

A Practical Approach to

Signals and Systems

D. Sundararajan

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A PRACTICAL APPROACH TO SIGNALS AND SYSTEMS

D. Sundararajan



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A PRACTICAL APPROACH TO SIGNALS AND SYSTEMS

Preface

The increasing number of applications, requiring a knowledge of the theory of signals and systems, and the rapid developments in digital systems technology and fast numerical algorithms call for a change in the content and approach used in teaching the subject. I believe that a modern signals and systems course should emphasize the practical and computational aspects in presenting the basic theory. This approach to teaching the subject makes the student more effective in subsequent courses. In addition, students are exposed to practical and computational solutions that will be of use in their professional careers. This book is my attempt to adapt the theory of signals and systems to the use of computers as an efficient analysis tool.

A good knowledge of the fundamentals of the analysis of signals and systems is required to specialize in such areas as signal processing, communication, and control. As most of the practical signals are continuous functions of time, and since digital systems are mostly used to process them, the study of both continuous and discrete signals and systems is required. The primary objective of writing this book is to present the fundamentals of time-domain and frequency-domain methods of signal and linear time-invariant system analysis from a practical viewpoint. As discrete signals and systems are more often used in practice and their concepts are relatively easier to understand, for each topic, the discrete version is presented first, followed by the corresponding continuous version. Typical applications of the methods of analysis are also provided. Comprehensive coverage of the transform methods, and emphasis on practical methods of analysis and physical interpretation of the concepts are the key features of this book. The well-documented software, which is a supplement to this book and available on the website (www.wiley.com/go/sundararajan), greatly reduces much of the difficulty in understanding the concepts. Based on this software, a laboratory course can be tailored to suit individual course requirements.

This book is intended to be a textbook for a junior undergraduate level one-semester signals and systems course. This book will also be useful for self-study. Answers to selected exercises, marked *, are given at the end of the book. A Solutions manual and slides for instructors are also available on the website (www.wiley.com/go/sundararajan). I assume responsibility for any errors in this book and in the accompanying supplements, and would very much appreciate receiving readers' suggestions and pointing out any errors (email address: d.sundararajan@yahoo.com).

I am grateful to my editor and his team at Wiley for their help and encouragement in completing this project. I thank my family and my friend Dr A. Pedar for their support during this endeavor.

D. Sundararajan

Abbreviations

dc: Constant

DFT: Discrete Fourier transform

DTFT: Discrete-time Fourier transform

FT: Fourier transform

FS: Fourier series

IDFT: Inverse discrete Fourier transform

Im: Imaginary part of a complex number or expression

LTI: Linear time-invariant

Re: Real part of a complex number or expression

ROC: Region of convergence

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1

Introduction

In typical applications of science and engineering, we have to process signals, using systems. While the applications vary from communication to control, the basic analysis and design tools are the same. In a signals and systems course, we study these tools: convolution, Fourier analysis, z -transform, and Laplace transform. The use of these tools in the analysis of linear time-invariant (LTI) systems with deterministic signals is presented in this book. While most practical systems are nonlinear to some extent, they can be analyzed, with acceptable accuracy, assuming linearity. In addition, the analysis is much easier with this assumption. A good grounding in LTI system analysis is also essential for further study of nonlinear systems and systems with random signals.

For most practical systems, input and output signals are continuous and these signals can be processed using continuous systems. However, due to advances in digital systems technology and numerical algorithms, it is advantageous to process continuous signals using digital systems (systems using digital devices) by converting the input signal into a digital signal. Therefore, the study of both continuous and digital systems is required. As most practical systems are digital and the concepts are relatively easier to understand, we describe discrete signals and systems first, immediately followed by the corresponding description of continuous signals and systems.

1.1 The Organization of this Book

Four topics are covered in this book. The time-domain analysis of signals and systems is presented in Chapters 2–5. The four versions of the Fourier analysis are described in Chapters 6–9. Generalized Fourier analysis, the z -transform and the Laplace transform, are presented in Chapters 10 and 11. State space analysis is introduced in Chapters 12 and 13.

The amplitude profile of practical signals is usually arbitrary. It is necessary to represent these signals in terms of well-defined basic signals in order to carry out

efficient signal and system analysis. The impulse and sinusoidal signals are fundamental in signal and system analysis. In Chapter 2, we present discrete signal classifications, basic signals, and signal operations. In Chapter 3, we present continuous signal classifications, basic signals, and signal operations.

The study of systems involves modeling, analysis, and design. In Chapter 4, we start with the modeling of a system with the difference equation. The classification of systems is presented next. Then, the convolution–summation model is introduced. The zero-input, zero-state, transient, and steady-state responses of a system are derived from this model. System stability is considered in terms of impulse response. The basic components of discrete systems are identified. In Chapter 5, we start with the classification of systems. The modeling of a system with the differential equation is presented next. Then, the convolution-integral model is introduced. The zero-input, zero-state, transient, and steady-state responses of a system are derived from this model. System stability is considered in terms of impulse response. The basic components of continuous systems are identified.

Basically, the analysis of signals and systems is carried out using impulse or sinusoidal signals. The impulse signal is used in time-domain analysis, which is presented in Chapters 4 and 5. Sinusoids (more generally complex exponentials) are used as the basic signals in frequency-domain analysis. As frequency-domain analysis is generally more efficient, it is most often used. Signals occur usually in the time-domain. In order to use frequency-domain analysis, signals and systems must be represented in the frequency-domain. Transforms are used to obtain the frequency-domain representation of a signal or a system from its time-domain representation. All the essential transforms required in signal and system analysis use the same family of basis signals, a set of complex exponential signals. However, each transform is more advantageous to analyze certain types of signal and to carry out certain types of system operations, since the basis signals consists of a finite or infinite set of complex exponential signals with different characteristics—continuous or discrete, and the exponent being complex or pure imaginary. The transforms that use the complex exponential with a pure imaginary exponent come under the heading of Fourier analysis. The other transforms use exponentials with complex exponents as their basis signals.

There are four versions of Fourier analysis, each primarily applicable to a different type of signals such as continuous or discrete, and periodic or aperiodic. The discrete Fourier transform (DFT) is the only one in which both the time- and frequency-domain representations are in finite and discrete form. Therefore, it can approximate other versions of Fourier analysis through efficient numerical procedures. In addition, the physical interpretation of the DFT is much easier. The basis signals of this transform is a finite set of harmonically related discrete exponentials with pure imaginary exponent. In Chapter 6, the DFT, its properties, and some of its applications are presented.

Fourier analysis of a continuous periodic signal, which is a generalization of the DFT, is called the Fourier series (FS). The FS uses an infinite set of harmonically related continuous exponentials with pure imaginary exponent as the basis signals.