ELEMENTS OF INFORMATION THEORY SECOND EDITION

THOMAS M.COVER
JOY A. THOMAS

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PREFACE TO THE SECOND EDITION

In the years since the publication of the first edition, there were many aspects of the book that we wished to improve, to rearrange, or to expand, but the constraints of reprinting would not allow us to make those changes between printings. In the new edition, we now get a chance to make some of these changes, to add problems, and to discuss some topics that we had omitted from the first edition.

The key changes include a reorganization of the chapters to make the book easier to teach, and the addition of more than two hundred new problems. We have added material on universal portfolios, universal source coding, Gaussian feedback capacity, network information theory, and developed the duality of data compression and channel capacity. A new chapter has been added and many proofs have been simplified. We have also updated the references and historical notes.

The material in this book can be taught in a two-quarter sequence. The first quarter might cover Chapters 1 to 9, which includes the asymptotic equipartition property, data compression, and channel capacity, culminating in the capacity of the Gaussian channel. The second quarter could cover the remaining chapters, including rate distortion, the method of types, Kolmogorov complexity, network information theory, universal source coding, and portfolio theory. If only one semester is available, we would add rate distortion and a single lecture each on Kolmogorov complexity and network information theory to the first semester. A web site, http://www.elementsofinformationtheory.com, provides links to additional material and solutions to selected problems.

In the years since the first edition of the book, information theory celebrated its 50th birthday (the 50th anniversary of Shannon's original paper that started the field), and ideas from information theory have been applied to many problems of science and technology, including bioinformatics, web search, wireless communication, video compression, and others. The list of applications is endless, but it is the elegance of the fundamental mathematics that is still the key attraction of this area. We hope that this book will give some insight into why we believe that this is one of the most interesting areas at the intersection of mathematics, physics, statistics, and engineering.

TOM COVER
JOY THOMAS

Palo Alto, California
January 2006

PREFACE TO THE FIRST EDITION

This is intended to be a simple and accessible book on information theory. As Einstein said, "*Everything should be made as simple as possible, but no simpler.*" Although we have not verified the quote (first found in a fortune cookie), this point of view drives our development throughout the book. There are a few key ideas and techniques that, when mastered, make the subject appear simple and provide great intuition on new questions.

This book has arisen from over ten years of lectures in a two-quarter sequence of a senior and first-year graduate-level course in information theory, and is intended as an introduction to information theory for students of communication theory, computer science, and statistics.

There are two points to be made about the simplicities inherent in information theory. First, certain quantities like entropy and mutual information arise as the answers to fundamental questions. For example, entropy is the minimum descriptive complexity of a random variable, and mutual information is the communication rate in the presence of noise. Also, as we shall point out, mutual information corresponds to the increase in the doubling rate of wealth given side information. Second, the answers to information theoretic questions have a natural algebraic structure. For example, there is a chain rule for entropies, and entropy and mutual information are related. Thus the answers to problems in data compression and communication admit extensive interpretation. We all know the feeling that follows when one investigates a problem, goes through a large amount of algebra, and finally investigates the answer to find that the entire problem is illuminated not by the analysis but by the inspection of the answer. Perhaps the outstanding examples of this in physics are Newton's laws and Schrödinger's wave equation. Who could have foreseen the awesome philosophical interpretations of Schrödinger's wave equation?

In the text we often investigate properties of the answer before we look at the question. For example, in Chapter 2, we define entropy, relative entropy, and mutual information and study the relationships and a few interpretations of them, showing how the answers fit together in various ways. Along the way we speculate on the meaning of the second law of thermodynamics. Does entropy always increase? The answer is yes and no. This is the sort of result that should please experts in the area but might be overlooked as standard by the novice.

In fact, that brings up a point that often occurs in teaching. It is fun to find new proofs or slightly new results that no one else knows. When one presents these ideas along with the established material in class, the response is "sure, sure, sure." But the excitement of teaching the material is greatly enhanced. Thus we have derived great pleasure from investigating a number of new ideas in this textbook.

Examples of some of the new material in this text include the chapter on the relationship of information theory to gambling, the work on the universality of the second law of thermodynamics in the context of Markov chains, the joint typicality proofs of the channel capacity theorem, the competitive optimality of Huffman codes, and the proof of Burg's theorem on maximum entropy spectral density estimation. Also, the chapter on Kolmogorov complexity has no counterpart in other information theory texts. We have also taken delight in relating Fisher information, mutual information, the central limit theorem, and the Brunn–Minkowski and entropy power inequalities. To our surprise, many of the classical results on determinant inequalities are most easily proved using information theoretic inequalities.

Even though the field of information theory has grown considerably since Shannon's original paper, we have strived to emphasize its coherence. While it is clear that Shannon was motivated by problems in communication theory when he developed information theory, we treat information theory as a field of its own with applications to communication theory and statistics. We were drawn to the field of information theory from backgrounds in communication theory, probability theory, and statistics, because of the apparent impossibility of capturing the intangible concept of information.

Since most of the results in the book are given as theorems and proofs, we expect the elegance of the results to speak for themselves. In many cases we actually describe the properties of the solutions before the problems. Again, the properties are interesting in themselves and provide a natural rhythm for the proofs that follow.

One innovation in the presentation is our use of long chains of inequalities with no intervening text followed immediately by the explanations. By the time the reader comes to many of these proofs, we expect that he or she will be able to follow most of these steps without any explanation and will be able to pick out the needed explanations. These chains of inequalities serve as pop quizzes in which the reader can be reassured of having the knowledge needed to prove some important theorems. The natural flow of these proofs is so compelling that it prompted us to flout one of the cardinal rules of technical writing; and the absence of verbiage makes the logical necessity of the ideas evident and the key ideas perspicuous. We hope that by the end of the book the reader will share our appreciation of the elegance, simplicity, and naturalness of information theory.

Throughout the book we use the method of weakly typical sequences, which has its origins in Shannon's original 1948 work but was formally developed in the early 1970s. The key idea here is the asymptotic equipartition property, which can be roughly paraphrased as "Almost everything is almost equally probable."

Chapter 2 includes the basic algebraic relationships of entropy, relative entropy, and mutual information. The asymptotic equipartition property (AEP) is given central prominence in Chapter 3. This leads us to discuss the entropy rates of stochastic processes and data compression in Chapters 4 and 5. A gambling sojourn is taken in Chapter 6, where the duality of data compression and the growth rate of wealth is developed.

The sensational success of Kolmogorov complexity as an intellectual foundation for information theory is explored in Chapter 14. Here we replace the goal of finding a description that is good on the average with the goal of finding the universally shortest description. There is indeed a universal notion of the descriptive complexity of an object. Here also the wonderful number Ω is investigated. This number, which is the binary expansion of the probability that a Turing machine will halt, reveals many of the secrets of mathematics.

Channel capacity is established in Chapter 7. The necessary material on differential entropy is developed in Chapter 8, laying the groundwork for the extension of previous capacity theorems to continuous noise channels. The capacity of the fundamental Gaussian channel is investigated in Chapter 9.

The relationship between information theory and statistics, first studied by Kullback in the early 1950s and relatively neglected since, is developed in Chapter 11. Rate distortion theory requires a little more background than its noiseless data compression counterpart, which accounts for its placement as late as Chapter 10 in the text.

The huge subject of network information theory, which is the study of the simultaneously achievable flows of information in the presence of noise and interference, is developed in Chapter 15. Many new ideas come into play in network information theory. The primary new ingredients are interference and feedback. Chapter 16 considers the stock market, which is the generalization of the gambling processes considered in Chapter 6, and shows again the close correspondence of information theory and gambling.

Chapter 17, on inequalities in information theory, gives us a chance to recapitulate the interesting inequalities strewn throughout the book, put them in a new framework, and then add some interesting new inequalities on the entropy rates of randomly drawn subsets. The beautiful relationship of the Brunn–Minkowski inequality for volumes of set sums, the entropy power inequality for the effective variance of the sum of independent random variables, and the Fisher information inequalities are made explicit here.

We have made an attempt to keep the theory at a consistent level. The mathematical level is a reasonably high one, probably the senior or first-year graduate level, with a background of at least one good semester course in probability and a solid background in mathematics. We have, however, been able to avoid the use of measure theory. Measure theory comes up only briefly in the proof of the AEP for ergodic processes in Chapter 16. This fits in with our belief that the fundamentals of information theory are orthogonal to the techniques required to bring them to their full generalization.

The essential vitamins are contained in Chapters 2, 3, 4, 5, 7, 8, 9, 11, 10, and 15. This subset of chapters can be read without essential reference to the others and makes a good core of understanding. In our opinion, Chapter 14 on Kolmogorov complexity is also essential for a deep understanding of information theory. The rest, ranging from gambling to inequalities, is part of the terrain illuminated by this coherent and beautiful subject.

Every course has its first lecture, in which a sneak preview and overview of ideas is presented. Chapter 1 plays this role.

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Palo Alto, California
June 1990

ACKNOWLEDGMENTS FOR THE SECOND EDITION

Since the appearance of the first edition, we have been fortunate to receive feedback, suggestions, and corrections from a large number of readers. It would be impossible to thank everyone who has helped us in our efforts, but we would like to list some of them. In particular, we would like to thank all the faculty who taught courses based on this book and the students who took those courses; it is through them that we learned to look at the same material from a different perspective.

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