

THERMAL STRESSES AND  
TEMPERATURE CONTROL  
**OF MASS CONCRETE**

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大体积混凝土温度应力  
**与温度控制**

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ZHU BOFANG

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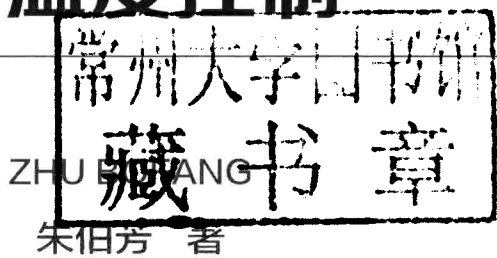
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## 内 容 简 介

大体积混凝土结构, 体积巨大, 通常不配置钢筋, 由混凝土本身承受拉应力。拉应力主要由温度变化引起, 因此控制温度应力是大体积混凝土结构设计和施工的一个重要问题。以混凝土坝为例, 体积巨大, 分层施工, 建造周期长达数年, 受环境温度变化和施工过程影响, 坝体温度应力变化十分复杂, 过去无法计算, 实际上是“无坝不裂”。本书介绍了作者建立的混凝土温度应力和温度控制的完整理论体系, 包括重力坝、拱坝、水闸、船坞、隧洞、浇筑块、地基梁等各种大体积混凝土结构温度场和应力场的精细计算方法, 控制温度防止裂缝的技术措施和设计与施工准则。在该理论指引下, 我国已在世界上首次建成数座无裂缝混凝土坝, 证明这一套理论是正确而切实可行的。在坝工技术上取得了比较重要的成就, 曾先后获得国家自然科学奖、国家科技进步奖和国际大坝会议荣誉奖。

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# **Thermal Stresses and Temperature Control of Mass Concrete**

# Preface

The cracking of massive concrete structures due to thermal stresses is a problem which had puzzled engineers for a long time. “No dam without crack” is the actual state of concrete dams in the world. The theory of thermal stress and temperature control of mass concrete is established by the writer in this book under the direction of which the problem of cracking of massive concrete structures had been solved, and several concrete dams without crack have been successfully constructed in China in recent years which indicates that the history of “No dam without crack” has ended.

Mass concrete is important for the economical construction of a country. For example, more than 10 million cubic meters of mass concrete are placed in the hydraulic engineering projects in China every year. In addition, a large amount of mass concrete is placed every year in the engineering of harbors, foundation of high buildings heavy machines, nuclear reactors, etc.

The thickness of a massive concrete structure is immense, e.g., the thickness of a concrete dam may be 100–200 m, the depth of the region under tension may be 10–30 m; if all the tensile stresses are undertaken by steel reinforcement, the amount of steel will be considerable, and the cost will be very high. In the process of construction, if there are many vertical steel reinforcements on the top of a concrete block, the spreading and placing of the new concrete lift will be very difficult. Thus in the design of massive concrete structures, such as concrete dams, generally it is required that the tensile stresses do not exceed the allowable tensile stress of concrete so that no steel reinforcement is used. If there are only concrete weight and water pressure acting on the dam, the above-mentioned requirement is easy to achieve, but the period of construction of a high concrete dam may be several years. Due to the heat of hydration of cement and the variation of the ambient temperature, large tensile stresses may appear in the massive concrete structure. As a result, cracks developed in almost all the concrete dams.

The concrete dams are divided into blocks and each block is constructed in horizontal lifts with thickness 1–3 m. The intermissions between two lifts are 5–10 days. As the mechanical and thermal properties of concrete vary with age and have different values in different layers, so the computing of thermal stresses in concrete dams is rather complicated. In the past, there were no methods to compute the thermal stresses in the period of construction of concrete dams, although some temperature control measures had been adopted, but the thermal stresses in the dam are unknown. Actually the tensile stresses are so large that many cracks developed in almost all the dams.

Now a perfect system of the theory of thermal stress and temperature control is established by the writer in this book which includes the following parts:

1. A series of methods for computing the temperature field and the thermal stress field, especially the simulation method for computing the temperature field and stress field of the structure taking account of the influences of all the factors including (a) the process of construction, (b) the mechanical and thermal properties varying with the age of concrete, (c) the variation of ambient air and water temperature, (d) the various measures of temperature control.
2. The law of variation and peculiarity of thermal stresses of different types of massive concrete structures, such as gravity dams, arch dams, buttress dams, concrete blocks, locks, sluices, concrete beams on elastic foundations, concrete pipes, and concrete linings of tunnels. Understanding these issues by engineers is favorable for the construction of massive concrete structures without crack.
3. Various technical measures to prevent cracking of mass concrete, such as choice of raw materials, precooling, pipe cooling, and superficial thermal insulation.
4. The experiences of many practical concrete dams, particularly the success of the construction of several concrete dams without crack in China in recent years.
5. Many new ideas and new methods for prevention of cracking and temperature control of mass concrete.
6. Comprehensive analysis of different schemes of construction of concrete dams with different combinations of the measures of temperature control.

In the design stage of a massive concrete structure, several schemes of temperature control may be given and computed the temperature field and stress field in detail by the methods given in this book, after comprehensive analysis, a rational scheme may be obtained. Otherwise, a new scheme with improved combination of temperature control may be given and analyzed, until a good scheme of temperature control is obtained which will lead to the possibility that there will be no crack in the dam in the construction and operation period. By this method, several concrete dams without crack have been constructed in China in recent years. This is an important and valuable experience in the construction of massive concrete structures.

Cracks in massive concrete structures, such as concrete dams, will reduce the safety, integrity, and durability of the structure. The repair of cracks in concrete dams is very difficult, e.g., a big crack developed in Norfolk dam, the engineers had attempted to repair the dam by grouting, but due to the worry that the crack may develop further under the pressure of grouting, the crack was not repaired and the dam has been working with a big crack in the dam body; as a result, the safety and durability of the dam are reduced remarkably.

The successful construction of several concrete dams without crack in China is an important achievement in technical science in the world.

Due to the needs of flood control, irrigation, and hydropower, many concrete dams have been constructed in China in the past 60 years. At present the amount of concrete dams higher than 15 m in China is over 40% of those elsewhere and the three highest concrete dams in the world (Jingping 305 m, Xiaowan 295 m, Xiluodu 284 m) are in China. In the process of large-scale construction of concrete dams in China, besides learning abroad experiences, systematic research works had

been carried out and new theory and new experiences were created; hence, the problem of cracking in mass concrete has been solved and several concrete dams without crack have been constructed in recent 10 years.

After graduating from university in 1951, the writer participated in the design and construction of the first three concrete dams in China (Fuzhiling dam, Meishang dam, and Xiang hongdian dam) in 1951–1957. Although some measures to prevent cracking had been adopted, cracks still appeared in these dams, which indicates that cracking of mass concrete is a complex problem. The writer began to research the problem in 1955 and published two papers in 1956 and 1957 which triggered research of thermal stress and temperature control of mass concrete in China.

In 1958, the writer was transferred to the China Institute of Water Resources and Hydropower Research where he was engaged in the research work of high concrete dams, particularly the thermal stresses and temperature control of concrete dams.

A vast amount of research works had been carried out under the direction of the writer for a series of important concrete dams in China, such as Three Gorges, Xiaowan, Longtan, Xiluodu, Sanmenxia, Liujiaxia, Xin'anjiang, and Gutian.

More than 120 papers had been published putting forward a series of new ideas, new calculating methods, and new technical measures, including (1) a new idea of “long time superficial thermal insulation together with comprehensive temperature control” which may prevent crack in mass concrete effectively, (2) methods for calculating the temperature field and thermal stresses in dams, docks, sluices, tunnels, concrete blocks, and beams on elastic foundations; (3) simulation thermal stress computation taking into account the influences of all the factors and simulating the process of construction; (4) method of back analysis for determining the practical thermal and mechanical properties of concrete from the observed results; (5) the new idea of numerical monitoring of mass concrete; (6) the new idea of semi-mature age of concrete; and (7) formulas for determining the water temperature in reservoirs and temperature loading of arch dams.

Hence, a perfect system of the theory of thermal stress and temperature control of mass concrete is established whereby several concrete dams without crack have been successfully constructed in China in the past 10 years, including the Sangianghe concrete arch dam and the third stage of the famous Three Gorges concrete gravity dam and hence “no dam without crack” is no longer a problem.

The solution of the problem of cracking is an important achievement in the technology of mass concrete.

More than 10 results of the author's scientific research were adopted in the specifications for design and construction of gravity dams, arch dams, docks, and massive concrete structures in China.

In order to summarize the experiences, the author published the book *Thermal Stresses and Temperature Control of Mass Concrete* (in Chinese) in 1999.

The Information Center of the China Academy of Science published two statistics in 2011: (1) According to the number of quotations, the first 10 books of each profession of China, *Thermal Stresses and Temperature Control of Mass Concrete* is one of the 10 most widely quoted books of civil engineering in China. (2) According to the number of quotations, the first 20 authors of scientific papers of each profession in

China, the writer is the first one of the 20 most widely quoted authors of hydraulic engineering.

The author was awarded the China National Prize of Natural Science in 1982 for research work in thermal stresses in mass concrete, the China National Prize of Scientific Progress in 1988 for research work in the optimum design of arch dams, the China National Prize of Scientific Progress in 2000 for research work in simulating computation and thermal stresses, and the International Congress on Large Dams Honorary Member at Saint Petersburg in 2007.

Outside China there are two books on temperature control of mass concrete: (1) US Bureau of Reclamation, *Cooling of Concrete Dams*, 1949, (2) Stuky A, Derron MH, *Problemes Thermiques Poses Par La Construction des Barrages-Reservoirs*, Lausanne, Sciences & Technique, 1957. Theoretical solutions and many graphs for determining the temperatures of concrete dams are given in these two books which are useful to engineers, but there is no method for computing the thermal stresses, no method for preventing crack except pipe cooling, no criterion for temperature control, no experiences for preventing cracks, particularly the successful experiences in China, thus, they are insufficient for engineers to design and construct mass concrete structures without crack.

A vast amount of mass concrete is placed in the world every year. How to prevent crack is still an important problem, thus *Thermal Stresses and Temperature Control of Mass Concrete* in English will be useful for engineers and professors of civil engineering.

In this book, consideration is given to both the theory and the practice. On one side, the methods for computing the temperature fields, thermal stresses, and the variation of temperatures and thermal stresses in various types of mass concrete structures are introduced in detail; on the other side, the technical measures to control temperature and to prevent cracking, the criterion of temperature control and the experiences of practical engineering projects, particularly, the successful experiences in China in the construction of several concrete dams without crack, are described. A series of new ideas and new techniques, e.g., the idea of "long time superficial thermal insulation together with comprehensive temperature control," MgO self-expansive concrete, etc., many useful methods, formulas, graphs, charts, and figures are given.

Apart from causing cracks, the change of temperature is an important and complex loading which has great influence on the stress state of concrete structures, particularly the arch dam. In the design and construction of mass concrete structures, particular attention should be paid to thermal stress and temperature control. I hope the publication of this book will give useful help to the engineers engaged in the design and construction of mass concrete structures and the professors and students of the department of civil engineering of universities.

I am grateful to Mr. Wu Longshen, Miss Hao Wengqian, and Mrs. Li Yue for their help given to me in the preparation of this book.

**Zhu Bofang**

July 2013



# About the Author

Zhu Bofang, the academician of the Chinese Academy of Engineering and a famous scientist of hydraulic structures and solid mechanics in China, was born in October 17, 1928 in Yujiang country, Jiangxi Province. In 1951, he graduated in civil engineering from Shanghai Jiaotong University, and then participated in the design of the first three concrete dams in China (Foziling dam, Meishan dam, and Xianghongdian dam). In 1957, he was transferred to the China Institute of Water Resources and Hydropower Research where he was engaged in the research work of high concrete dams. He was elected the academician of the Chinese Academy of Engineering in 1995. He is now the consultant of the technical committee of the Ministry of Water Resources of China, the consultant of the technical committee of water transfer from south part to north part of China, and a member of the consultant group of the Xiaowang dam, the Longtan dam, and the Baihetan dam. He was a member of the Eighth and the Ninth Chinese People's Consultative Conference, the board chairman of the Computer Application Institute of China Civil Engineering Society, and a member of the standing committee of the China Civil Engineering Society and the standing committee of the China Hydropower Engineering Society.

He is the founder of the theory of thermal stresses of mass concrete, the shape optimization of arch dams, the simulating computation of concrete dams, and the theory of creep of concrete in China.

He has established a perfect system of the theory of thermal stress and temperature control of mass concrete, including two basic theorems of creep of nonhomogeneous concrete structures, the law of variation, and the methods of computation of the thermal stresses of arch dams, gravity dams, docks, sluices, tunnels, and various massive concrete structures, the method of computation of temperature in reservoirs and pipe cooling, thermal stress in beams on foundation, cold wave, heightening of gravity dams, and the methods and criteria for control of temperatures. He proposed the idea of "long time thermal insulation as well as comprehensive temperature control" which ended the history of "no concrete dam without crack" and some concrete dams without crack had been constructed in China in recent years, including the Sanjianghe concrete arch dam and the third stage of the famous Three Gorges concrete gravity dam.

He proposed the mathematical model and methods of solution for shape optimization of arch dams, which was realized for the first time in the world and to date has been applied to more than 100 practical dams, resulting in a 10–30% saving of dam concrete and raising enormously the efficiency of design.

He developed the simulating computation of concrete dams and proposed a series of methods, including the compound element, different time increments in different regions, the equivalent equation of heat conduction for pipe cooling, and the implicit method for computing elastocreeping stresses by FEM.

He proposed the equivalent stress for FEM and its allowable values which had been adopted in the design specifications of arch dams in China, thus the condition for substituting the trial load method by FEM is provided.

The reason why houses and bridges were destroyed but no concrete dam was destroyed by strong earthquakes is explained. It is due to the fact that concrete dams must resist large horizontal water loads with large coefficients of safety in the ordinary loading case.

The instrumental monitoring can give the displacement field but cannot give the stress field and the coefficient of safety of concrete dams. In order to overcome this defect, a new idea for numerical monitoring has been proposed which can give the stress field and the coefficient of safety and raise the level of safety control of concrete dams.

The new idea for the semimature age of concrete has been proposed. The crack resistance of concrete may be promoted by changing its semimature age.

A vast amount of scientific research work has been conducted under his direction for a series of important concrete dams in China, such as Three Gorges, Xiaowan, Longtan, Xiluodu, Sanmenxia, Liujiaxia, and Xing'anjiang. More than 10 results of his scientific research were adopted in the design specifications of gravity dams, arch dams, docks, and hydraulic concrete structures.

He has published eight books: *Theory and Applications of the Finite Element Method* (1st ed. in 1979, 2nd ed. in 1998, 3rd ed. in 2009), *Thermal Stresses and Temperature Control of Mass Concrete* (1999), *Thermal Stresses and Temperature Control of Hydraulic Concrete Structures* (1976), *Theory and Applications of Structural Optimization* (1984), *Design and Research of Arch Dams* (2002), *Collected Works on Hydraulic Structures and Solid Mechanics* (1988), *Selected Papers of Academician Zhu Bofang* (1997), and *New Developments in Theory and Technology of Concrete Dams* (2009). He has published more than 200 scientific papers.

Academician Zhu was awarded the China National Prize of Natural Science in 1982 for his research work in thermal stresses in mass concrete, the China National Prize of Scientific Progress in 1988 for his research work in the optimum design of arch dams, and the China National Prize of Scientific Progress in 2001 for his research works in simulating computation and thermal stresses. He was awarded the ICOLD Honorary Member at Saint Petersburg in 2007.

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