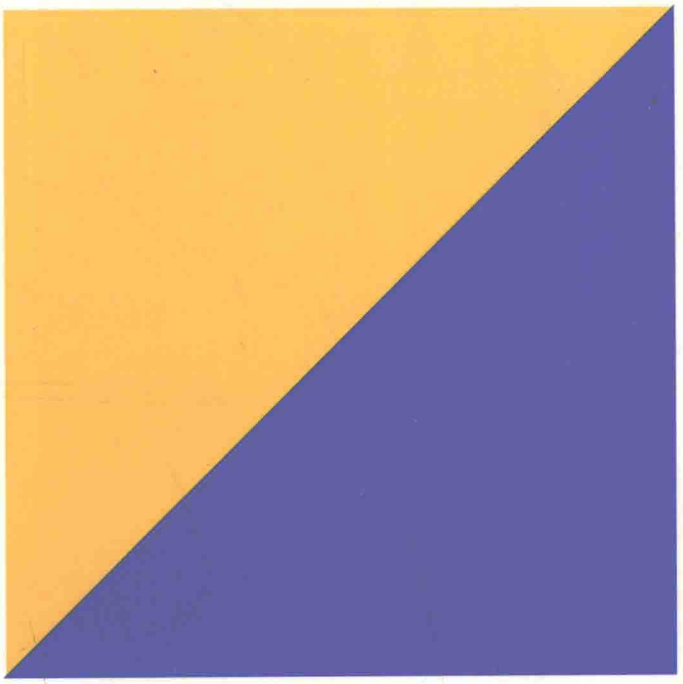
Study Guide

Elementary Algebra

Larson ■ Hostetler



Second Edition

This study guide provides easy-to-follow, comprehensive support throughout the course.

- Items keyed to the text
- Text section highlights summary
- Examples with detailed step-by-step solutions and side comments
- Starter exercises combined with feedback as students check their work
- Additional exercises
- Chapter tests
- Warm-up exercises
- Answers

Jay R. Wiestling

Study Guide

to accompany

Elementary Algebra

Second Edition

Larson/Hostetler

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Dedication

To my parents, Ron and Nova, without whom this would not have been possible.

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PREFACE

This study guide has been prepared to supplement the sections of each chapter of *Elementary Algebra*, Second Edition, by Roland E. Larson and Robert P. Hostetler.

Features

Each section starts with *Section Highlights,* which review definitions, facts, formulas, and properties that students should understand in order to be proficient in the section.

The workbook contains many examples with detailed step-by-step solutions. These are followed by *Starter Exercises*, in which a partial solution has been provided and students are expected to complete the solution. Detailed solutions to the Starter Exercises are provided at the end of each section.

Each section has an *exercise set.* Answers to all exercises have been included at the end of each chapter.

Each chapter has a *Cumulative Practice Test.* These tests contain exercises on the topics covered in that chapter and all chapters back through Chapter 1. Answers to all of these exercises have also been provided at the end of each chapter.

At the end of the Study Guide you will find a set of ten Warm-Up exercises for each section of the text starting with Chapter 1. These allow you to practice the skills you learned previously that are necessary to master the "new skills" presented in the section. All Warm-Up exercises are answered in the section following the Warm-Ups in the back of the text.

How to Use This Workbook

It is assumed that the corresponding sections in the text have been read.

Each section of the workbook corresponds to a text section. After reading a section in the text, read the examples in this workbook and work the accompanying Starter Exercises. Then work the exercises in the text. If more exercises (with answers) are desired, then work the exercises in this workbook.

The Cumulative Practice Test exercises can be used as a study aid for exams. These exercises will help you maintain proficiency with all the material covered in the course.

How to Study Mathematics

A lot of students find success in working with a study partner. Partners can help by answering questions and pushing each other to become better. Study partners seem to work particularly well at exam time. When reviewing for an exam, questions may arise (when an instructor is not available) and someone in the study group may know the answer.

Here are four steps for section-by-section studying.

1. Read the section in your textbook before you go to a lecture on that section. You may not completely understand the material at this point, but you will be able to pinpoint your problems and ask precise questions in class.
2. Go to class regularly and take good notes. Your notes should include the important ideas and examples of these ideas. Try recopying your notes after class. This gives you a complete, understandable set of notes to review later. Also, you may be surprised at how much more information you retain.
3. Reread the section in your textbook before starting the exercises. This allows you to see how ideas relate to each other and enhances your understanding of the material. In addition, you may want to read the examples and try the Starter Exercises in this study guide as they correspond to the text material.
4. Now it is time to work the exercises. Some students try to work the exercises before having adequately prepared and find it to be a frustrating experience. The exercises are there to improve your understanding of the subject; if you are overly frustrated, you will not

benefit from your work. You may also want to work the exercises in this study guide. The combination should give you the maximum benefit.

Finally, a few remarks on *studying for exams*. You should start studying well in advance of the exam (at least a week). The first day or two, only study for about two hours. Gradually increase the study time each day. Be completely prepared for the exam two days in advance. Spend the final day just building confidence so you can be relaxed during the exam.

The first things to look at when studying for an exam are all definitions, properties, and formulas. After these are known, then work as many exercises as you can. Working exercises is very important in the learning process as well as studying for an exam. Make sure you work a lot of the exercises that have given you trouble in the past.

When taking the exam itself, go through the exam and answer only the questions that you know immediately. Do not struggle with any one question on the first time through. If you do, you may not get to all the questions that you know by the end of the exam. Also, you may begin to panic if you struggle and not be able to recall the things you know.

After answering all the questions that you know immediately, then go back and work on the questions you skipped. After doing this, go back and check your work.

Good luck.

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CONTENTS

EXAMPLE 2 ■ Ordering Real Numbers

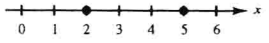
Place the correct inequality symbol ($<$ or $>$) between the two numbers.

(a) $2 \square 5$ (b) $-3 \square 2$ (c) $-\frac{1}{2} \square -6$

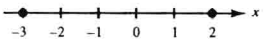
(d) $\frac{1}{12} \square -\frac{1}{13}$ (e) $-2.7 \square 2.6$

Solution

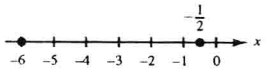
(a) $2 < 5$



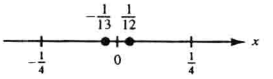
(b) $-3 < 2$



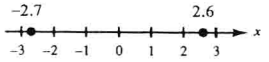
(c) $-\frac{1}{2} > -6$



(d) $\frac{1}{12} > -\frac{1}{13}$



(e) $-2.7 < 2.6$



■

Starter Exercise 2 *Fill in the blanks.*

Place the correct inequality symbol ($<$ or $>$) between the two numbers.

(a) $-2 \square 1$ (b) $\frac{1}{2} \square \frac{1}{3}$ (c) $-1.1 \square -1$

EXAMPLE 3 ■ Evaluating Absolute Values

Evaluate the following expressions.

(a) $|2|$ (b) $|-3|$ (c) $-|-5|$

Solution

(a) $|2| = 2$ because the distance between 2 and 0 is 2.

(b) $|-3| = 3$ because the distance between -3 and 0 is 3.

(c) $-|-5| = -(5) = -5$

■

Starter Exercise 3 *Fill in the blanks.*

Correctly place a positive sign (+) or a negative sign (−) in the blank box.

(a) $|6| = \square 6$ (b) $|-1| = \square 1$ (c) $-|-3| = \square 3$

■ Solutions to Starter Exercises ■

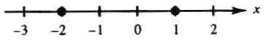
1. (a) $\{13, 3\}$

(b) $\{13, -\frac{3}{1} = -3, -16, 3\}$

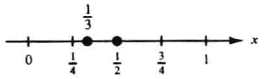
(c) $\{-\frac{4}{11}, 13, \frac{1}{12}, -\frac{3}{1}, -16, 3\}$

(d) $\{-\frac{4}{11}, 13, \frac{1}{12}, -\frac{3}{1}, -16, 3\}$

2. (a) $-2 \boxed{<} 1$



(b) $\frac{1}{2} \boxed{>} \frac{1}{3}$



(c) $-1.1 < 1$

3. (a) $|6| = \boxed{+}6$ (b) $|-1| = \boxed{+}1$ (c) $-|-3| = \boxed{-}3$

P.1 EXERCISES

In Exercises 1–8, evaluate the expression.

1. $|2|$ **2.** $|\frac{1}{2}|$ **3.** $|-3|$ **4.** $|-16|$

5. $-|4|$ **6.** $-|7|$ **7.** $-|-14|$ **8.** $-|-6|$

In Exercises 9–26, show each number on the real number line, and place the correct symbol ($<$, $>$, or $=$) between the two real numbers.

9. $2 \square 4$ **10.** $5 \square 1$ **11.** $\frac{1}{2} \square 3.5$

12. $0.96 \square \frac{3}{4}$ **13.** $-6 \square -2$ **14.** $-4 \square -1$

15. $|3| \square |-3|$ **16.** $2 \square |-2|$ **17.** $|-\frac{1}{2}| \square |-4|$

18. $|-1| \square |-5|$ **19.** $-|-\frac{3}{5}| \square -\frac{3}{5}$ **20.** $-|0.6| \square -|-0.6|$

21. $-\frac{5}{2} \square -|3|$ **22.** $-|3| \square |3|$ **23.** $0 \square |0|$

24. $|0| \square -|0|$ **25.** $|0.75| \square |-\frac{3}{4}|$ **26.** $-|0.5| \square \frac{1}{2}$

In Exercises 27–31, write the letter (a, b, c, d, or e) that corresponds to the phrase that best completes the given statement. (Use each letter only once.)

(a) not positive
(b) positive
(c) a rational number
(d) a fraction
(e) the distance from that number to the origin on the real number line

27. A rational number can be written as _____ .

28. Every integer is _____ .

29. The absolute value of a number is _____ .

30. The absolute value of a negative number is _____ .

31. The opposite of the absolute value of a real number is _____ .

P.2 Integers and Prime Factorization

Section Highlights

1. If a and b are positive integers, then a is a factor of b if and only if there is a positive integer c such that $a \cdot c = b$.
2. A positive integer greater than 1 with no factors other than 1 and itself is called a prime number.
3. The greatest common factor of two (or more) positive integers a and b is the largest positive integer that is a factor of both a and b.
4. The least common multiple of two (or more) positive integers a and b is the smallest positive integer that is a multiple of both a and b.

EXAMPLE 1 ■ Prime Factorization

Find the prime factorization of each of the following.

(a) 72 (b) 84

Solution

(a) We know that $72 = 8 \cdot 9$. Thus $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

(b) We can see that 4 is a factor of 84. Thus $84 = 4 \cdot 21 = 2 \cdot 2 \cdot 3 \cdot 7$.

Starter Exercise 1 *Prime Factorization*

Find the prime factorization of each of the following.

(a) 42 (b) 65

Solution

(a) $42 = 6 \cdot \square = 2 \cdot \square \cdot \square$ (b) $65 = 5 \cdot \square$

EXAMPLE 2 ■ Finding the Greatest Common Factor

Find the greatest common factor of the following.

(a) 28 and 63 (b) 12, 48, and 54

Solution

(a) The prime factorization of these numbers is $28 = 2 \cdot 2 \cdot 7$ and $63 = 3 \cdot 3 \cdot 7$. Both numbers have a common factor of 7, and nothing else. Hence, the greatest common factor is 7.

(b) The prime factorization of these numbers is $12 = 2 \cdot 2 \cdot 3$, $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ and $54 = 2 \cdot 3 \cdot 3 \cdot 3$. The factors common to all three are 2 and 3. Hence, the greatest common factor is $2 \cdot 3 = 6$.

Starter Exercise 2 *Finding the Greatest Common Factor*

Find the greatest common factor of the following.

(a) 36 and 84 (b) 72 and 90

Solution

(a) Prime Factorization: $36 = 2 \cdot 2 \cdot 3 \cdot 3$ and $84 = 2 \cdot 2 \cdot 3 \cdot 7$
Common Factors: 2, ☐, and ☐
Greatest Common Factor is $2 \cdot$ ☐ $\cdot$ ☐ = ☐

(b) Prime Factorization: $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ and $90 = 2 \cdot 3 \cdot 3 \cdot 5$
Common Factors: ☐, ☐, and 3 Greatest common factor is ☐ $\cdot$ ☐ $\cdot$ ☐ = ☐

EXAMPLE 3 ■ Finding the Least Common Multiple

Find the least common multiple of the following.

(a) 6 and 15 (b) 10, 18, and 20

Solution

(a) Prime Factorization: $6 = 2 \cdot 3$ and $15 = 3 \cdot 5$. A number can be a multiple of 6 only if it has 2 and 3 as factors. Similarly, to be a multiple of 15, a number must have factors of 3 and 5. The smallest number that satisfies both of these conditions is $2 \cdot 3 \cdot 5$. Thus the least common multiple is $2 \cdot 3 \cdot 5 = 30$.

(b) Prime Factorization: $10 = 2 \cdot 5$, $18 = 2 \cdot 3 \cdot 3$, and $20 = 2 \cdot 2 \cdot 5$. The least common factor must contain at least two factors of 2 (from 20), at least two factors of 3 (from 18) and at least one factor of 5. Thus the least common multiple is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$.

Starter Exercise 3 *Finding the Least Common Multiple*

Find the least common multiple of the following.
(a) 4 and 5 (b) 21 and 28

Solution

(a) Prime Factorization: $4 = 2 \cdot$ ☐ and $5 = 5 \cdot$ ☐. The least common multiple needs at least ☐ factors of 2 and ☐ factor of 5. Thus, the least common multiple is $2 \cdot$ ☐ $\cdot$ ☐ = ☐.

(b) Prime Factorization: $21 = 3 \cdot$ ☐ and $28 =$ ☐ $\cdot$ ☐ $\cdot 7$. Least common multiple is $2 \cdot$ ☐ $\cdot 3 \cdot$ ☐ = ☐.

■ Solutions to Starter Exercises ■

1. (a) $= 6 \cdot \boxed{7} = 2 \cdot \boxed{3} \cdot \boxed{7}$

(b) $= 65 = 5 \cdot \boxed{13}$

2. (a) Common factors: 2, $\boxed{2}$ and $\boxed{3}$.

Greatest common factor is $2 \cdot \boxed{2} \cdot \boxed{3} = 12$

(b) Common factors: $\boxed{2}$, $\boxed{3}$ and 3.

Greatest common factor is $\boxed{2} \cdot \boxed{3} \cdot \boxed{3} = \boxed{18}$

3. (a) $4 = 2 \cdot \boxed{2}$ and $5 = 5 \cdot \boxed{1}$

The least common multiple needs at least $\boxed{2}$ factors of 2 and $\boxed{1}$ factor of 5.

Thus, the least common multiple is $2 \cdot \boxed{2} \cdot \boxed{5} = 20$

(b) $21 = 3 \cdot \boxed{7}$ and $28 = \boxed{2} \cdot \boxed{2} \cdot 7$

Least common multiple is $2 \cdot \boxed{2} \cdot 3 \cdot \boxed{7} = \boxed{84}$

P.2 EXERCISES

In Exercises 1–6, is the number prime or composite?

1. 121 **2.** 43 **3.** 625

4. 1073 **5.** 143 **6.** 97

In Exercises 7–12, write the prime factorization.

7. 78 **8.** 338 **9.** 196

10. 2352 **11.** 982 **12.** 2445

In Exercises 13–18, find the greatest common factor.

13. 76, 57 **14.** 60, 48 **15.** 63, 105

16. 96, 192 **17.** 15, 25, 55 **18.** 112, 168, 252

In Exercises 19 and 20, list the first four positive integer multiples of the number.

19. 8 **20.** 23

In Exercises 21–26, find the least common multiple.

21. 6, 10 **22.** 12, 15 **23.** 60, 90

24. 105, 30 **25.** 4, 6, 8 **26.** 24, 42, 48

In Exercises 27–30, determine whether the numbers are relatively prime.

27. 70, 121

28. 429, 350

29. 69, 75

30. 63, 72, 95

P.3 Adding and Subtracting Integers

Section Highlights

1. To add two real numbers with like signs, add their absolute values and attach the common sign of the two terms to this sum.
2. To add two real numbers with unlike signs, find the absolute value of each term, subtract the smaller absolute value from the larger absolute value, and attach the sign of the term with the larger absolute value to the result.
3. To subtract one number (the subtrahend) from another number (the minuend), add the opposite of the subtrahend to the minuend.

EXAMPLE 1 ■ Adding Integers

Find the following sums.

(a) $-12 + (-6)$ (b) $13 + (-16)$ (c) $-6 + 9$

Solution

(a) $-12 + (-6) = -18$ (b) $13 + (-16) = -3$ (c) $-6 + 9 = 3$ ■

Starter Exercise 1 *Fill in the blanks.*

Find the following sums.

(a) $-3 + 11 =$ ☐ (b) $-2 + (-5) =$ ☐ (c) $4 + (-9) =$ ☐

EXAMPLE 2 ■ Subtracting Integers

Find the following differences.

(a) $-2 - (-4)$ (b) $16 - (-36)$ (c) $-10 - 9$

Solution

(a) $-2 - (-4) = -2 + 4 = 2$

(b) $16 - (-36) = 16 + 36 = 52$

(c) $-10 - 9 = -10 + (-9) = -19$ ■

Starter Exercise 2 *Fill in the blanks.*

Find the following differences.

(a) $-17 - 5 = -17 + (-5) = \square$ (b) $13 - (-2) = \square + \square = 15$

(c) $-6 - (-8) = -6 + \square = \square$

■ **Solutions to Starter Exercises** ■

1. (a) $-3 + 11 = \boxed{8}$

(b) $-2 + (-5) = \boxed{-7}$

(c) $4 + (-9) = \boxed{-5}$

2. (a) $-17 - 5 = -17 + (-5) = \boxed{-22}$

(b) $13 - (-2) = \boxed{13} + \boxed{2} = 15$

(c) $-6 - (-8) = -6 + \boxed{8} = \boxed{2}$

P.3 EXERCISES

In Exercises 1–9, find the sum.

1. $-2 + (-3)$

2. $24 + (-16)$

3. $-42 + 9$

4. $-37 + 17$

5. $21 + (-12)$

6. $|6| + |-3|$

7. $|-4| + |-15|$

8. $7 + (-|-13|)$

9. $(-|5|) + (-7)$

In Exercises 10–18, find the difference.

10. $-7 - 12$

11. $-8 - 6$

12. $-19 - (-6)$

13. $47 - (-19)$

14. $119 - (-5)$

15. $|10| - |-10|$

16. $|-3| - |-9|$

17. $17 - (-|-16|)$

18. $-26 - (-|-4|)$

P.4 Multiplying and Dividing Integers

Section Highlights

1. To multiply two numbers with like signs, find the product of their absolute values.
2. To multiply two numbers with unlike signs, find the product of their absolute values, and attach a negative sign to this product.
3. To divide two numbers, multiply the dividend by the reciprocal of the divisor.

EXAMPLE 1 ■ Multiplying Integers

Find the following products.

(a) $(-3)(-6)$ (b) $-4 \cdot 7$ (c) $24 \cdot (-36)$

Solution

(a) $(-3)(-6) = 18$ (b) $-4 \cdot 7 = -28$

(c) The vertical algorithm is the best method for finding the product of the two absolute values.

$$\begin{array}{rl} & 24 \\ \times & 36 \\ \hline & 144 \quad (6 \times 24) \\ & 72 \quad (3 \times 24) \\ \hline & 864 \quad \text{(sum of columns)} \end{array}$$

$24 \cdot (-36) = -864$ ■

Starter Exercise 1 *Fill in the blanks.*

Find the following products.

(a) $-3 \cdot 7 = \square$ (b) $(-5) \cdot (-4) = \square$

EXAMPLE 2 ■ Division of Integers

Find the following quotients.

(a) $-12 \div 4$ (b) $-32 \div (-2)$ (c) $\dfrac{480}{-32}$

(d) $0 \div (-6)$ (e) $-15 \div 0$

Solution

(a) $-12 \div 4 = -3$ because $-12 = 4 \cdot (-3)$.

(b) $-32 \div (-2) = 16$ because $-32 = (-2) \cdot (16)$.

(c) We will use the long division algorithm.

$$\begin{array}{r} 15 \\ 32\overline{)\,480} \\ \underline{32} \\ 160 \\ \underline{160} \\ 0 \end{array}$$

$$\frac{480}{-32} = -15$$

(d) $0 \div (-6) = 0$ because $0 = -6 \cdot 0$.

(e) $-15 \div 0$ is undefined because division by zero is undefined. ■

Starter Exercise 2 *Fill in the blanks.*

Find the following quotients.

(a) $\frac{-15}{-3} =$ ☐ (b) $14 \div (-7) =$ ☐ (c) $-312 \div 13 =$ ☐

Use long division algorithm.

Solutions to Starter Exercises

1. (a) $-3 \cdot 7 =$ $\boxed{-21}$

(b) $(-5)(-4) =$ $\boxed{20}$

2. (a) $\frac{-15}{-3} =$ $\boxed{5}$

(b) $14 \div (-7) =$ $\boxed{-2}$

(c) $-312 \div 13 =$ $\boxed{-24}$

$$\begin{array}{r} 24 \\ 13\overline{)312} \\ \underline{26} \\ 52 \\ \underline{52} \\ 0 \end{array}$$

P.4 EXERCISES

In Exercises 1–9, find the product.

1. $(-9) \cdot (-4)$ **2.** $(-2) \cdot (-7)$ **3.** $(-5) \cdot (10)$

4. $4 \cdot (-3)$ **5.** $|-19| \times 2$ **6.** $4 \times (-|8|)$

7. $|-7| \cdot |-1|$ **8.** -22×19 **9.** $91 \cdot (-21)$

In Exercises 10–18, find the quotient.

10. $-\frac{8}{4}$ **11.** $12 \div (-4)$ **12.** $(-30) \div (-3)$

13. $\frac{-24}{-6}$ **14.** $-|20| \div (-4)$ **15.** $|-32| \div (-4)$

16. $27 \div (-|-3|)$ **17.** $1235 \div (-19)$ **18.** $-744 \div (-31)$

19. *Area of a Rectangle* The area of a rectangle is the product of its length and its width. Joe has a rectangular patio that has a length of 17 feet and a width of 16 feet. Find the area of Joe's patio.

20. *Area of a Rectangle* A room is 30 feet long by 24 feet wide. How much carpet is needed to cover the entire floor?