美国MCM/ICM竞赛指导丛书

Mathematical Modeling for the MCM/ICM Contests

Volume 1

MCM/ICM数学建模竞赛

第 1 卷

Jay Belanger
Amanda Beecher
Jie Wang

高等教育出版社

导丛书

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Foreword by Sol Garfunkel

While it is hard for me to believe, the Mathematical Contest in Modeling (MCM) is fast approaching its 30th year. During this time we have grown from 90 US teams to over 5000 teams representing 25 countries from all across the globe. We have been especially buoyed by the enthusiasm shown by our international colleagues and the rapid growth in international participation. COMAP welcomes your involvement with open arms.

COMAP runs three contests in mathematical modeling; they are MCM, ICM (the Interdisciplinary Contest in Modeling), and HiMCM (the High School Mathematical Contest in Modeling). The purpose of all of these contests has never been simply to reward student efforts – as important as that is. Rather, our objective from the beginning has been to increase the presence of applied mathematics and modeling in education systems at all levels worldwide. Modeling is an attempt to learn how the world works and the use of mathematics can help us produce better models. This is not a job for one country, but for all. The COMAP modeling contests were conceived and evolved to be strong instruments to help achieve this much larger goal.

It is my supreme hope that through this excellent book series students will learn more about COMAP contests and more about the process of mathematical modeling. I hope that you will begin to work on the exciting and important problems you see here, and that you will join the MCM/ICM contests and the rewarding work of increasing the awareness of the importance of mathematical modeling.

Sol Garfunkel, PhD
Executive Director
COMAP
November 2014

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Foreword by Chris Arney

Undergraduate students who receive instruction and experiences in mathematical modeling become better and more creative problem solvers and graduate students. This book series is being published to prepare and educate students on the topics and concepts of mathematical modeling to help them establish a problem solving foundation for a successful career.

Mathematical modeling is both a process and a mindset or philosophy. As a process, students need instruction and experience in understanding and using the modeling process or framework. As part of their experience, they need to see various levels of sophistication and complexity, along with various types of mathematical structures (discrete, continuous, linear, nonlinear, deterministic, stochastic, geometric, and analytic). As a mindset, students need to see problems that are relevant, challenging, and interesting so they build a passion for the process and its utility in their lives. A major goal in modeling is for students to want to model problems and find their solutions. Recipes for structured or prescribed problem solving (canned algorithms and formulas) do exist in the real world, but mathematical modelers can do much more than execute recipes or formulas. Modelers are empowered to solve new, open, unsolved problems.

In order to build sufficient experience in modeling, student exposure must begin as early as possible – definitely by the early undergraduate years. Then the modeling process can be reinforced and used throughout their undergraduate program. Since modeling is interdisciplinary, students from all areas of undergraduate study benefit from this experience.

The articles and chapters in this series expose the readers to model construction, model analysis, and modeling as a research tool. All these areas are important and build the students' modeling skills. Modeling is a challenging and advanced skill, but one that is empowering and important in student development. In today's world, models are often complex and require sophisticated computation or simulation to provide solutions or insights into model behavior. Now is an exciting time to be a skilled modeler since methodology to provide visualization and find solutions are more prevalent and more powerful than ever before.

I wish the students well in their adventure into modeling and I likewise wish faculty well as they use the examples and techniques in this book series to teach the modeling process to their students. My advice to all levels of modelers is to build your confidence and skills and use your talents to solve society's most challenging and important problems. Good luck in modeling!

Chris Arney, PhD
United States Military Academy at West Point
Professor of Mathematics
Director of the Interdisciplinary Contest in Modeling
October, 2011

Preface

This book series is a collection and expositions of the ideas, background knowledge, and modeling methodologies for solving the problems for the Mathematical Contest in Modeling (MCM) and the Interdisciplinary Contest in Modeling (ICM). It is intended to help promote, enrich, and advance mathematical modeling education for undergraduate students. It is also intended to provide guidance for students to participate in the MCM/ICM contests. It can be used not only as a reference book in mathematical modeling, but also as supplementary materials for teaching an undergraduate course on modeling.

This book series is co-published by the Higher Education Press (HEP) and the Consortium for Mathematics and Its Applications (COMAP), making it accessible worldwide to students and their faculty advisors, as well as to readers interested in modeling.

This volume addresses Problem A and Problem B in MCM 2014, and Problem C in ICM 2014. Problem A asks how to devise rules to increase traffic throughput, Problem B asks how to rank the top coaches of a popular sport, and Problem C asks how to use networks to measure influence and impact. In addition to the expositions of these problems, this book also presents a brief history of the MCM/ICM contests, offers ideas to help students prepare for the MCM/ICM contests, presents general modeling framework and methodologies, describes the judging procedure of the MCM/ICM papers, explains how to write attractive MCM/ICM papers, and presents a sample scheduling of tasks during the contest. A number of exercise problems are also included to help students understand the materials presented in the book.

Jay Belanger drafted Chapters 1 to 4. Amanda Beecher drafted Chapter 5. Jie Wang devised the book, contributed to some of the writings, and edited and unified all the chapters. The authors thank Sol Garfunkel, Chris Arney, and Bill Fox for inviting them to participate in judging the MCM/ICM 2014 contests, and the judges of the MCM/ICM 2014 contests and the COMAP staff for making judging

the contests a rewarding experience.

We would like to thank Ying Liu of HEP and Sol Garfunkel of COMAP for their insights, support, and guidance. Without them this book series would not have been published. We welcome and appreciate feedback from our readers. Please email your comments and suggestions to the following address: micmbooks@gmail.com.

Jie Wang
Editor-in-Chief
November 27, 2014

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Dr. Jay Belanger is a mathematics professor at Truman State University in Kirksville, Missouri, USA. He received his PhD degree in Mathematics from Princeton University in 1987 and his BS degree in Mathematics from the University of Michigan at Ann Arbor in 1983. He has published research papers in complex analysis, computational complexity theory, mathematical computing, and the history of mathematics. He has judged for the MCM contest and co-authored two books in the HEP series on MCM/ICM Contests Guides and Solutions. Since 2011 he has served as a member of the editorial board of the series.

Dr. Amanda Beecher is a mathematics professor at Ramapo College of New Jersey in Mahwah, New Jersey, USA. She received her PhD (2007), MS (2003), and BS (2001) degrees in Mathematics from the University at Albany, State University of New York. She publishes in commutative algebra. She taught at the United States Military Academy at West Point for three years, where she began involvement with the MCM/ICM contests. She was a judge for the ICM contest over the past seven years, and wrote commentaries to provide teams with additional feedback for the past four years. She was a final judge for the ICM Problem C in 2013 and 2014. She is serving as Head Judge for the 2015 ICM contest Problem D.

Dr. Jie Wang is Editor-in-Chief of the HEP book series on MCM/ICM Contests Guides and Solutions and COMAP's Director for China Partnerships. He is Professor and Chair of Computer Science at the University of Massachusetts in Lowell, Massachusetts, USA. He has written problems for the MCM and served as a final judge multiple times, including ICM 2014. He received his PhD degree in Computer Science from Boston University in 1990, MS degree in Computer Science from Zhongshan University in 1985, and BS degree in Computational Mathematics from Zhongshan University in 1982. His research interests include big data modeling and applications, algorithms and computational optimization, and network security. His research has been funded continuously by the National Science Foundation since 1991. IBM, Intel, Google, and the Natural Science Foundation of China also funded his research. He has published over 160 journal and conference papers, 6 books and 4 edited books. He is active in professional service, including chairing conference program committees, organizing workshops, and serving as an editor for a number of journals.

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1 Introduction

Beyond winning and losing, mathematical competitions can have a profound effect on both the contestants and the mathematical community at large. Listed below are some notable benefits of contests:

- Challenge the better students.
- Drum up interest for the subject.
- Encourage students to study more math and think about its applications.
- Encourage students to discuss math outside of classes.
- Encourage the schools to have more varied math classes.
- Encourage the schools to have more extracurricular math.
- Guide the schools about what sort of math to offer.
- Improve a student's resume.
- Provide students with competition experience, and maybe even provide some fun.

1.1 A Brief History of Math Contests

One of the earliest organized math contests, the Eötvös Contest for graduating secondary school students, began in Hungary in 1894. It changed its name to the the Kürschák Competition after World War II. The contestants have four hours to solve three problems, and the problems are designed to require cleverness rather than sophisticated mathematics. This contest has been cited as one of the reasons that Hungarian mathematics rose to prominence at the turn of the century. Winners of the contest include Fejér, Haar, Riesz, Szegö, and Radó, who were not only great mathematicians but also inspired their students to greatness. Hungary's early entrance into mathematical contests may have led to its success in the International Mathematical Olympiad (IMO). Since its inception in 1959, China, the United States, and Hungary are the countries with the most winners.

Despite their undeniable positive outcomes, competitions such as the Eötvös

Contest are not without criticisms. Math contests of the sort can be disheartening to contestants who fare poorly. They can also emphasize the wrong things and give a false impression of mathematical work by inflating the importance of quick and clever solutions rather than a deeper understanding of the material. In 1949, Hungary established a contest for college students, the Miklós Schweitzer Competition, which is almost the antithesis of the Eötvös Contest. Rather than requiring quickness and cleverness with basic mathematics, the Miklós Schweitzer Competition requires a deeper understanding of higher-level mathematics. The contestants are given ten days to solve ten problems, and they can use books and notes.

The William Lowell Putnam Mathematical Competition (Putnam), beginning in 1938, is the premier college level math contest in the United States. The contest takes place over a day, where the students have a three-hour morning session consisting of six problems, and a three-hour afternoon session consisting of another six problems. Like the Eötvös competition, solving the problems typically relies on cleverness rather than a knowledge of advanced mathematics and the winners include a veritable Who's Who of twentieth century mathematicians. The students do the work individually with no resources. Low scores are the norm for the Putnam, and the medium is often zero point.

1.2 The MCM

Ben Fusaro, a math professor at Salisbury State University, noticed that his students were reluctant to prepare for and take the Putnam exam; many considered it a chore, the emphasis on pure mathematics did not appeal to many of the students, and the low scores added to the lack of enthusiasm. Ben Fusaro thought that a contest similar to the Putnam, but with problems from applied rather than pure mathematics, might appeal to some students. As he was the chair of the Education Committee of the Society for Industrial and Applied Mathematics (SIAM), he sent an outline of a proposal to the committee. Similar to the Putnam, his original idea had the contest taking place over a day, with a three-hour morning and three-hour afternoon session, but with only one problem in each session (one problem involving continuous mathematics and one involving discrete mathematics). This was feasible largely because of the rise in personal computers, which the contestants would be allowed to use, but he was still concerned that this would not allow enough time for the students to work on interesting applied problems and he later changed the plan to have a team of students work on one problem over a weekend. Since the committee concentrated on pre-college education, they declined the proposal.

Solomon Garfunkel, the executive director of COMAP, the Consortium for Mathematics and Its Applications, told Ben Fusaro that he should ask for funding from the Department of Education's Fund for the Improvement of Postsecondary Education (FIPSE). From the proposal:

> The purpose of this competition is to involve students and faculty in clarifying, analyzing, and proposing solutions to open-ended problems. We propose a structure which will encourage widespread participation and emphasize the entire modeling process. Major features include:
>
> - The selection of realistic open-ended problems chosen with the advice of working mathematicians in industry and government.
> - An extended period for teams to prepare solution papers within clearly defined format.
> - The ability of participants to draw on outside resources including computers and texts.
> - An emphasis on clarity of exposition in determining final awards with the best papers published in professional mathematics journals.
>
> As the contest becomes established in the mathematics community, new courses, workshops, and seminars will be developed to help students and faculty gain increased experience with mathematical modeling.

This resulted in a three year grant from FIPSE.

The contest, called the Mathematical Contest in Modeling (MCM), was founded in 1984 and the first contest took place in 1985. For the contest, teams of three students choose between one of two problems called Problem A and Problem B. They are expected to create mathematical models to describe the problem and use them to come up with their solutions. Problem A is designed to require creating a model involving continuous mathematics, and Problem B is designed to require creating a model involving discrete mathematics. The team has three days to develop a model to solve the problem and write up their solution. While working on the problem, the team can use any inanimate resources, but they cannot get help from other people. The MCM became a highly-successful large international event that was later extended to a four-day contest.

Ben Fusaro expected about 55 teams the first year, but it turned out that 158

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