlinna Institute for her enormous help in preparing this volume. Without her reliable and efficient work this book would still be a long way from print.

Lappeenranta, April 1991 Matti Heiliö

TABLE OF CONTENTS

INVITED LECTURES

Mini-symposium:
OPTIMAL SUPPLY AND DISTRIBUTION OF ENERGY

CONTRIBUTED PAPERS

INVITED LECTURES

SOME PDE AND STATISTICAL PROBLEMS FROM THE VLSI INDUSTRY

by

Ellis Cumberbatch

My talk consisted of descriptions of problems and results for three projects submitted to the Claremont Mathematics Clinic by local industry. They were (i) temperature estimation for Joule heating in current-carrying metal lines; (ii) an experimental design problem for the optimum choice of the width and spacing of metal lines; (iii) an asymptotic result for the resistance of current flowing into a small contact in a MOSFET source/drain region. I also described a number of results obtained in W. Fang's thesis at CGS concerning the inverse problem of the estimation of interfacial contact resistance and of the location/size/shape of the contact region from boundary measurements. Due to limitations on space, and the fact that the latter two topics will be published elsewhere, I shall write about (i) and (ii) only.

Heat Transfer in Transistor Lines

Transistors are getting smaller and more of them are being packed on a chip. The lines which carry current between devices are also getting narrower, thereby increasing current densities and the heat generated by current. A consequence of this is an increase in electromigration, the migration of ions of polycrystalline materials, [1], [2]. When there is differential migration

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sufficient to create a void across a wire, current flow is interrupted. Electromigration is found to be dependent on temperature gradients, and the goal of the Clinic project, [3], was to obtain temperature profiles in regions of large gradients, say where the line cross-sectional area changes rapidly, giving large current density changes.

Typical cross-sectional and plan-view geometries are shown in Figures 1,2. The metal line is sheathed in an electrical insulator (SiO_2) and is above an insulating pad on a silicon base.

The steady-state heat conduction equation applies, with a source term modeling the Joule heating effect. The latter is proportional to the product of the square of the current density and the material resistivity, which is taken linear in the temperature, [4]. Hence the field equations are

$$(1)\quad \Delta u_1 = -au_1 + b \text{ in the metal region,}$$

$$(2)\quad \Delta u_2 = 0 \text{ in the insulator,}$$

$$\text{where}\quad a = \frac{I^2 r_1}{K_{AL}\delta^2\omega^2}, \quad b = \frac{I^2 r_2}{K_{AL}\delta^2\omega^2}.$$

I is the current input, K_{AL} the thermal conductivity of aluminum, δ, ω the thickness and width of the rectangular line, and $r_1, r_2 > 0$ two resistivity constants. Typical dimensions are $\omega = 2$ microns, and δ and the insulator thicknesses fractions of microns.

Boundary conditions are taken as

$$
\begin{aligned}
(3) \qquad u &= u_0 \ \text{ on } \ \Gamma_1, \Gamma_3, \\
(4) \quad K_i \frac{\partial u}{\partial n} &= -h(u - u_0) \ \text{ on } \ \Gamma_2,
\end{aligned}
$$

where u_0 is the ambient air temperature, Γ_1, Γ_3 represent upstream and downstream locations, and Γ_2 is the insulator-air interface where a linear convective model is taken with coefficient h, see [4]. At the $A\ell - SiO_2$ interface the temperature and heat flux are continuous, giving

$$
(5) \qquad u_1 = u_2 \ , \quad K_{AL} \frac{\partial u_1}{\partial n} = K_i \frac{\partial u_2}{\partial n} \ \text{ on } \ \Gamma_4.
$$

It is clear that the complicated geometry inhibits analytic approaches, so a numerical investigation was begun. There are powerful 3-D elliptic solvers, but they are restrictive in the geometries and boundary conditions they can handle. ELLPACK provides a sub-routine for modifications necessary for a jump discontinuity in normal derivative, present on Γ_4 since $K_{A\ell} \neq K_i$, but the numerical precision was found poor, giving false output near corners of the domains. These difficulties forced a reduction to less complicated geometries. Three 2-D problems were solved: (I) in the cross-section plane, (II) in the plan-view plane where the effect of varying cross-section could be modeled, with convective boundary conditions taken on the sides of the region, and (III) a hybrid model with the same geometry as in (II) but a constant temperature sink taken on one side of the region to gauge the effect of the base. These solutions were obtained using PLTMG, a finite-element 2-D elliptic solver of good precision.

Interesting results for problems (II) and (III) were obtained for a wedge-shaped region of unit slope narrowing from a width of 103 to a rectangular region of width 3, see Figure 2. Of particular note are the appearance at high current densities of local maxima followed by an extremely fast decrease in temperature towards a local minimum inside the narrow rectangular region. The temperature then increases again. The minimum has a negative temperature which indicates meaningless results for the model. The threshold for the minimum is for currents around 0.4 amps for II, 1.0 amps for III (larger than operating currents). Speculation on the cause of this focused on the possibility of the appearance of multiple solutions. However no definite results were obtained in the short time available.

Experimental Design

The determination of the width of electrical lines is required for quality control in the VLSI industry. One method is to measure line resistance, the inverse of which is proportional to width. A test apparatus consists of a number of lines laid parallel to one another; the width, W, of each line, and the spacing, S, change from one section of the line to another. Current is passed along the lines and resistances of sections of the line measured. The model adopted to relate the difference between W_a, the design linewidth, and its measured value $W_{\alpha\beta}$, when it is located at a spacing S_β to its neighbor, is

$$W_{\alpha\beta} - W_\alpha = \Delta W + \frac{a}{W_\alpha} + \frac{b}{S_\beta} + \frac{c}{W_\alpha S_\beta} + \xi_\alpha \tag{6}$$

where ΔW is a constant called the over/under etch; a, b, c are process effect coefficients and ξ represents an error term. There are upper and lower bounds on W and S. The aim of this investigation is to determine optimum choices for W, S within these bounds for a set of tests used to obtain the constants $\Delta W, a, b, c$ by regression analysis.

Most Mathematics Clinics are year projects. Due to exceptional circumstances this investigation involved three different teams over three semesters. Their reports are [5], [6], [7]. This material is taken from [7].

In the standard form of regression analysis, (6) is written

$$y_i = \beta_0 + \frac{\beta_1}{x_{1i}} + \frac{\beta_2}{x_{2i}} + \frac{\beta_3}{x_{1i}x_{2i}} + \epsilon_i \tag{7}$$

where $i = 1, 2, \cdots, n$ indexes the number of experiments. Models are called linear if they are linear in the parameters $\beta_0, \cdots, \beta_3$. The experimental region is a rectangle in the $x_1 - x_2$ plane, and the design problem is to specify n-points in this rectangle in order that the measurements y_i yield estimates of the parameters under some optimality criteria.

The experimental design problem is usually referred to the linear model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_r x_{ri} + \epsilon_i \tag{8}$$

or, in vector form,

$$Y = X\beta + \epsilon \tag{9}$$

where X is an $n \times (r+1)$ design matrix, and ϵ is the error vector assumed to be of zero mean, uncorrelated and to have uniform variance and normal distribution.

Using the least squares method, the maximum likelihood method, or searching for the best unbiased linear estimater, provides the same estimator for β, viz

$$\hat{\beta} = [X^T X]^{-1} X^T Y \tag{10}$$

so that

$$\hat{Y} = HY \quad \text{where} \quad H = X[X^T X]^{-1} X^T \tag{11}$$

Given a design region R, a design X is thought of as a probability distribution over R. If X gives weight $\frac{1}{n}$ to each of n not necessarily distinct points in R then X is called an exact design, since it can be realized in practice exactly.

There is considerable literature on the general experimental design problem, see [8] for a recent review. Most optimality criteria are based on minimizing some form of "variance" associated with $\hat{\beta}$ or $\hat{Y}$. The designs are said to be A, D, E, G or V optimal depending on the criterion chosen. The Clinic concentrated on: