



# COLLEGE ALGEBRA

Charles Henry Brase  
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D. C. Heath and Company  
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*This book is dedicated to an inspiring teacher,  
humanitarian, and friend*

Frederick T. Daly, S.J.  
Professor Emeritus, Regis College



# Preface

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This text is designed to build skills through problem solving. Our approach is based on the premise that the student must be an *active participant* in the learning process. To aid student learning, we have included the following special features.

*Diagnostic Test:* A test covering the review topics of basic algebra with problems keyed to sections of Chapter 1 is included.

*Single-Concept Units:* Each section is divided into single-concept units with *examples* in each unit.

*Examples:* The text includes a great many carefully selected examples. These examples are worked out in detail with justification given for each major step.

*Graded Section Exercises:* Exercises at the end of each section require the student to use the new concepts mastered in that section. The exercises are broken into two groups. Group A exercises are fairly routine and are keyed to the single concepts in each section. Group B exercises are more challenging; since they are *not* keyed to the single concepts, the student must first determine an appropriate solution method. Some of the problems in Group B are more general; others are more advanced. *Answers* to the odd-numbered problems in both Group A and Group B exercises are included at the back of the book. Answers to the even-numbered problems are included in the Instructor's Guide.

*Chapter Review Problems:* Each chapter includes chapter review problems. While section exercises cover material from only one section, the chapter review problems are taken from the entire chapter. A section key indicating the section affiliation of each review problem appears at the end of each chapter. Answers to *all* chapter review problems (except proofs) are included at the back of the book.

*A Large Number of Exercises and Problems:* There are more than 2400 exercises and problems in all, many of which have several parts.

*Applications:* A wide variety of applications to real-life situations is included in the examples, exercises, and problems whenever appropriate. Every effort has been made to include realistic and interesting applications.

*Calculator and Computer Usage:* Where appropriate, we have included exercises to utilize the advantages of a calculator. These are designated with a †. The chapter on logarithms encourages calculator use, although logarithm tables are provided in the



appendix. Since many students are interested in computers and some are acquainted with the widely used computer language BASIC, we have included occasional examples and exercises that refer to this language. Although no knowledge of a computer language is assumed, Chapter 9 contains a program to approximate real zeros of polynomials. Students can use this program to explore some polynomial functions on any computer that accepts BASIC.

*Flexibility:* The text can easily be tailored to fit the particular needs of each class. There are alternative paths through the text that allow the option of teaching different chapter sequences during a course. If the Diagnostic Test indicates certain student weaknesses in basic algebra, the appropriate sections in Chapter 1 should start the course. However, if the test is successfully completed, the sequence can start with Chapter 2 if time is short. Chapter 4 and Chapter 9 can be either omitted or taught in the usual order depending on time and preference. Chapter 9 can also be covered following Chapter 5 instead of Chapter 8.

*Instructor's Guide:* An instructor's guide is available. It includes a test bank with two versions of tests for each chapter. Answers to even-numbered problems are also included.

**Acknowledgments:** It is with pleasure that we thank the many people who assisted us in the writing of this book. In particular, we wish to thank Carol Jacobs, who carefully read the manuscript, and Diane Wagner, who gave us some valuable advice on the computer examples of Chapter 9.

We also thank the following reviewers for their helpful comments and suggestions: Richard A. Fritz, Moraine Valley Community College; Lotus Hershberger, Illinois State University; George W. Johnson, University of South Carolina; William Perry, Texas A & M University; and Robert G. Savage, North Carolina State University.

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# To the Student

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Studies about the ways students learn mathematics show that you will learn college algebra more quickly and retain it better if you start working with the subject right away. You cannot be a passive agent in the learning process. Years of experience show that immediate reinforcement of new material through active problem solving is the best way for you to learn (or review) college algebra.

This book has several features to help you learn college algebra better, and build confidence as you go. Each section in the book is broken into a number of single-topic units. In each unit you will find explanations and examples about the material you are to learn. After you have finished several topics, you will come to the end of a section and find section problems. Section problems are broken into two groups. The group A problems are keyed according to topic headings of the section. If you need help with a group A problem, refer to the indicated topic heading of the section for explanations and examples. Group B problems are not grouped by topics, and some of the B problems may be a little harder. Work over these problems carefully; they will give you needed maturity for later work.

Answers to odd-numbered section problems are in the back of the book. Be honest with yourself; don't peek in the back of the book before you try to solve the problems. If you try the problems first and then check your work you will soon find you are learning college algebra more rapidly and retaining it better. If your solutions check, reward yourself with a little study break. Then come back refreshed and confident to study the next section. If any solution doesn't check, read your own work carefully and see if you can spot the error. Carefully read, and if necessary reread, the explanations and examples under the topic heading of the section corresponding to the topic heading of the problem. This process will normally help you work the problem correctly. If you still need help, ask your instructor for assistance.

Be sure to look over the review problems at the end of each chapter. They will help you prepare for exams over larger stretches of material. Remember, answers to all chapter review problems and all odd-numbered section problems can be found in the back of the book so that you can check your progress.

We sincerely hope the features of this book will help you master and enjoy college algebra.



## Diagnostic Test with Problems Keyed to Sections of Chapter 1

Chapter 1 reviews basic algebra. To see how much basic algebra you need to review, work the following problems. Each problem is worked out completely as an example in the section indicated to the right of the problem. For your convenience, the answers are also listed at the end of the text.

<u>Problem</u>	<u>Section</u>
1. Simplify by writing the product $8(2x + 5)$ as a sum of products.	1.1
2. Simplify $ -8 $ .	1.1
3. Simplify $-4 - (-8)$ .	1.2
4. Reduce to lowest terms: $-\frac{60}{96}$ .	1.2
5. Simplify and reduce $6\left(-\frac{5}{81}\right)\left(\frac{9}{25}\right)$ .	1.2
6. Simplify and reduce $\frac{9}{5} \div \frac{3}{4}$ .	1.2
7. Simplify and reduce $\frac{2}{15} - \frac{4}{9}$ .	1.2
8. Identify the base and exponent and write in factored form: $(2x + 1)^5$ .	1.3
9. Simplify $(-3x^2y)(2xy^4)(-xy^3)$ .	1.3
10. Simplify $\frac{16x^7y}{8x^2y^5}$ .	1.3
11. Simplify $\left(-\frac{3}{y}\right)^3$ .	1.4
12. Simplify $\left(\frac{12x^4y^2}{3xy^9}\right)^3$ .	1.4
13. Simplify and write with positive exponents $(5x^4)(2x^{-6})$ .	1.4
14. Simplify $4\sqrt{9} - \sqrt{121}$ .	1.5

*continued on next page*



*(continued)*

<u>Problem</u>	<u>Section</u>
15. Simplify $\sqrt{0.04}$ .	1.5
16. Assume $x > 0$ and $y > 0$ and simplify $\sqrt{\frac{3x^3}{2y}} \sqrt{\frac{3xy}{8}}$ .	1.5
17. Simplify $4x(x + 3) + (-3)(x^2 - 6x + 7)$ .	1.6
18. Simplify $19x^5 + 3x^4 + 2x^2 - 1 - (17x^5 + x^4 - 4x^3 + x^2)$ .	1.6
19. Given $2x^4 - 9x^5 + x + 4$ , identify the leading term, the constant term, and the degree of the polynomial.	1.6
20. Multiply and simplify $(x + 2)(x + 5)$ .	1.6



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# Chapter 1

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## Review of Basic Algebra

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### 1.1 Properties of Real Numbers

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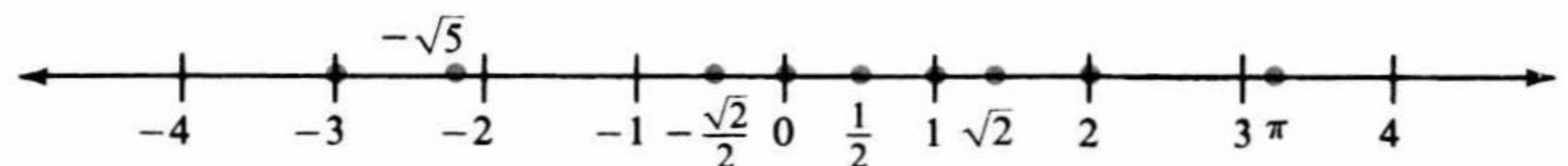
We use numbers almost every day to describe quantities such as prices, class size, grade averages, cost effectiveness, energy production, population changes, and so forth. An important collection of numbers is the *real numbers*.

#### Real Number Line

A useful model of the real numbers is given by the number line. Every real number corresponds to a point on the real number line, and every point on the number line corresponds to a real number.

From the number line (Figure 1.1), we see that the following are examples of real numbers:  $0$ ,  $1$ ,  $\frac{1}{2}$ ,  $\sqrt{2}$ ,  $2$ ,  $\pi$ ,  $-3$ ,  $-\sqrt{5}$ ,  $-\sqrt{2}/2$ .

Figure 1.1 Real number line.



Real numbers include the following types of numbers:

Natural numbers:  $1, 2, 3, \dots$

Whole numbers:  $0, 1, 2, 3, \dots$

Integers:  $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$



Rational numbers: Numbers that can be written in the form

$$\frac{a}{b}$$

where  $a$  and  $b$  are integers and  $b \neq 0$ . In decimal representation, rational numbers have decimal parts that terminate, such as  $\frac{1}{4} = 0.25$ , or that begin to repeat, such as  $\frac{1}{3} = 0.33\overline{3}$  or  $\frac{1}{6} = 0.1\overline{6}$ , where the bar is over the repeating portion.

Irrational Numbers: Numbers that cannot be expressed as a ratio of two integers. In decimal form, the decimal parts are nonterminating and nonrepeating. For example,  $\sqrt{2}$  is an irrational number.

### Order of Real Numbers

The number line shows the order of the real numbers. As we move to the *right* on the number line, the numbers *increase*. We see (Figure 1.2) that  $-2$  is larger than  $-3$ ,  $0$  is larger than any negative number, and so forth. In symbols we have

$x > y$  read “ $x$  is greater than  $y$ ” if  $x$  lies to the right of  $y$  on the number line  
 $x < y$  read “ $x$  is less than  $y$ ” if  $x$  lies to the left of  $y$  on the number line

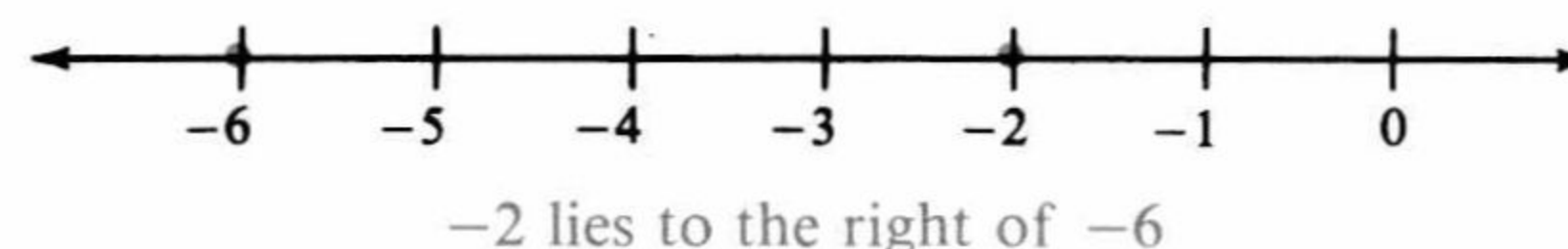
Figure 1.2 Real number line.



#### Example 1

Compare  $-6$  and  $-2$ .

Figure 1.3 Comparing  $-6$  and  $-2$  on the real number line.



On the number line (Figure 1.3) we see that  $-2$  lies to the right of  $-6$ , so  $-2$  is greater than  $-6$  and we can write  $-2 > -6$ . But we can also observe that  $-6$  lies to the left of  $-2$ , so  $-6$  is less than  $-2$  and we can write  $-6 < -2$ . The two expressions  $-6 < -2$  and  $-2 > -6$  express the same relationship between  $-6$  and  $-2$ .

### Properties of Real Numbers

The real numbers obey several properties when we combine them with either addition or multiplication. These properties provide a basis for doing arithmetic with real numbers and for simplifying algebraic expressions representing real numbers.

#### 1. Closure Properties

The sum of any two real numbers is again a real number. The product of any two real numbers is again a real number.