Mechanical Science IV

M. A. Rix

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M. A. Rix ACP CEng MIMechE was formerly Senior
Lecturer in Mechanical Engineering at Richmond
upon Thames College.

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Preface

This book is written to cover the objectives of the Business and Technician Education Council's standard unit U82/041, but it could also be of use to students on many courses covering similar topics. The relevant sections of the standard unit covered by each chapter are indicated in the contents pages. Although Chapter 1 contains a large amount of material normally covered by a student at Level III it is included for the benefit of students who enter the Higher BTEC courses from other courses of study.

A series of self-assessment questions are included at the end of each chapter. These refer to the theory and they will have greater value if, initially, they are attempted without reference back to the text. Reasoned answers, where appropriate, are given at the end of the book and large number of unworked examples are included at the end of each chapter. These are designed to be of a standard required for phase or end tests.

In connection with Chapter 6, where graphical solutions to examples are required, such solutions are printed to a much smaller scale than would be required to ensure reasonable accuracy. In these cases guidance is given with regard to the minimum actual scale which is recommended for accurate solutions.

The abbreviations and symbols for units, and the units themselves, are as far as possible those recommended by the BSI and Business and Technician Education Council.

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1 Loading, Shear Force and Bending Moment Diagrams

1.1 Introduction

The term 'beam' is normally associated with structures, although machine components, pipes carrying fluids etc., can often act as beams and they must be designed accordingly. The theory concerned with this design, *bending theory*, depends for its application, upon a knowledge of the maximum bending moment as well as the beam dimensions.

1.2 The diagrams

Loading diagram. This is a sketch or a scaled drawing of the beam indicating the nature and the positions of the supports and the loads. It is usually drawn prior to the other two diagrams.

Shear force diagram. This is a sketch or a scaled drawing showing the variation in the shear force from section to section of the beam due to the particular loading.

Bending moment diagram. Since the bending moment varies according to the positions and types of load, their combined effect is portrayed by a scaled drawing or sketch called a bending moment diagram.

1.3 Bending moments, shear force definitions and sign convention

Bending moment. This is the total moment, at any section of a beam, due to all the forces *either* to the right of the section *or* to the left of the section.

Shear force. This is the algebraic sum of all the forces *either* to the right of the section *or* to the left of the section.

Sign convention. A bending moment may have a clockwise or an anticlockwise sense and a shear force may be in one of two directions, so it is necessary to adopt a consistent method of distinguishing between them. The convention which is to be used is illustrated in Fig. 1.1. It should be noted that other conventions, if consistently applied, are equally valid.

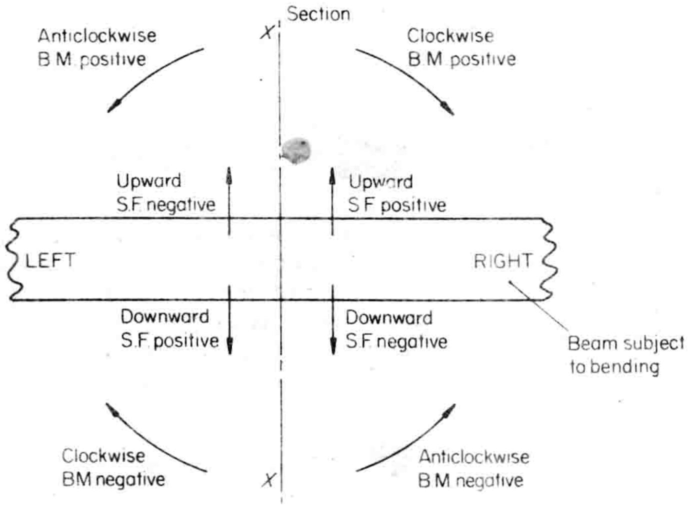


Fig. 1.1 Bending Moment (B.M.) and Shear Force (S.F.) sign convention

At any section of a beam, such as XX' in Fig. 1.1, the bending moment and the shearing force can be calculated by considering all the forces to the *right* of the section and ignoring those to the *left*. Then:

a *clockwise* bending moment is *positive*
an *upward* shear force is *positive*.

If the forces to the left of the section are used, and those to the right are ignored, then:

an *anticlockwise* bending moment is *positive*
a *downward* shear force is *positive*.

1.4 Diagram construction; simple cases

In the examples which follow, the various diagrams are sketched only approximately to scale as any significant values are calculated for accuracy. Loads in newtons are preferred to masses in kilograms since it is the gravity force on the mass which gives rise to the bending moment and shear force.

Example 1.1
A simply supported beam 0.8 m long carries a concentrated central load of 3 kN. Draw the load diagram and sketch the bending moment and shear force diagrams. Determine:

(a) the bending moment and shear force at a section 0.2 m from the right-hand support;
(b) the value and position of the maximum bending moment.

Load Diagram, Fig. 1.2 (a). The load is central so the support reactions are each equal to half the load.

$$\therefore \quad R_1 = R_2 = 1.5 \text{ kN}.$$

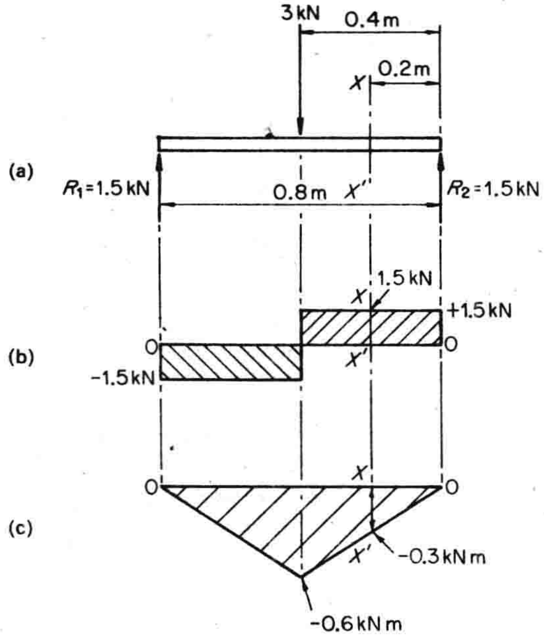


Fig. 1.2 (a) Load diagram
(b) Shear force diagram
(c) Bending moment diagram

Shear force diagram, Fig. 1.2 (b). Working from the right-hand end (RHE), the shear force (SF) at the RHE changes from 0 to +1.5 kN due to the support reaction.

SF at section XX' = +1.5 kN due to the forces to the right of XX'.

Considering the forces to the left of section XX',

$$\text{SF} = +3 \text{ kN} - 1.5 \text{ kN} = +1.5 \text{ kN}.$$

The SF, therefore remains constant from the RHE since there are no other forces between section XX' and the RHE.

SF at the centre = +1.5 kN, changing abruptly to −1.5 kN due to the 3 kN concentrated load at this point.

The shear force will then remain constant at −1.5 kN up to the left-hand support when it changes abruptly to zero due to the support reaction.

Bending moment diagram, Fig. 1.2 (c). The BM (bending moment) at the RHE = 0 since there are no forces to the right of this. If the forces to the left are considered then:

BM at RHE = $+(3 \times 0.4) - (1.5 \times 0.8) = 0$ as expected.

Considering the forces to the right of XX',

$$\text{BM at section } XX' = -1.5 \times 0.2 = -0.3 \text{ kN m}.$$

Considering forces to the left of XX',

$$\text{BM at section } XX' = (3 \times 0.2) - (1.5 \times 0.6) = -0.3 \text{ kN m}.$$

Considering either the forces to the right or to the left of the centre,

$$\text{BM at the centre} = -(1.5 \times 0.4) = -0.6 \text{ kN m}.$$

The bending moment is obviously proportional to the distance from either end and it is a maximum under the centre load.

From this simple example it should be apparent that:

(i) between concentrated loads or forces the bending moment increases uniformly and the bending moment diagram is a sloping straight line;

(ii) between concentrated loads or forces, the shearing force remains constant but changes abruptly at those loads or forces;

(iii) the bending moment has a maximum value where the shear force changes from + to − or passes through zero.

The next example illustrates the differences in both the BM and the SF diagrams when the loading is evenly distributed.

Example 1.2

A beam is simply supported at its ends over a span of 0.8 m and carries a distributed load of 3 kN/m. Draw the load diagram and sketch the bending moment and shear force diagrams. Determine and indicate the value of the bending moment and shear force at:

(a) a section XX' 0.2 m from the right-hand end;

(b) at the centre of the beam.

Load diagram, Fig 1.3 (a). Since the load is evenly distributed it will be shared equally between the supports.

$$\text{Total load} = 0.8 \times 3 = 2.4 \text{ kN}$$

$$\therefore \quad R_1 = R_2 = \frac{2.4}{2} = 1.2 \text{ kN}.$$

Shear force diagram, Fig 1.3 (b). Working from the RHE:

SF at RHE = 0 changing abruptly to + 1.2 kN due to the reaction R_2

Considering the load to the right of XX'

$$\text{SF at } XX' = +1.2 - (3 \times 0.2) = 0.6 \text{ kN}.$$

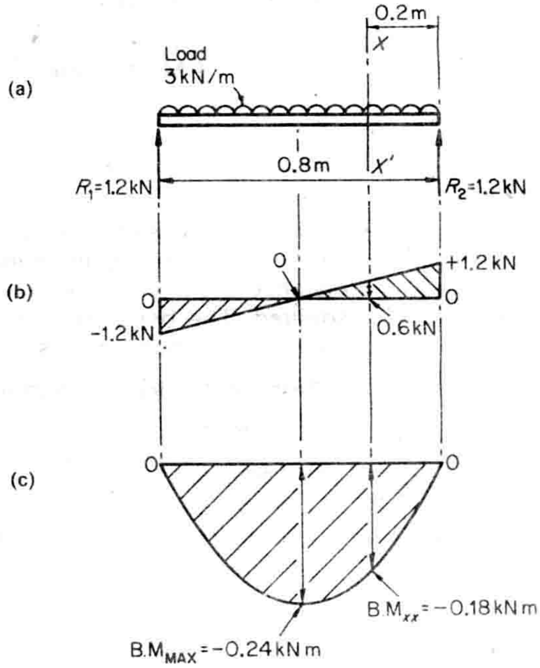


Fig. 1.3 (a) Load diagram
(b) Shear force diagram
(c) Bending moment diagram

Considering the forces to the left of XX',

$$\text{SF at } XX' = -1.2 + (3 \times 0.6) = 0.6 \text{ kN.}$$

$$\text{SF at centre} = +1.2 - (3 \times 0.4) \text{ or } -1.2 + (3 \times 0.4) = 0$$

$$\text{SF at LHE} = -1.2 \text{ kN, changing abruptly to zero.}$$

Bending moment diagram, Fig. 1.3 (c). Working from the RHE:

BM at RHE = 0 since there are no forces to the right of R_2.

A similar result is obtained by considering the forces to the left of R_2.

$$\therefore \quad \text{BM at RHE} = -(1.2 \times 0.8) + \left(\frac{3 \times 0.8^2}{2}\right) = -0.96 + 0.96 = 0$$

Considering the forces to the right of XX',

$$\text{BM at } XX' = -(1.2 \times 0.2) + \left(\frac{3 \times 0.2^2}{2}\right) = -0.24 + 0.06 = -0.18 \text{ kN m.}$$

Considering the forces to the left of XX',

$$\text{BM at } XX' = -(1.2 \times 0.6) + \left(\frac{3 \times 0.6^2}{2}\right) = -0.72 + 0.54 = -0.18 \text{ kN m.}$$

Considering the forces to the right of centre,

$$\text{BM at centre} = -(1.2 \times 0.4) + \left(\frac{3 \times 0.4^2}{2}\right)$$
$$= -0.48 + 0.24$$
$$= -0.24 \text{ kN m.}$$

A similar result is obtained by considering the forces to the left of centre. The bending moment no longer varies linearly with the distance from either end but in proportion to the distance squared. The bending moment diagram is thus sketched as a curve between the points.

From this second example it is possible to conclude the following points.

(i) The shear force for an evenly distributed load varies directly with distance from one end and the diagram is thus a sloping straight line. With end supports this line passes through zero at the centre.

(ii) The bending moment for an evenly distributed load varies with the square of the distance from one end and the bending moment diagram is a curved line.

(iii) The bending moment is a maximum at the centre of the simply supported beam where, more significantly, the shear force passes through zero i.e. it changes sign.

1.5 Standard cases

Examples 1.1 and 1.2 used specific loads and beam dimensions. However, as these types of loading are commonly encountered it would be far more useful to develop the solutions in general terms. They then become known as *standard cases*. Five such standard cases are summarised in Table 1.1.

It is worthwhile noting from Table 1.1 of standard cases that both the shear force and bending moment can be expressed in terms of the load (w or W), the length of the beam (l) and the distance of a beam section from one end (x). The ability to write the expressions in this way, particularly the bending moment, becomes very important later when calculating the deflections caused by different loads.

Table 1.1.

Bending moment and shear force diagrams: standard cases.

(a) Simply supported beam; centre load

(i) *Load diagram*

$$R_1 = R_2 = \frac{W}{2}$$

(ii) *Shear force diagram*

$$\text{SF}_{\text{MAX}} = \mp\frac{W}{2}$$

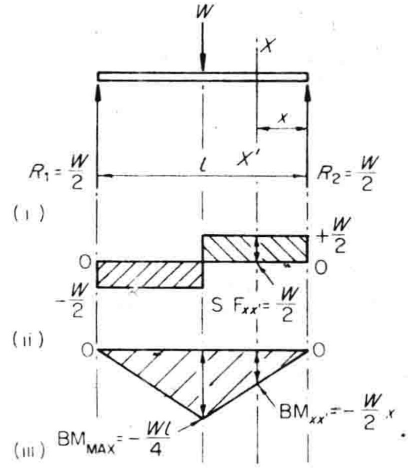
$$\text{SF}_{XX'} = \mp\frac{W}{2}$$

(iii) *Bending moment diagram*

$$\text{BM}_{\text{MAX}} \text{ (at the centre)} = -\frac{W}{2} \times \frac{l}{2} = -\frac{Wl}{4}$$

$$\text{BM}_{XX'} = -\frac{Wx}{2}$$

N.B. In this case when $x > \frac{l}{2}$ the BM is calculated from the opposite end of the beam.



(b) Simply supported beam; distributed load

(i) *Load diagram*

$$R_1 = R_2 = \frac{wl}{2}$$

(ii) *Shear force diagram*

$$\text{SF}_{XX} = +wl - \frac{wx}{2}$$

$$\text{SF}_{\text{MAX}} = \mp\frac{wl}{2}$$

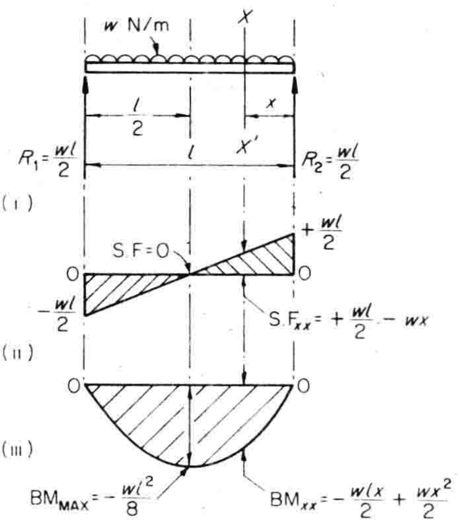
(iii) *Bending moment diagram*

$$\text{BM}_{XX} = -\frac{wlx}{2} + \frac{wx^2}{2}$$

$$\text{BM}_{\text{MAX}} = -\frac{wl^2}{4} + \frac{w}{2}\left(\frac{l}{2}\right)^2$$

$$= -\frac{wl^2}{8} \text{ when } x = l/2$$

$$-\frac{Wl}{8} \text{ if } wl = W.$$



(c) Simply supported beam; concentrated offset load

(i) *Load diagram*
Taking moments about R_1:

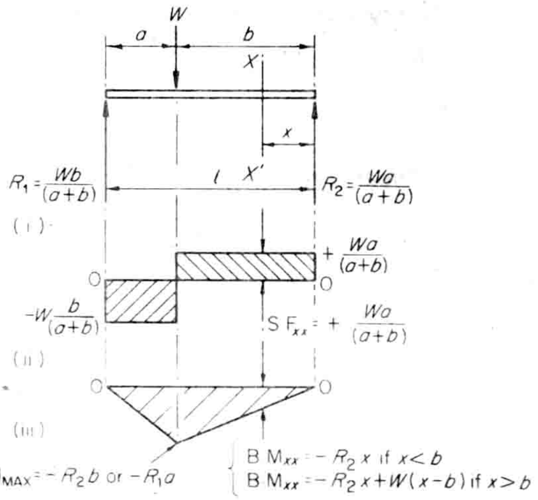
$$Wa = R_2(a+b)$$

$$R_2 = W\frac{a}{(a+b)}$$

Taking moments about R_2,

$$Wb = R_1(a+b)$$

$$R_1 = W\frac{b}{(a+b)}$$



(ii) *Shear force diagram*
Working from the RHE.
For values of $x < b$:

$$SF_{XX} = +R_2 \text{ or } +W\frac{a}{(a+b)}$$

For values of $x > b$:

$$SF_{XX} = -R_1 \text{ or } -W\frac{b}{(a+b)}$$

(iii) *Bending moment diagram*
Working from the RHE.
For values of $x < b$:

$$BM_{XX} = -R_2x$$

For values of $x > b$:

$$BM_{XX} = -R_2x + W(x-b)$$

BM_{MAX} occurs at the load

$$= -R_2b \text{ or } -R_1a.$$

(d) Cantilever; single concentrated end-load

(i) *Load diagram*

(ii) *Shear force diagram*

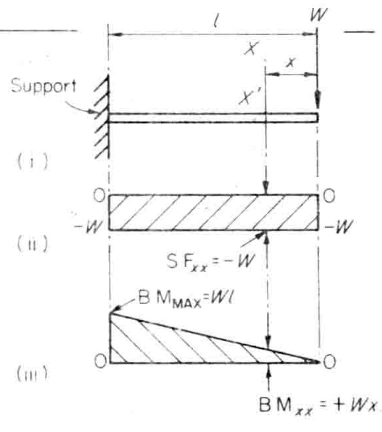
$$SF_{XX} = -W \text{ (Constant)}$$

(iii) *Bending moment diagram*

$$BM_{XX} = +Wx$$

$$BM_{MAX} = +Wl$$

at the support when $x = l$.



(e) Cantilever with distributed load

(i) *Load diagram*

(ii) *Shear force diagram*

$$SF_{XX} = -wx$$

$$SF_{MAX} = -wl$$

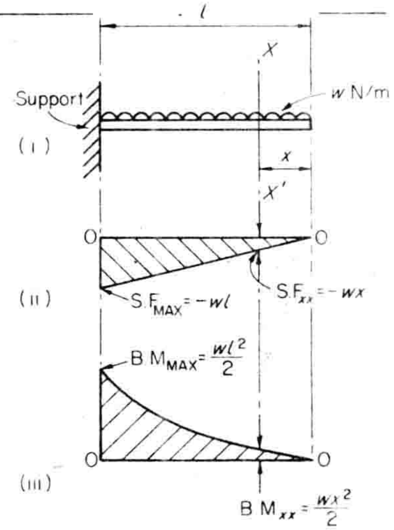
(at the support)

(iii) *Bending moment diagram*

$$BM_{XX} = +\frac{wx^2}{2}$$

$$BM_{MAX} = \frac{wl^2}{2} \text{ or } \frac{Wl}{2}$$

where $W = wl$, and occurs at the support.



1.6 General cases; mixed loads

In practice, beam loadings will not always consist of a single concentrated or distributed load. The weight of the beam itself may constitute the distributed load, to which may be added concentrated loads at various points. To determine the bending moment and the shear force distributions in such cases a more general treatment is necessary.

Example 1.3 Two concentrated loads

A horizontal shaft is simply supported (in spherical bearings) over a span of 0.5 m, and carries two gear wheels of weight 200 N and 400 N at distances of 0.1 m and 0.3 m from the right-hand end respectively. Ignoring the weight of the shaft, determine:

(a) the shear force at each load;

(b) the bending moment at each load;

(c) the maximum bending moment, and where it occurs.

Sketch, approximately, the *load, shear force* and *bending moment* diagrams.

Load diagram, Fig. 1.4(a). Taking moments about R_2,

$$0.5R_1 = 0.1 \times 200 + 0.3 \times 400$$

$$R_1 = \frac{20 + 120}{0.5} = 280 \text{ N}$$

$$R_2 = (400 + 200) - 280 = 320 \text{ N}$$

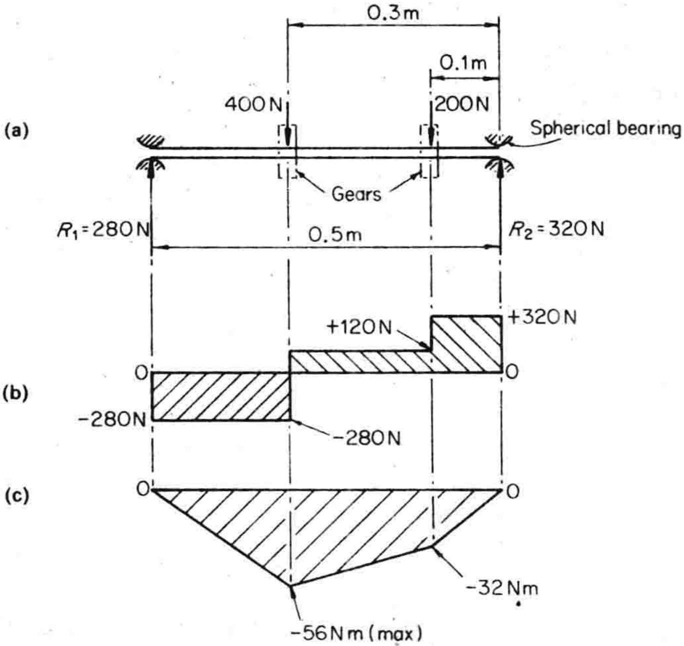


Fig. 1.4 (a) Load diagram
(b) Shear force diagram
(c) Bending moment diagram

Shear force diagram, Fig. 1.4(b). At the RHE, where the reaction R_2 occurs, the shear force changes abruptly from 0 to +320 N. It is then constant up to the 200 N downward force where it will change abruptly to +120 N, i.e. +320 − 200. The shear force is once again constant up to the 400 N load, where it changes abruptly to −280 N i.e. +120 − 400. It then remains constant as far as the left-hand support where a further abrupt change to 0 N occurs i.e. −280 + 280.

Note that the shear force passes through zero at the section where the 400 N load is applied.

Bending moment diagram, Fig. 1.4(c). Working from the RHE, at R_2 the BM = 0. At the 200 N load section the BM due to the forces to the right of this section

$$= -R_2 \times 0.1$$
$$= -320 \times 0.1$$
$$= -32 \text{ N m.}$$

Since all the loads are concentrated, the change in bending moment between the two points on the diagram is uniform. At the 400 N load section, considering the forces to the right of this section,

$$\text{BM} = -R_2 \times 0.3 + 200\,(0.3 - 0.1)$$
$$= -320 \times 0.3 + 200 \times 0.2$$
$$= -56 \text{ N m.}$$

Once again, because the loads are concentrated, the change from −32 N m to −56 N m is uniform as shown on the diagram. At the left-hand support, considering all the forces to the right,

$$\begin{aligned} BM &= -R_2 \times 0.5 + 200(0.5-0.1) + 400(0.5-0.3) \\ &= -320 \times 0.5 + 200 \times 0.4 + 400 \times 0.2 \\ &= 0. \end{aligned}$$

From -56 N m to the left-hand support the bending moment changes uniformly to zero.

The maximum bending moment is thus -56 N m and it occurs at a section under the 400 N load. This is also the section at which the shear force *passes through zero*. As suggested by the standard cases there is, apparently, a connection between the section where the maximum bending moment occurs and the section at which the shear force passes through zero (i.e. changes sign).

The next example illustrates the effect of adding to the concentrated loads an evenly distributed load representing the weight of the shaft.

Example 1.4 Mixed loads

If the shaft in Example 1.3 was of uniform cross-section and had a weight of 100 N determine once again:

(a) the shear force at each load;

(b) the bending moment at each load;

(c) the maximum bending moment and where it occurs.

Sketch the approximate load, shear force and bending moment diagrams.

Load diagram, Fig. 1.5(a). The shaft weight is evenly distributed and it is equivalent to $\frac{100}{0.5}$ or 200 N/m. Taking moments about R_2:

$$\begin{aligned} 0.5R_1 &= 0.1 \times 200 + 0.3 \times 400 + \frac{(200 \times 0.5^2)}{2} \\ R_1 &= \frac{20 + 120 + 25}{0.5} = 330 \text{ N}. \\ R_2 &= (400 + 200 + 100) - 330 = 370 \text{ N}. \end{aligned}$$

This, as is to be expected, shows that half the shaft weight is added to each reaction as calculated in Example 1.3.

Shear force diagram, Fig. 1.5(b). At R_2 the shear force changes abruptly from 0 to $+370$ N. Between R_2 and the 200 N load it will decrease uniformly by 200 N/m, due to the distributed load (i.e. the weight of the shaft). At the 200 N load, therefore,

$$SF = +370 - (0.1 \times 200) = 350 \text{ N}.$$

At this same section the shear force then changes abruptly to 350 $-$ 200 or 150 N due to the effect of the 200 N load. Between the 200 N and 400 N loads the same uniform decrease due to the distributed load will occur so that at the 400 N load,

$$SF = 150 - (0.2 \times 200) = 110 \text{ N}.$$

The abrupt change due to the 400 N load itself causes the shear force to then become 110 − 400 N or −290 N at the same section. At the left-hand support,

$$SF = -290 + (0.2 \times 200)$$
$$= -330\ \text{N}.$$

This then changes abruptly to zero due to the effect of the reaction R_1.

Bending moment diagram, Fig. 1.5(c). Working from the RHE, at R_2 the BM = 0 (No forces to the right of R_2). At the 200 N load section, due to the forces to the right of the section,

$$BM = -R_2 \times 0.1 + \left(200 \times 0.1 \times \frac{0.1}{2}\right)$$
$$= -370 \times 0.1 + \left(200 \times \frac{0.1^2}{2}\right)$$
$$= -36\ \text{N m}.$$

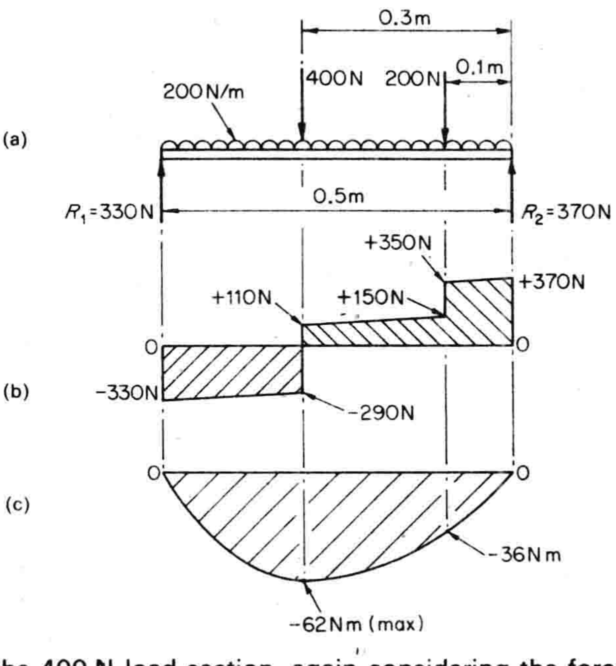


Fig. 1.5 (a) Load diagram
(b) Shear force diagram
(c) Bending moment diagram

At the 400 N load section, again considering the forces to the right of the section,

$$BM = -370 \times 0.3 + 200(0.3 - 0.1) + 200 \times \frac{0.3^2}{2}$$
$$= -111 + 40 + 9 = -62\ \text{N m}.$$

At the left-hand support, the BM should be zero since there are no forces to its left. However, as a check consider the forces to the right: