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Calculus

WITH ANALYTIC GEOMETRY AND LINEAR ALGEBRA

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Textbooks in Mathematics

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Preface

This book presents the material for a first course in calculus and analytic geometry, together with an introduction to linear algebra, and is optimally covered in three semesters. It is written on a level somewhat above that found in the average text on elementary calculus, for much valuable time can be saved if the student's first course in calculus is sufficiently rigorous so that no serious backtracking will be necessary when he studies advanced calculus. In this connection, the current Committee on the Undergraduate Program in Mathematics says pointedly: "The basic concepts should be introduced in the same spirit in which they are used by working mathematicians, and proofs ought to have the same clarity and elegance which distinguishes all first rate mathematics." I have tried to keep these views constantly in mind while writing this book.

But rigor alone is not enough! For what is rigor in mathematics other than accuracy and precision in the formulation of new definitions and concepts, and correctness in the proof of theorems? The formal presentation of each new concept in this book is always preceded by an intuitive discussion, and the formal proofs are generally preceded by heuristic ones, much in the fashion in which one lectures.

This text was planned to be used along two different tracks, according to the aptitudes and interests of the student and the breadth of the curriculum. Track I is the principal track for this book, and corresponds to the standard introductory calculus course taken by the majority of freshmen and sophomores in our colleges and universities today. It is intended for those students who need a thorough familiarity with the calculus as a scientific tool, and for those mathematics majors who do not follow an honors program. The student is led to an intimate acquaintance with all the leading concepts and ideas of the calculus. Proofs are given for all except a few of the most abstract theorems, and the course on this level is designed for most students majoring in physics, chemistry, or any of the engineering disciplines, as well as for a portion of those majoring in mathematics.

Track II is the honors course. It is designed for those students who seek an early mastery of the concepts and ideas of the calculus as a stage toward more advanced work in mathematics and related fields. Here, ideas are presented in their modern form and proofs are given in detail. This course will meet the needs of the more talented students majoring in mathematics, physics, and chemistry, as well as those students who have had some advanced preparation in calculus on the high school level.

The complete description of the material covered by Track I is given in a table to be found immediately following this preface. It is assumed that students following Track II will cover the entire volume.

This double track arrangement incorporates a number of significant advantages:

(1) It facilitates the switching back and forth of students who have originally been placed in courses above or below their own level. If the same text is used on the two different tracks between which the student is switching, the transfer invariably goes along much more smoothly than otherwise.

(2) It provides the students on Track I with built-in "outside reading." It is folly of course to imagine that this arrangement can serve as a total replacement for outside reading, but it does give the more eager or more talented student an opportunity to seek out advanced ideas or detailed proofs which are already presented in the familiar language and spirit of his own text.

(3) There is an advantage for the honors student, who is traveling along Track II. For even on this track it is the rare student who can digest every new idea in its most sophisticated form without the softening cushion of a relaxed intuitive introduction, as is provided in this book.

After a brief introduction to set theory, the book begins with a postulational presentation of the real number system. As is well known, the theorems of the calculus are all rooted in the properties of the real number system, and it is thus impossible to give a rigorous treatment of the former without first giving an appropriate treatment of the latter. Of the various approaches to the real number system, the postulational presentation seems to me to be pedagogically superior to all the others for use in a first course in calculus. After the absolute value of a real number is defined, the concept of a neighborhood of a real number is introduced. This is followed by a well-motivated, simple, yet logically adequate development of the topology on the space of real numbers.

Since the central theorem of the calculus is embodied in the statement concerning the relation between the derivative and the integral, these two concepts are presented together. The derivative is presented historically, as an outgrowth of the effort to solve the problem of tangents and the problem of instantaneous velocities. The integral, in turn, is presented as an outgrowth of the method of exhaustions. The existence of the integral of a continuous function is proved.

A chapter on linear algebra has been included, partly because the discussion of functions of several variables makes use of some results about linear systems and quadratic forms, and partly because the modern abstract theory of vector spaces serves so well to illuminate the subject of vectors in Euclidean spaces. A thoroughgoing vectorial presentation is made of the analytic geometry of 3-dimensional Euclidean space.

The customary treatments of such geometric concepts as area, volume, centroid, and moment of inertia make no attempt to show that these concepts are independent of the particular frame of reference and of the mode of decomposition. I feel that such an omission is out of consonance with the modern view in mathematics, and have on this account included in the chapter on multiple integrals a detailed though elementary discussion of Jordan content.

The subject of surface area is one that is almost invariably treated very superficially in texts on the calculus, both on the elementary and advanced levels. An impression is thereby unwittingly created that the subject is so profound that it were best for the student to be satisfied with the rather arbitrary treatment that is being given. I feel that this is both unfortunate and unnecessary, and in the chapter on multiple integrals I have presented the subject of surface area in a manner that I believe is more natural and more satisfying.

While a high level of rigor is maintained throughout the text, I have generally preferred the simple and direct approach to the sophisticated one when the latter is neither necessary nor particularly convenient.

It is my earnest hope that this book will be of help to all those who seek a mastery of the basic concepts of the calculus.

I am grateful to my wife Gloria and to my son Lee for the encouragement they gave me during the writing of this book. To Mrs. Sadelle Wladaver, I wish to express my thanks for her fine job of typing the manuscript. I have benefited from discussions which I had with Professor Fred Ficken, chairman of the mathematics department at New York University, and with many of my colleagues at New York University, especially, with Professor Hilbert Levitz. I wish to express my deep feeling of gratitude to Professor Herman Gluck of Harvard University and to Professor Paul Sally of the University of Chicago. Professor Gluck suggested many improvements in the presentation of the material. Professor Sally read thoroughly and critically the entire manuscript, pointed out many incongruities, and suggested some very valuable emendations. I am very grateful to the staff of Academic Press for their unfailing cooperation.

L. V. TORALBALLA

New York, New York

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2	Introd., Secs. 2.1 (Omit pf. of Thm. 2.1), 2.2 (omit pfs. of Thms. 2.2–2.4), 2.3–2.9 (omit all pfs. in Sec. 2.9), 2.10
3	Introd., Secs. 3.1–3.5, 3.8, 3.9 (omit pf. of Thm.3.6)
4	All
5	All
6	All
7	All
8	All (omit pf. of L'Hôpital's Thm.)
9	All (omit pfs. of Thms.)
10	All
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9 *The Search for Primitives. Applications of the Integral*

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11 *Infinite Series of Constants*

12 *Infinite series of Functions. Power Series*

13 *Linear Algebra*

16 Functions of Two or More Real Variables

17 Multiple Integrals

18 Applications of the Multiple Integral

Introduction. Basic Concepts of Set Theory

0.1. *Propositions, propositional functions, sets*

A *proposition* is a statement or sentence of which it can be asked meaningfully whether the statement or sentence is true or false. For instance, the sentences

> Kennedy was president of the United States
> 5 is greater than 10

are propositions. On the other hand the sentences

> Virtue is green
> A circle is honest

are not propositions.

A *propositional function* is a sentence which contains one and only one *variable*. For instance

> x was president of the United States
> x is greater than 10

are propositional functions. They are not propositions. One sees, however, that if *Nixon* is substituted for x in the first sentence or *15* is substituted in the second sentence, one obtains a proposition.

Now, one finds it very convenient to associate with every given propositional function a *class*, the class of the objects, each of which, when substituted for the variable in the propositional function, yields a proposition that is true. Thus, to the propositional function

> x is a friend of Richard

we associate the class of people who are friends of Richard. If John is a friend of Richard then we say that John is a member or an element of this class.