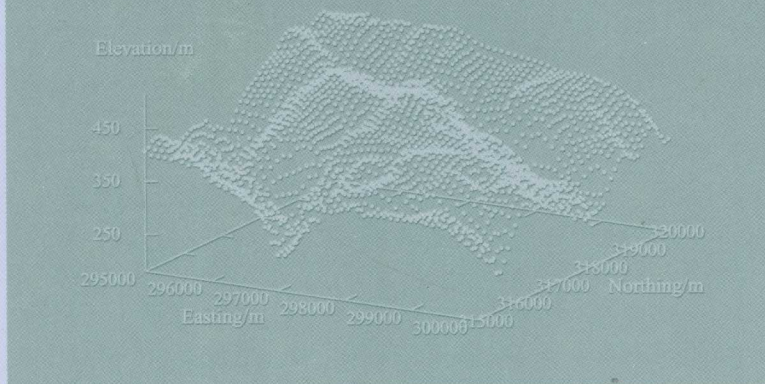


Michael J. Barnsley

ENVIRONMENTAL MODELING



A Practical Introduction



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A Practical Introduction

Preface

The initial motivation to write this book stemmed from the perception of a growing divide between the increasingly sophisticated computer-based models that are being developed to represent various aspects of Earth's environmental systems (including those pertaining to its climate, ecosystems, biogeochemical cycles and hydrological processes) and the ability of many undergraduate students, and even some graduate students, of the environmental sciences to engage constructively with these models. The aim of this book, therefore, is to provide a practical introduction to the various methods, techniques and skills involved in computerized environmental modeling, including (i) representing an environmental problem in conceptual terms (i.e., developing a conceptual model), (ii) formalizing the conceptual model using mathematical expressions (i.e., formulating a mathematical model), (iii) converting the mathematical model into a program that can be run on a desktop or a laptop computer (i.e., implementing a computational model) and (iv) examining the results produced by the computational model (i.e., visualizing the output from a model and checking the model's validity in comparison with observations of the target system).

The contents of this book are based on a course that I have taught for many years to honors degree undergraduate students at Swansea University. The objectives of the course, and of this book, are to introduce the student to the broad arena of environmental modeling, to show how computational models can be used to represent environmental systems, and to illustrate how such models can improve our understanding of the ways in which environmental systems function. Equally important, the book also aims to impart a set of associated analytical and practical skills, which will allow the reader to develop, implement and experiment with a range of computerized environmental models. The emphasis is, therefore, on active engagement in the modeling process rather than on passive learning about a suite of well-established models. A practical approach is adopted throughout, one that tries not to get bogged down in the details of the underlying mathematics and that encourages learning through "hands on" experimentation. To this end, a set of software tools and data sets are provided free-of-charge under the General Public License (GPL) and Gnuplot License so that the reader can work through the various examples and exercises presented in each chapter.

Most of the data sets used in this book relate to an area immediately south of Llyn Efyrynwy, Powys in mid-Wales, UK. Apart from the fact that this is a particularly beautiful part of the world, I chose this site because it is one of a relatively small

number of locations in the UK at which the solar irradiance measurements used in Chapter 5 are routinely recorded. I should also confess to deriving a certain amount of innocent amusement thinking about the additional challenge that the pronunciation of this particular Welsh place name will present to many readers. If nothing else, it will help to take the reader's mind off the demands of environmental modeling, every now and then.

I am deeply grateful to the UK Ordnance Survey, and in particular Ed Parson, for making available the digital elevation data used in Chapters 2 and 10. Thanks are also due to the UK MetOffice for permission to use the various meteorological data sets pertaining to Llyn Efyrynwy, and also to the staff at the British Atmospheric Data Centre (BADC), which is operated by the UK's Natural Environmental Research Council (NERC), for providing the excellent service through which I was able to access these data.

This book was put together using a range of "open source" software, including the GNU/Linux operating system. Most of the figures were produced using gnuplot (<http://www.gnuplot.info/>); the majority of the remainder were created using the PSTricks class in \LaTeX . \LaTeX (<http://www.lyx.org/>) and \LaTeX (<http://www.latex-project.org/>) were used to produce the camera-ready copy, and the Beamer class in \LaTeX was used to create the presentation files. I should like to thank the developers of all of these software packages.

I should also acknowledge the many cohorts of undergraduate students at Swansea University who have acted as a test bed for much of the material presented in this book: a sea of blank faces is undoubtedly the most immediate and effective signal that the material being presented is inadequately explained or otherwise confusing, and I hope that the salutary lessons that my students have taught me along the way have resulted in a clearer exposition in this book. On a more positive note, I am deeply gratified by those students who, having been introduced to computer-based environmental modeling for the first time, have honed their newly acquired skills and gone on to greater things. I hope that this is also the case for the readers of this book.

Finally, I should like to acknowledge the support of various friends and colleagues who have offered help and advice, and above all provided much-needed injections of humor at numerous points during the production of this book: to Mat Disney, Philip Lewis ("Lewis") and Tristan Quaife at University College London, to Tim Fearnside, Sietse Los, Adrian Luckman, Peter North and Rory Walsh at Swansea, and to Paul Mather at the University of Nottingham, *diolch yn fawr iawn* (thank you very much). Paul Mather deserves special mention for kindly reading through the drafts of this book and for providing many detailed comments and helpful suggestions. As it has become conventional to say at this point, though, any oversights, omissions or errors that remain are mine alone. I understand that in the world of "closed source" computer software such things are often described as "features"; I hope, however, that "features" of this type are few and far between herein. *Pob lwc!* (Good luck!)

Mike Barnsley

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