STUDENT'S STUDY GUIDE AND SOLUTIONS MANUAL

Colm Mulcahy ■ William L. Briggs

Using and Understanding Mathematics

A Quantitative Reasoning Approach

SECOND EDITION

Jeffrey O. Bennett ■ William L. Briggs

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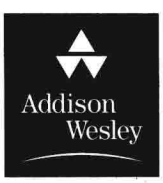

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STUDENT'S STUDY GUIDE

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Introduction

Welcome to the *Study Guide and Solutions Manual* for *Using and Understanding Mathematics*. Hopefully, with the help of this book, your course in quantitative reasoning will be both enjoyable and successful. The goal of this guide is not to add to your workload for the course, but to give you a set of concise notes that will make your studying as effective as possible. If you work with this guide as you read and do exercises, you should get the most from the course.

This guide is organized according to the units in the textbook. For each unit you will find the following features:

Overview

This section provides a brief survey of the unit, its major points and its goals. This feature is not a substitute for reading the text!

Key Words and Phrases

This section is simply a list of the important words and phrases used in the unit. You can use this section for review, study, and self-testing. You should be able to explain or define all of the terms on this list.

Key Concepts and Skills

In this section you will find a summary of the most important concepts in each unit. These concepts may be general ideas (for example, the distinction between deductive and inductive arguments) or basic skills (for example, creating the equation of a straight line). This section should also be helpful for review, study, and self-testing.

Important Review Boxes

If a unit has one or more Review Boxes, they are listed. These boxes are quite important for providing background skills or knowledge needed for the unit.

Solutions to (Most) Odd Problems

Following all of the unit summaries, the second half of this Guide provides full solutions to most of the odd-numbered problems in the textbook. Mathematics is not a spectator sport! Reading the solutions is never a substitute for working the problems. *You are strongly advised to work the problems first and then check the solutions.*

How to Succeed In This Course

Using *This* Book

Before we get into more general strategies for studying, here are a few guidelines that will help you use *this* book most effectively.

- Before doing any assigned problems, read assigned material *twice*:
  On the first pass, read quickly to gain a feel for the material and concepts presented.
  On the second pass, read the material in more depth, and work through the examples carefully.
- During the second reading, take notes that will help you when you go back to study later. In particular:
  Use the margins! The wide margins in this textbook are designed to give you plenty of room for making notes as you study.
  Don't highlight — underline! Using a pen or pencil to underline material requires greater care than highlighting, and therefore helps to keep you alert as you study.
- After you complete the reading, and again when studying for exams, make sure you can answer the *review questions* at the end of each unit.
- You'll learn best by *doing*, so work plenty of the end-of-unit exercises. Don't be reluctant to work more than the exercises that your instructor assigns.

Budgeting Your Time

A general rule of thumb for college courses is that you should expect to study about 2 to 3 hours per week *outside* class for each unit of credit. For example, a student taking 15 credit hours should spend 30 to 45 hours each week studying outside of class. Combined with time in class, this works out to a total of 45 to 60 hours per week — not much more than the time required of a typical job. If you find that you are spending fewer hours than these guidelines suggest, you can probably improve your grade by studying more. If you are spending more hours than these guidelines suggest, you may be studying inefficiently; in that case, you should talk to your instructor about how to study more effectively for a mathematics class.

General Strategies for Studying

- Don't miss class. Listening to lectures and participating in discussions is much more effective than reading someone else's notes. Active participation will help you retain what you are learning.
- Budget your time effectively. An hour or two each day is more effective, and far less painful, than studying all night before homework is due or before exams.
- If a concept gives you trouble, do additional reading or problem solving beyond what has been assigned. If you still have trouble, *ask for help*: you surely can find friends, colleagues, or teachers who will be glad to help you learn. Never be reluctant to ask questions or ask for help in this

course. If you have a question or problem, it is extremely unlikely that you will be alone!

- Working together with friends can be valuable; you improve your own understanding when discussing concepts with others. However, be sure that you learn *with* your friends and do not become dependent on them.

Preparing for Exams

- Rework exercises and other assignments; try additional exercises to be sure you understand the concepts. Study your assignments, quizzes, and exams from earlier in the semester.
- Study your notes from lectures and discussions. Pay attention to what your instructor expects you to know for an exam.
- Reread the relevant sections in the textbook, paying special attention to notes you have made in the margins.
- Study individually before joining a study group with friends. Study groups are effective only if *every* individual comes prepared to contribute.
- Try to relax before and during the exam. If you have studied effectively, you are capable of doing well. Staying relaxed will help you think clearly.

Finally, good luck! We wish you an enjoyable and rewarding experience with quantitative reasoning.

1 Thinking Critically

Overview

Before discussing Chapter 1, we urge you to take a few minutes to read the prologue to the textbook. This short chapter sets the stage for the entire book. It presents the idea of quantitative reasoning and discusses the importance of interdisciplinary thinking. It gives a high altitude picture of mathematics and how it impacts many other subjects that you will encounter either in other courses or in your career. Finally, it gives some advice on using the book and studying for your course. It's worth a quick reading. Now on to Chapter 1.

In teaching this course to many students of over many years, we know that often the most serious weakness that students bring to the course is not poor mathematical skills, but poor reasoning skills. Often it's not multiplying two numbers that creates problems, but deciding *when* to multiply! For this reason, the book opens with a chapter that contains virtually no mathematics. The emphasis of the chapter is critical thinking and logical skills.

In this chapter you will encounter some introductory logic, but don't worry; we don't get carried away with symbolic logic and heavy-duty truth tables. In fact, much of this chapter may be familiar to you from previous courses in logic or philosophy.

Unit 1A Recognizing Fallacies

Unit 1A opens the chapter by explaining that a logical **argument** (as opposed to an everyday argument) is a set of facts or assumptions, called **premises**, that lead to a **conclusion**. A **fallacy** is an argument that is either deceptive or wrong. This unit explores common fallacies that you might encounter in advertising or (bad) news reports. We present ten different so-called informal fallacies, some of which may seem quite obvious, others of which may be quite subtle. Critical reading and thinking will help you avoid becoming a victim of these fallacies!

Key Words and Phrases

logic	argument	premise
conclusion	fallacy	appeal to popularity
false cause	appeal to ignorance	hasty generalization
limited choice	appeal to emotion	personal attack
circular reasoning	diversion	straw man

Key Concepts and Skills

- identify the premise and conclusion of an argument.
- recognize informal fallacies in advertisements and news report.

Unit 1B Propositions and Truth Values

Overview

In this unit formal logic is introduced in a somewhat casual way. We start with **propositions** — statements that make a claim that can be true or false. Then we look at the **connectors** that can be used with propositions to make more complex propositions. The connectors that you will encounter are

- *not* (negation)
- *or* (disjunction)
- *and* (conjunction)
- *if ... then* (implications).

Whereas many logic books make heavy use of symbolic logic and truth tables, we will use truth tables primarily for fairly simple propositions that involve one, two, or three connectors. So our excursion into symbolic logic will be limited and designed to provide only an introductory glimpse.

The *if ... then* connector is quite important in both logic and everyday speech (for example, *if I pass this course, then I will graduate*). For this reason, we spend a little time discussing other forms of the proposition *if P, then Q*. These other forms are the

- **converse** (*if Q, then P*),
- **inverse** (*if not P, then not Q*), and
- **contrapositive** (*if not Q, then not P*).

This particular discussion may seem a bit technical, but it's also extremely practical. For example, suppose it's true that *if I read the book, then I will pass the course*. Does it follow that *if I don't read the book, then I won't pass the course*? You will see!

Key Words and Phrases

proposition	truth table	negation
conjunction	disjunction	conditional
antecedent	consequent	converse
inverse	contrapositive	logical equivalence

Key Concepts and Skills

- understand negation, conjunction, disjunction, and conditionals and their truth tables.
- use truth tables to evaluate the truth of compound propositions that use two or more connectors.
- analyze various forms of *if ... then* propositions.

Unit 1C Sets and Venn Diagrams

Overview

You may have encountered Venn diagrams before now as a way to illustrate the relationships between collections of objects, or **sets**. In this unit, we review some of the most basic properties of sets and then discuss how Venn diagrams can be used to work with sets. Throughout the unit, the emphasis is on practical applications of sets and Venn diagrams. One of the important applications of Venn diagrams is to illustrate what are called **categorical propositions** of logic. We will study four basic categorical propositions. Given a **subject set** S and a **predicate set** P, they may be related in the following ways:

- All S are P (for example, all whales are mammals)
- No S are P (for example, no fish are mammals)
- Some S are P (for example, some doctors are women)
- Some S are not P (for example, some teachers are not men).

As we will see, each form of categorical proposition has a specific Venn diagram. Equally important, the negation of each categorical proposition is one of the other propositions in the list above. Specifically, we have the following relations between the four propositions and their negations.

Proposition	Negation
All S are P	Some S are not P
No S are P	Some S are P
Some S are P	No S are P
Some S are not P	All S are P

The unit concludes with a nice collection of other uses of Venn diagrams. As you will see, Venn diagrams with two or more circles can be used to organize all sorts of information.

Key Words and Phrases

set	Venn diagram	subset
disjoint sets	overlapping sets	categorical propositions

Key Concepts and Skills

- use set notation.
- construct Venn diagrams for categorical propositions.
- put propositions in standard form.
- negate categorical propositions.
- construct Venn diagrams for three or more sets.

Important Review Box

- A Brief Review of Sets of Numbers

Unit 1D Analyzing Arguments

Overview

The propositions that we studied in the previous unit can be combined in various ways to form arguments. Of primary importance is the distinction between **deductive** and **inductive** arguments. Deductive arguments generally proceed from general premises to a more specific conclusion. As we will see, in a deductive argument all of the premises are needed to reach the conclusion. By contrast, inductive arguments generally proceed from specific premises to a general conclusion; in an inductive argument the premises independently support the conclusion.

To analyze inductive arguments, we ask about the **strength** or **weakness** of the argument. Determining the strength of an inductive argument is often a subjective judgment, and so there are no systematic methods to apply.

Most of the unit is spent analyzing three-line deductive arguments using Venn diagrams. They can consist of the four types of categorical propositions or they may involve conditional propositions (studied in Unit 1C). Of fundamental importance in this business is the distinction between valid and invalid arguments. An argument is **valid** if, based on the Venn diagram analysis, it is logically solid and consistent. An argument that fails the Venn diagram analysis must contain a fallacy and is **invalid**. Validity has nothing to do with the truth of the premises or conclusion; it is a measure of the logical structure of the argument.

Having shown that an argument is valid, we can then ask if it is sound. A **sound** argument is valid *and* has true premises; a sound argument must lead to a true conclusion. Soundness is the highest test of a deductive argument.

Fallacies can arise in deductive arguments in many ways. Perhaps the most common fallacies occur in arguments that involve conditional (*if...then*) propositions. These fallacies appear in the everyday arguments of advertising and news reports. There are four different forms of conditional arguments; two are valid and two are invalid:

- affirming the antecedent (valid)
- affirming the consequent (invalid)
- denying the antecedent (invalid)
- denying the consequent (valid).

The unit closes with examples of how inductive and deductive arguments are used in mathematics.

Key Words and Phrases

deductive	inductive	strength/weakness
valid/invalid	sound	affirming the

antecedent

affirming the consequent | denying the antecedent | denying the consequent

Pythagorean theorem

Key Concepts and Skills

- know the distinction between deductive and inductive arguments.
- determine the strength of inductive arguments.
- assess the validity of three-line deductive arguments consisting of categorical propositions, using Venn diagrams.
- assess the validity of three-line deductive arguments involving conditional propositions, using Venn diagrams.
- identify fallacies that arise in conditional arguments.
- understand various combinations of valid/invalid and sound/unsound that can occur in deductive arguments.
- determine the soundness of three-line deductive arguments.

Unit 1E Critical Thinking in Everyday Life

Overview

Critical thinking is an approach to problem solving and decision-making that involves careful reading (or listening), sharp thinking, logical analysis, good visualization, and healthy skepticism. In this chapter, we present several guidelines design to sharpen critical thinking skills as they apply to practical problems. The following guidelines are all accompanied by one or more specific examples.

- Read or listen carefully
- Look for hidden assumptions
- Identify the real issue
- Use visual aids (pictures, diagrams, tables)
- Understand all the options
- Watch for fine print and misinformation
- Are other conclusions possible?

Key Concepts and Skills

- apply the guidelines of the unit to practical decisions and problems

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2 Approaches to Problem Solving

Overview

The first chapter of the book was devoted to *qualitative* issues — topics that don't require extensive use of numbers and computation. In this chapter (and the remainder of the book) we turn to *quantitative* matters. Perhaps it's not surprising that we begin our study of quantitative topics with problem solving. The first two units of the chapter deal with a very basic and important problem solving technique, the use of units. The last unit of the chapter presents various problem-solving strategies. It is a valuable chapter whose lessons run through the rest of the book.

Unit 2A The Problem-Solving Power of Units

Overview

Nearly every number that you encounter in the real world is a measure of *something*: 6 billion *people*, 5280 *feet*, 5 trillion *dollars*, 26 *cubic feet*. The quantities that go with numbers are called **units**. And the message of this unit of the book is that using units can simplify problem solving immensely. Indeed the use of units is one of the most basic problem solving tools.

We begin by considering the most basic **simple units** for the fundamental types of measurement. Here are a few examples of simple units.

- length — inches, feet, meters
- weight — pounds, grams
- capacity — quarts, gallons, liters
- time — seconds, hours.

From these simple units, we can build endless **compound units**.

The rest of the unit is spent illustrating a wide variety of such compound units. Among the many you will meet and work with are units of

- units of area such as square feet and square yards
- units of volume such as cubic inches and cubic feet
- units of speed such as miles per hour
- units of price such as dollars per pound
- units of gas mileage such as miles per gallon.

One of the realities of life is that there are many units for the same quantity. For example, we can measure lengths in inches, centimeters, feet, meters, miles, or kilometers. For example, we measure rooms in square feet, but have to buy carpet in square yards. Thus one of the necessities of problem solving is being able to convert from one unit to another consistent unit. The key to doing conversions between units is to realize that there are three equivalent ways to express a conversion factor. For example, we can say 1 foot = 12

inches, or we can say 1 foot *per* 12 inches, or we can say 12 inches *per* foot. Mathematically, we can write

$$1 \text{ foot} = 12 \text{ inches} \quad \text{or} \quad \frac{1 \text{ foot}}{12 \text{ inches}} \quad \text{or} \quad \frac{12 \text{ inches}}{1 \text{ foot}}.$$

These three forms of the same conversion factor are absolutely equivalent. The key to a happy life with units is choosing the appropriate form of the conversion factor for a given situation.

Additional problem solving skills can then be built on these fundamental skills. You will see and learn how to make a chain of conversions factors to solve more complex problems. For example, the time required to count a billion dollars at the rate of one dollar per second is

$$\$1{,}000{,}000{,}000 \times \frac{1 \text{ sec}}{\$1} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365 \text{ days}} = 31.7 \text{ years}.$$

The fact that the units cancel and give an answer in *years* tells you that the problem has been set up correctly.

Finally, we look at a very practical type of unit conversion problem, those associated with currency. If you travel to France you learn that 1 franc is equal to about 18 cents. From this fact, you might need to answer questions such as

- which is larger, 1 franc or 1 dollar?
- how many francs in a dollar?
- how many dollars in a franc?
- how many dollars in a 23.45 francs?
- if apples cost 23 francs per kilogram, what is the price in dollars per pound?

Here are a few final words of advice: Never was the motto *practice makes perfect* more true than with problem solving and units. You should work all assigned exercises, and then some, in order to master these techniques. And unless your instructor tells you otherwise, there is no need to memorize hundreds of conversion factors. It's helpful to know a few essential conversion factors off the top of your head. As for the rest, it's easiest just to know how to find them quickly in the book.

Key Words and Phrases

simple units	compound units	area
volume	conversion factor	

Key Concepts and Skills

- convert from one simple unit to another; for example, from inches to yards.
- convert from one unit of area to another unit of area; for example, from square inches to square yards.
- convert from one unit of volume to another unit of area; for example, from cubic inches to cubic yards.

- solve problems involving chains of conversion factors; for example, finding the number of seconds in a year.
- convert from one unit of currency to another; for example, from dollars to pesos.

Important Review Box

- A Brief Review of Working with Fractions

Unit 2B Standardized Units: More Problem-Solving Power

Overview

In this unit we continue the study of units and conversion factors, but now explore the two standard systems of units: The U.S. Customary System of Measurement (or USCS system, which is used primarily in the Unites States) and the metric system (which is used everywhere else in the world).

We first proceed systematically and survey the USCS units for length, weight, and capacity. Tables 2.2, 2.3, and 2.4 contain many conversion factors, but you should focus on *using* these conversion factors, not memorizing them! Having seen the complications of the USCS system, the metric system should come as a welcome relief. Next we present the metric units for length, weight and capacity. As you will see, the system is based on powers of ten and standard prefixes (Table 2.5), which makes conversions between units relatively simple.

Unfortunately, for people living in the United States, conversions between the metric system and the USCS system are necessary. (If you would like to avoid doing such exercises, then you should work on getting the United States to go metric!) Table 2.6 has a few of the essential conversion factors between the metric and USCS systems. You may want to write other useful conversion factors in the margin near this table.

Finally, we explore two more categories of units. **Energy** (what makes things move or heat up) and **power** (the rate at which energy is used) are incredibly important concepts in understanding the world around us. Uses of energy and power units are presented in matters such as utility bills, diet, and the environment. The other category of units is **density** and **concentration**. In measuring population density, the capacity of computer discs, levels of pollution, and blood alcohol content, these units are indispensable.

Armed with all of these units and conversion factors (and don't forget currency conversion factors as well), we can do even more elaborate problem solving; this is the goal of the remainder of the unit and the problems at the end of the unit.

Key Words and Phrases

USCS system metric system meter