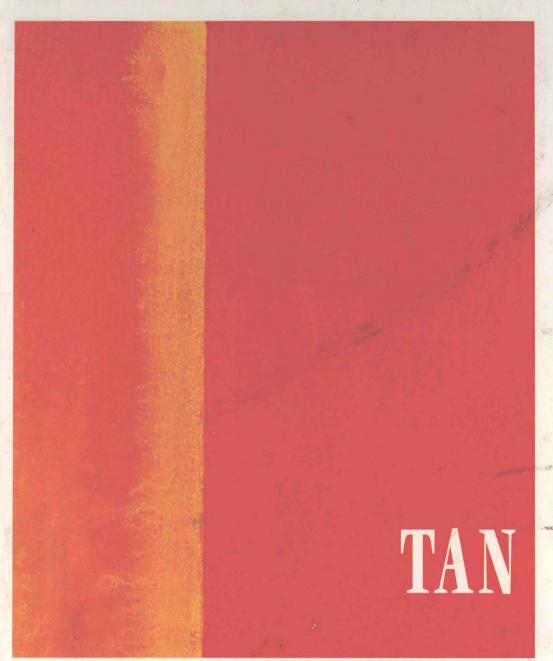
APPLIED

# CALCULUS



FOURTH EDITION

# APPLIED CALCULUS

**FOURTH EDITION** 

S. T. TAN

Stonehill College



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Applied Calculus, Fourth Edition, is suitable for use in a two-semester or three-quarter introductory calculus course for students in the managerial, life, and social sciences. As with the previous editions, our objective in Applied Calculus is twofold: (1) to write a textbook that is readable by students and (2) to make the book a useful teaching tool for instructors. We hope that with the present edition we have come one step closer to realizing our goal. The fourth edition of this text incorporates many suggestions by users of the earlier editions.

#### **FEATURES**

The following list includes some of the many important features of the book:

- Coverage of Topics The book contains more than enough material for the usual applied calculus course. Optional sections have been marked with an asterisk in the table of contents, thereby allowing the instructor to be flexible in choosing the topics most suitable for his or her course.
- Approach The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to clarify each new concept or result in order to facilitate students' comprehension of new material. Figures are used extensively to help students visualize concepts and ideas.
- Level of Presentation Our approach is intuitive and we state the results informally. However, we have taken special care to ensure that this approach does not compromise the mathematical content and accuracy. Proofs of certain results are given, but they may be omitted if desired.
- Applications The text is application oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one goal of the text is to include at least one real-life application in each section (whenever feasible).
- Exercises Each section of the text is accompanied by an extensive set of exercises containing ample problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics. Self-check exercises are also

included at the end of each section. These exercises give the students a chance to test themselves on their understanding of the material.

• **Portfolios** These interviews are designed to convey to the student the real-world experiences of professionals who have a background in mathematics and use it in their professions.

#### **TECHNOLOGY**

In this book our primary emphasis lies in the use of the graphing utility, which is employed in two ways:

1. It is used to explore mathematical concepts and also to shed further light on selected worked examples in the text. In this capacity, it serves to augment the analytic solution obtained in the text with a graphical and/or numerical solution, thus adding to a greater understanding of the problem. Exercises in this category appear under the heading:



### EXPLORING WITH TECHNOLOGY

2. It is used to solve problems whose solutions require a prodigious amount of calculations. Indeed, it is the availability of the graphing utility that makes it possible to include the many real-life problems in this book. Such exercises are found at the end of many subsections in the book under the heading:

/// USING TECHNOLOGY

#### **NEW IN THE 4TH EDITION**



#### **Exploring with Technology Questions**

These optional questions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Complete solutions to these exercises are given in the *Instructor's Complete Solutions Manual*.

#### /// Using Technology Subsections

These pages contain optional material and are placed at the end of the sections for which their use is appropriate. The subsections are written in the traditional example–exercise format with answers given at the back of the book. They may be used in the classroom if desired or as material for self-study by the student.

As many up-to-date and relevant applications have been introduced in these subsections, they provide students with an opportunity to interpret results in a real-life setting.



#### **Group Discussion Questions**

These are optional questions, appearing throughout the main body of the text, that can be discussed in class or assigned as homework. These questions generally require more thought and effort than the usual exercises. Complete solutions to these exercises are given in the *Instructor's Complete Solutions Manual*.

#### **Critical Thinking Questions**

These new exercises go beyond the usual exercises involving computation and provide the student with a little more challenge. Many of these questions call for the interpretation of results or the theory presented in the section.



#### **New Internet-Linked Supplements**

Students and instructors will now have access to these additional materials at the Brooks/Cole World Wide Web site: http://www.brookscole.com/math/authors/tans

- Study hints, review material, and practice chapter tests for students
- Group projects and extended problems for each chapter
- Instructions, including keystrokes, for the procedures referenced in the text for specific calculators (TI-82, TI-85, and other popular models)
- Modified "Using Technology" sections for CAS systems, including the command statements for Mathematica, Maple, and other popular systems

#### Sources

We have included sources for those applications that are based on real-life data.

#### **New Exercises**

A wealth of innovative and timely exercises have been added to pique student interest. More exercises (both rote and applied) calling for graphical interpretations have also been added.

#### **Helpful Changes**

A new real-life example—the motion of a magnetic levitation train—is used to motivate topics in limits, derivatives, and integration. The use of the motion of a familiar object in the different settings serves to enhance comprehension of the topics.

Chapter 2: A new illustrative example, "Spending by businesses on computer security equipment," has been added to the discussion of mathematical models in Section 2.3. The motion of a maglev is now used to motivate an intuitive discussion of the Intermediate Value Theorem.

Chapter 3: The use of the application "Estimating the Size of the Rings

of Neptune" in Section 3.7, Differentials, is yet another example of the importance attached to our philosophy of motivating topics with real-life examples. A discussion of relative error has also been added to this section.

Chapter 4: The chapter has been reorganized and condensed. Sections 4.1 to 4.3 now cover the material previously covered in five sections. Topics are now organized into Applications of the First Derivative, Applications of the Second Derivative, and Curve Sketching.

Chapter 6: An early introduction to differential equations and a discussion of initial value problems have been added.

#### **SUPPLEMENTS**

- Student's Solutions Manual, available to both students and instructors, includes the solutions to odd-numbered exercises, ISBN 0-534-95558-4
- Instructor's Complete Solutions Manual, available only to instructors, includes solutions to all exercises. ISBN 0-534-95559-2
- Test Bank with Chapter Tests, free to adopters of the book, contains sample tests for each chapter. ISBN 0-534-35093-3
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S. T. Tan Stonehill College







In **Applied Calculus** we attempt to solve a wide variety of problems arising from many diverse fields of study. A small sample of the types of practical problems we will consider follows.

**POPULATION GROWTH** A study prepared for a Sunbelt town's Chamber of Commerce projected that the town's population in the next 3 years would grow according to the rule

$$P(x) = 50,000 + 30x^{3/2} + 20x$$

where P(x) denotes the population x months from now. How fast will the population be increasing nine months from now? Sixteen months from now?

AIR POLLUTION According to the South Coast Air Quality Management District, the level of nitrogen dioxide, a brown gas that impairs breathing, present in the atmosphere on a certain May day in downtown Los Angeles is approximated by

$$A(t) = 0.03t^3(t-7)^4 + 60.2$$
  $(0 \le t \le 7)$ 

where A(t) is measured in pollutant standard index and t is measured in hours, with t = 0 corresponding to 7 A.M. How fast is the level of nitrogen dioxide increasing at 11 A.M.?

**OPTIMAL DRIVING SPEED** A truck gets 400/x miles per gallon when driven at a constant speed of x miles per hour (between 50 and 70 miles per hour). If the price of fuel is \$1 per gallon and the driver is paid \$8 an hour, at what speed between 50 and 70 miles per hour is it most economical to drive?







**LEARNING CURVES** The Eastman Optical Company produces a 35-mm single-lens reflex camera. Eastman's training department determined that after completing the basic training program, a new, previously inexperienced employee would be able to assemble

$$Q(t) = 50 - 30e^{-0.5t}$$

model F cameras per day, t months after the employee began work on the assembly line. How many model F cameras can a new employee assemble per day after basic training? How many model F cameras can the average experienced employee assemble per day?

SUBWAY FARES A city's Metropolitan Transit Authority (MTA) operates a subway line for commuters from a certain suburb to the downtown metropolitan area. Currently, an average of 6000 passengers a day take the trains, paying a fare of \$1.50 per ride. The board of the MTA, contemplating raising the fare to \$1.75 per ride in order to generate a larger revenue, engaged the services of a consulting firm. The firm's study revealed that for each 25-cent increase in fare, the ridership would be reduced by an average of 1000 passengers a day. The consulting firm recommended that MTA stick to the current fare of \$1.50 per ride, which already yields a maximum revenue. Show that the consultants' recommendations were correct.

**ADVERTISING EXPENDITURES** The Ross-Simons Company has a monthly advertising budget of 60,000. Their marketing department estimates that if they spend x dollars on newspaper advertising and y dollars on television advertising, then the monthly sales will be given by

$$z = f(x, y) = 90x^{1/4}y^{3/4}$$

dollars. Determine how much money Ross-Simons should spend on newspaper ads and on television ads per month to maximize its monthly sales.

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# APPLIED CALCULUS

The first two sections of this chapter contain a brief review of algebra. We then introduce the Cartesian coordinate system, which allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. This chapter also covers straight lines. The slope of a straight line plays an important role in the study of calculus.

What sales figure can be predicted for next year? In Example 10, page 43, you will see how the manager of a local sporting goods store used sales figures from the previous years to predict the sales level for next year.

