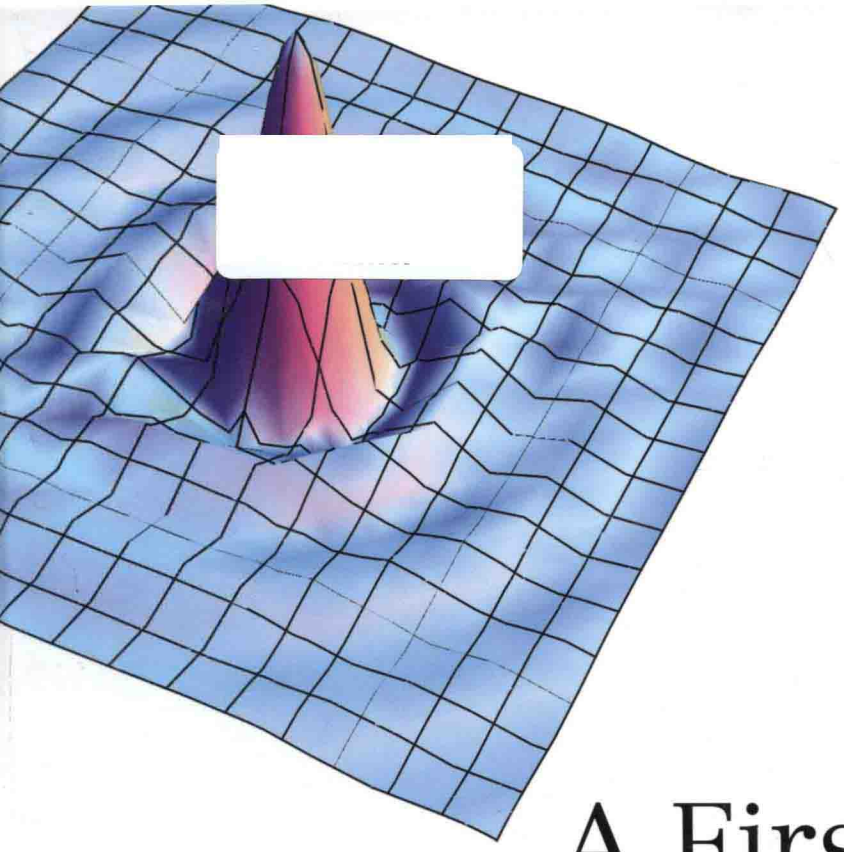

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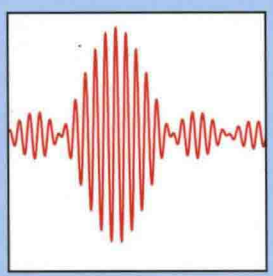

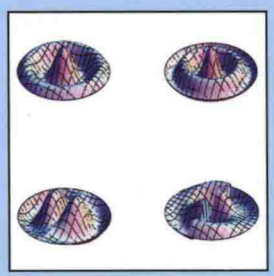

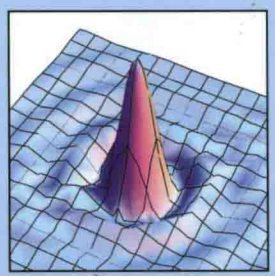

Mohammad Samiullah

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Truman State University

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A FIRST COURSE IN VIBRATIONS AND WAVES

This book is dedicated to the memory of my sister Mrs. Rehana Sultana Ilyas whose wit and warmth brightened the lives she touched.

Preface

The study of vibrations and waves is central to physics and engineering. A course dealing with vibrations and waves often forms a bridge course between introductory physics courses and more advanced physics and engineering courses. This book is written for such a course, which I have taught regularly for a number of years at Truman. The book emphasizes the understanding of physics based on the fundamental principles expressed through mathematical equations.

While teaching the Vibs and Waves course I noticed that the mathematical maturity of students assumed by the existing textbooks was not realistic. Two textbooks that I tried in the course posed undue mathematical difficulties to students, even though students had already completed the Calculus sequence. To aid students I began to write subsidiary modules which they found to be very helpful. Some of them even relied on them completely rather than on the textbook. These modules have evolved into the present book. To make the book more complete and more useful to a broader audience I have expanded it by including some topics which I normally do not cover in my course due to a lack of time in a one-semester course.

The book follows the standard logical progression from simple harmonic motion to waves. It is organized into three parts. Part I contains a preliminary chapter that serves as a review of relevant ideas of mechanics and complex numbers. Although this chapter is just a review of the basics, I have found that most students need a review, especially, to learn or re-learn the language of equations of motion and exponential complex notation.

Part II is devoted to a detailed discussion of vibrations of mechanical systems. Chapters 2–4 are devoted to free harmonic motions in increasingly complicated systems. In Chapter 2, I present various systems which can be approximated by simple harmonic motion. They include the classic mass/spring system, the plane pendulum, the physical pendulum, fluids, and electric circuits. Both undamped and damped motion of free simple harmonic oscillators are presented here. In Chapter 3, we study the motion of two coupled oscillators. Here we learn various techniques for obtaining normal modes and the use of normal coordinates for understanding arbitrary motion. Chapter 4 extends the treatment of two coupled oscillators to N-bodies and continuous systems. The Fourier series is introduced in Chapter 4 along with expansion in the normal modes of a continuous system. Chapter 5 is devoted to driven oscillators. In this chapter we study driven oscillations and resonance in systems with one degree of freedom as well as many degrees of freedoms. The discussions of Chapter 5 are limited to closed systems in which the driven system occupies a finite space so that driving the system does not lead to the propagation of waves but rather to the excitement of standing waves.

Part III is concerned with waves. Here, the emphasis is on the discussion of common aspects of all types of waves. The basic language of traveling waves is more easily visualized in a one-dimensional situation of a transverse wave on a taut string. In Chapter 6 we make use of this example to study fundamental aspects of waves. Although the wave on a string is easy to visualize and serves to introduce the idea of waves well, the one-dimensional nature of the string limits its scope. In Chapter 7 we study waves in three dimensions, where we also introduce the fundamental nature of sound, electromagnetic, and matter waves.

We also learn about polarization of waves. In Chapter 8 we address the issue of the reaction of a wave with the boundary between different media. The reflection and transmission of waves is illustrated for waves on a string as well as the more complicated case of electromagnetic waves. Chapter 9 contains the important topic of interference of waves with applications in interferometry. The book concludes with a chapter on diffraction, where the application to diffraction gratings is presented.

The book includes many examples to illustrate main ideas. The exercises at the end of chapters are integral to the text. There are some simple exercises that help clarify concepts or familiarize students with important formulas and some challenging ones that help explore ideas further. Many exercises require the student to think of simplifying aspects and use analogies to solve them. The solutions to exercises are included in the Appendix. However, to get the full benefit of the exercises, a student should first try the exercises before consulting the solution.

Preliminary versions of this book were used as a textbook for my course and I would like to thank numerous students who have given feedback that has improved the book. My wife Huping deserves special thanks for encouraging me to convert my notes into a useful textbook and for giving me considerable free time to pursue this project.

M. Samiullah

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