S.T.(P)
# Technology Today Series

A Series for Technicians

MATHEMATICS FOR TECHNICIANS

A. Greer & G. W. Taylor

Level II
Analytical Mathematics

S.T.(P)
# Technology Today Series

A Series for Technicians

MATHEMATICS FOR TECHNICIANS

Level II Analytical Mathematics

A. Greer
C.Eng., M.R.Ae.S.
Senior Lecturer
City of Gloucester College of Technology

G. W. Taylor
B.Sc.(Eng.), C.Eng., M.I.Mech.E.
Principal Lecturer
City of Gloucester College of Technology

Stanley Thornes (Publishers) Ltd.

First published in 1978 by:
Stanley Thornes (Publishers) Ltd
EDUCA House
Liddington Industrial Estate
Leckhampton Road
CHELTENHAM GL53 0DN
England

Reprinted 1979

ISBN 0 85950 064 0

Typeset in Monotype 10/12 Times New Roman by
Gloucester Typesetting Co Ltd
Printed and bound in Great Britain by
The Pitman Press, Bath

S.T.(P)
# Technology Today Series

A Series for Technicians

MATHEMATICS FOR TECHNICIANS

Level II
Analytical Mathematics

AUTHORS' NOTE ON THE SERIES

Arising from the recommendations of the Haslegrave Report, the Technician Education Council has been set up. The Council has devised 'standard units', leading in various subjects to the award of its Certificates and Diplomas. The units are constructed at three levels, I, II and III.

A major change of emphasis in the educational approach adopted in T.E.C. Standard Units has been introduced by the use of 'objectives' throughout the courses, the intention being to allow student and lecturer to achieve planned progress through each unit on a step-by-step basis.

This set of books provides all the mathematics required for the T.E.C. Standard Units at each of the three levels. Each book follows a standard pattern, and each chapter opens with the words "After reaching the end of this chapter you should be able to:-" and this statement is followed by the objectives for that particular topic as laid down in the Standard Unit. Thereafter each chapter contains explanatory text, worked examples, and copious supplies of further exercises. As planned at present the series comprises:-

AN INTRODUCTORY COURSE	Level I (full unit)
MECHANICAL ENGINEERING MATHEMATICS	Level II (half-unit)
PRACTICAL MATHEMATICS	Level II (half-unit)
ANALYTICAL MATHEMATICS	Level II (half-unit)
ELECTRICAL ENGINEERING MATHEMATICS	Level II (full-unit)
MATHEMATICS FOR ENGINEERING TECHNICIANS	Level III (full unit)

A. Greer
G. W. Taylor

Gloucester, 1978

CONTENTS

1. SOLUTION OF EQUATIONS

On reaching the end of this chapter you should be able to:-

1. *Solve a pair of simultaneous equations in two unknowns by (i) substitution, by (ii) elimination, and (iii) graphically.*
2. *Define the roots of an equation.*
3. *Determine the equation which is satisfied by a given pair of roots.*
4. *Plot the graph of a given quadratic function with specified intervals and range.*
5. *Determine the real roots of a quadratic equation by factorisation, by formula and graphically.*
6. *Solve a quadratic and a linear equation simultaneously (i) algebraically and (ii) graphically.*
7. *Plot the graph of a given cubic function with specified intervals and range.*
8. *Solve the equation using 7*

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

Consider the equations:

$$3x+2y = 7$$

$$4x+y = 6$$

The unknown quantities x and y appear in both equations. To solve the equations we have to find values of x and y so that *both* equations are satisfied. Such equations are called *simultaneous equations.*

Three methods are available for solving simultaneous equations.

1. Substitution Method

EXAMPLE 1

Solve the equations:

$$2x+y = 10 \quad (1)$$

$$3x+2y = 17 \quad (2)$$

We can write equation (1) above as:

$$y = 10-2x \quad (3)$$

and, substituting this value of y into equation (2), we have:

$$3x+2(10-2x) = 17$$

and we now have an equation with x the only unknown.

$\therefore$ $$3x+20-4x = 17$$

$\therefore$ $$x = 3$$

Substituting this value for x in equation (3),

$$y = 10-2(3)$$

$\therefore$ $$y = 4$$

The solutions are therefore,

$$x = 3 \quad \text{and} \quad y = 4$$

The solutions should always be checked by substituting the values found into each of the original equations:

Equation (1) has:

$$\text{L.H.S.} = 2(3)+4 = 10 = \text{R.H.S.}$$

and equation (2) has:

$$\text{L.H.S.} = 3(3)+2(4) = 17 = \text{R.H.S.}$$

EXAMPLE 2

Solve the equations:

$$2x+3y = 16 \qquad (1)$$

$$3x+2y = 14 \qquad (2)$$

From equation (1):

$$3y = 16-2x$$

$\therefore$ $$y = \frac{16-2x}{3} \qquad (3)$$

Substituting this value in equation (2),

$$3x+\frac{2(16-2x)}{3} = 14$$

and, multiplying through by 3, we get:

$$9x+2(16-2x) = 42$$

from which $$x = 2$$

Substituting this value for x in equation (3), we have:

$$y = \frac{16-2(2)}{3} = \frac{16-4}{3}$$

$$\therefore \quad y = 4$$

The solutions are therefore:

$$x = 2 \quad \text{and} \quad y = 4$$

Checking these values by substituting into the original equations we have:

Equation (1) has:

$$\text{L.H.S.} = 2(2)+3(4) = 16 = \text{R.H.S.}$$

Equation (2) has:

$$\text{L.H.S.} = 3(2)+2(4) = 14 = \text{R.H.S.}$$

2. Elimination Method

This method is most generally used in solving equations which contain the first power only of the unknown quantities.

EXAMPLE 3

Solve the equations:

$$3x+4y = 11 \qquad (1)$$

$$x+7y = 15 \qquad (2)$$

If we multiply equation (2) by 3 we shall have the same coefficient of x in each of the equations:

$$3x+21y = 45 \qquad (3)$$

We can now eliminate x by subtracting equation (1) from equation (3).

$$(3x+21y)-(3x+4y) = 45-11$$

$$\therefore \quad 17y = 34$$

$$\therefore \quad y = 2$$

To find x we may substitute in either of original equations.

Substituting in equation (1):

$$3x+4(2) = 11$$

$$\therefore \quad x = 1$$

Therefore the solutions are:

$$x = 1 \quad \text{and} \quad y = 2$$

To check these values substitute them in equation (2). (There would be no point in substituting them in equation (1) for this was used in finding x from the y value.) Substituting in equation (2),

$$\text{L.H.S.} = 1+7(2) = 15 = \text{R.H.S.}$$

EXAMPLE 4

Solve the equations:

$$5x+3y = 29 \qquad (1)$$

$$4x+7y = 37 \qquad (2)$$

The same coefficient of x can be obtained if equation (1) is multiplied by 4 and equation (2) by 5. As before, we may then subtract and x will disappear.

Multiplying equation (1) by 4,

$$20x+12y = 116 \qquad (3)$$

Multiplying equation (2) by 5,

$$20x+35y = 185 \qquad (4)$$

Subtracting equation (3) from equation (4),

$$(35-12)y = 185-116$$

$$\therefore \quad y = 3$$

Substituting in equation (1),

$$5x+3(3) = 29$$

$$\therefore \quad x = 4$$

Therefore the solutions are:

$$x = 4 \quad \text{and} \quad y = 3$$

A check on these values is made by substituting them into equation (2):

$$\text{L.H.S.} = 4(4)+7(3) = 37 = \text{R.H.S.}$$

Frequently, in practice, the coefficients of the unknowns are not whole numbers. The same methods apply but care must be taken with the arithmetic.

EXAMPLE 5

Solve the equations:

$$3.175x+0.238y = 6.966 \qquad (1)$$

$$2.873x+4.192y = 11.804 \qquad (2)$$

To eliminate, say, x we must arrange for x to have the same coefficient in both equations. To achieve this we multiply equation (1) by the coefficient of x in equation (2) and then equation (2) by the coefficient of x in equation (1).

Multiplying equation (1) by 2.873,

$$9.122x+0.683\,8y = 20.02 \qquad (3)$$

Multiplying equation (2) by 3.175,

$$9.122x+13.31y = 37.48 \qquad (4)$$

Subtracting equation (3) from equation (4),

$$12.63y = 17.46$$

$$\therefore \quad y = 1.383$$

Substituting this value in equation (1),

$$3.175x+0.238(1.383) = 6.966$$

$$\therefore \quad x = \frac{6.966-0.329\,7}{3.175}$$

$$\therefore \quad x = 2.089$$

Therefore the solutions are:

$$x = 2.089 \quad \text{and} \quad y = 1.383$$

A check on these values may be made by substituting them into equation (2):

$$\text{L.H.S.} = 2.873(2.089)+4.192(1.383) = 11.804 = \text{R.H.S.}$$

EXAMPLE 6

Solve the equations:

$$\frac{2x}{3}-\frac{y}{4}=\frac{7}{12} \qquad (1)$$

$$\frac{3x}{4}-\frac{2y}{5}=\frac{3}{10} \qquad (2)$$

In this example it is best to clear each equation of fractions before attempting to solve simultaneously.

Multiplying equation (1) by 12,

$$8x-3y=7 \qquad (3)$$

Multiplying equation (2) by 20,

$$15x-8y=6 \qquad (4)$$

We can now proceed in the usual way.

Multiplying equation (4) by 8,

$$120x-64y=48 \qquad (5)$$

Multiplying equation (3) by 15,

$$120x-45y=105 \qquad (6)$$

Subtracting equation (5) from equation (6),

$$-45y-(-64)y=105-48$$

$$\therefore \qquad y=3$$

Substituting in equation (3),

$$8x-3(3)=7$$

$$\therefore \qquad x=2$$

Hence solution is:

$$x=2 \quad \text{and} \quad y=3$$

A check on these values will necessitate substitution into both equations (1) and (2) since both were modified before any elimination took place:

Equation (1) has:

$$\text{L.H.S.}=\frac{2(2)}{3}-\frac{3}{4}=\frac{7}{12}=\text{R.H.S.}$$

Equation (2) has:

$$\text{L.H.S.}=\frac{3(2)}{4}-\frac{2(3)}{5}=\frac{3}{10}=\text{R.H.S}$$

3. Graphical Method

EXAMPLE 7

Solve graphically: $y-2x = 2$ (1)

$3y+x = 20$ (2)

Equation (1) can be rewritten as: $y = 2x+2$

Equation (2) can be rewritten as:

$$y = \frac{20-x}{3}$$

Drawing up the following table, we can plot the two equations on the *same axes.*

x	-3	0	3
$y = 2x+2$	-4	2	8
$y = \frac{20-x}{3}$	7.7	6.7	5.7

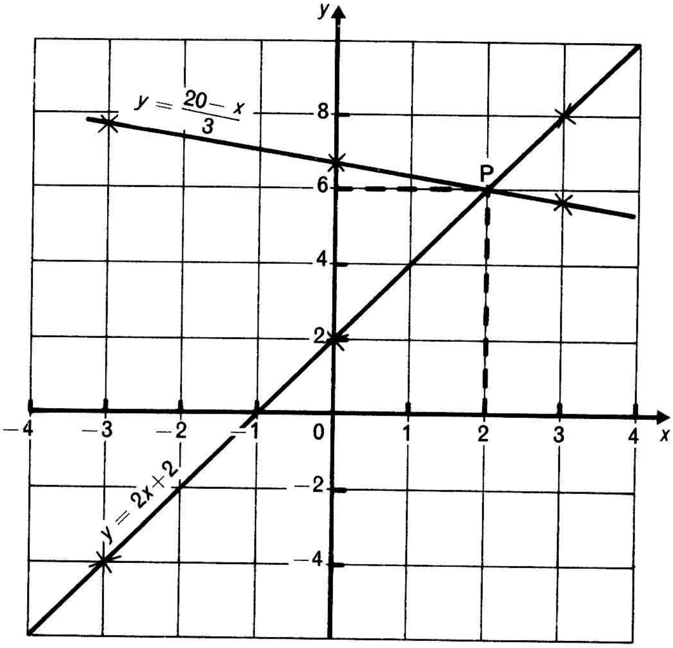

Fig. 1.1

The solutions of the equations will be given by the co-ordinates of the point where the two lines cross (that is, point P in Fig. 1.1). The co-ordinates of P are $x = 2$ and $y = 6$. Hence the solutions are:

$$x = 2 \quad \text{and} \quad y = 6$$

PROBLEMS INVOLVING SIMULTANEOUS EQUATIONS

In problems which involve two unknowns it is first necessary to form two separate equations from the given data and then solve as shown previously.

EXAMPLE 8

In a certain lifting machine it is found that the effort E N and the load W N which is being raised are connected by the equation $E = aW+b$. An effort of 26 N raises a load of 80 N, whilst an effort of 38 N raises a load of 120 N. Find the values of constants a and b and also determine the effort required to raise a load of 150 N.

Substituting $E = 26$ and $W = 80$ into the given equation, we have

$$26 = 80a+b \qquad (1)$$

Substituting $E = 38$ and $W = 120$ into the given equation, we have

$$38 = 120a+b \qquad (2)$$

Subtracting equation (1) from equation (2),

$$12 = 40a$$

$$\therefore \quad a = 0.3$$

Substituting in equation (1),

$$26 = 80(0.3)+b$$

$$\therefore \quad b = 2$$

The given equation therefore becomes

$$E = (0.3)W+2$$

Hence when $W = 150$,

$$E = (0.3)150+2$$

$$\therefore \quad E = 47.$$

Hence an effort of 47 N is required to raise a load of 150 N.

EXAMPLE 9

Two lathe operators are producing the same components. Their total output per week is 220 components. If the ratio of their individual outputs is 6 : 5, find the number of components per week that each operator produces.

Let the number of components produced by the faster operator be x and let the number of components produced by the slower operator by y. Then

$$x+y = 220 \qquad (1)$$

and

$$\frac{x}{y} = \frac{6}{5} \qquad (2)$$

$$\therefore \quad x = \frac{6y}{5} \qquad (3)$$

We now have two simultaneous equations for x and y. Substituting the value of x from equation (3) into equation (1), we get

$$\frac{6y}{5}+y = 220$$

Multiplying by 5,

$$6y+5y = 1\,100$$

$$\therefore \quad y = 100$$

Substituting for y in equation (1),

$$x+100 = 220$$

$$\therefore \quad x = 120$$

Hence the operators produce 100 and 120 each respectively per week.

Exercise 1

Solve the following equations for x and y:

1) $3x+2y = 7$
$x+y = 3$

2) $4x-3y = 1$
$x+3y = 19$

3) $x+3y = 7$
$2x-2y = 6$

4) $7x-4y = 37$
$6x+3y = 51$

5) $4x-6y = -2.5$
$7x-5y = -0.25$

6) $\frac{x}{2}+\frac{y}{3} = \frac{13}{6}$
$\frac{2x}{7}-\frac{y}{4} = \frac{5}{14}$

7) $\frac{x}{8}-y = -2.5$
$3x+\frac{y}{3} = 13$

8) $\frac{(x-2)}{3}+\frac{(y-1)}{4} = \frac{13}{12}$
$\frac{(2-x)}{2}+\frac{(3+y)}{3} = \frac{11}{6}$

9) $x+y = 17$
$\frac{x}{5}-\frac{y}{7} = \frac{1}{1}$

10) $0.2x+1.5y = 0.6$
$1.3x+0.6y = 2.07$

11) $1.24x-0.56y = 1.00$
$0.47x+0.18y = 0.026$

12) $3.17x+9.30y = 3.445$
$1.25x-2.67y = 0.091$

13) Two quantities M and N are connected by the formula $M = aN+b$ in which a and b are constants. When $N = 3$, $M = 6.5$, and when $N = 7$, $M = 12.5$. Find the constants a and b.

14) In an experiment to determine the friction force F N between two metallic surfaces when the load is W N, the law connecting the two quantities was of the type $F = mW+b$. When $F = 25$ N, $W = 60$ N, and when $F = 31$ N, $W = .90$ N. Find the values of constants m and b and find also the friction force when the load is 120 N.

15) A motorist travels x km at 40 km/h and y km at 50 km/h. The total time taken is $2\frac{1}{2}$ hours. If the time taken to travel $6x$ km at 30 km/h and $4y$ km at 50 km/h is 14 hours find x and y.

16) An alloy containing 8 cm³ of copper and 7 cm³ of tin has a mass of 122.3 g. A second alloy containing 9 cm³ of copper and 7 cm³ of tin has a mass of 131.2 g. Find the densities of copper and tin respectively in g/cm³.

17) Find the values of x and y from the following equations:

$$\frac{3}{x}-\frac{2}{y} = \frac{1}{2}$$

$$\frac{5}{x}+\frac{3}{y} = \frac{29}{12}$$

Hint: put $X = \frac{1}{x}$ and $Y = \frac{1}{y}$ and first solve for X and Y.

18) In a certain fraction the denominator exceeds the numerator by 26. If both the numerator and the denominator are increased by 3 the fraction becomes 11/24. What is the original fraction?

QUADRATIC EQUATIONS

An equation of the type $ax^2+bx+c = 0$, involving x in the second degree and containing no higher power of x, is called a *quadratic equation.* The constants a, b and c can have any numerical values. Thus,

$$x^2-9 = 0 \quad \text{where} \quad a = 1, \quad b = 0 \quad \text{and} \quad c = -9,$$

$$x^2-2x-8 = 0 \quad \text{where} \quad a = 1, \quad b = -2 \quad \text{and} \quad c = -8,$$

$$2.5x^2-3.1x-2 = 0 \quad \text{where} \quad a = 2.5, \quad b = -3.1 \quad \text{and} \quad c = -2,$$

are all examples of quadratic equations. A quadratic equation may contain only the square of the unknown quantity, as in the first of the above equations, or it may contain both the square and the first power as in the other two.

Three methods are available for solving quadratic equations.

1. Solution by Factors

If the product of two factors is zero, then one factor or the other must be zero. Thus if $M \times N = 0$ then either $M = 0$ or $N = 0$.

We make use of this fact in solving quadratic equations as the following examples show.

The solutions of a quadratic equation are also called the roots of the equation.

EXAMPLE 10

Solve the equation $(2x+3)(x-5) = 0$.

Since the product of the two factors $2x+3$ and $x-5$ is zero then either,

$$2x+3 = 0 \quad \text{or} \quad x-5 = 0$$

Hence: $$x = -\frac{3}{2} \quad \text{or} \quad x = 5$$

EXAMPLE 11

Solve the equation $6x^2+x-15 = 0$.

Factorising, $(2x-3)(3x+5) = 0$

$\therefore$ either $2x-3 = 0$ or $3x+5 = 0$

Hence: $$x = \frac{3}{2} \quad \text{or} \quad x = -\frac{5}{3}$$

EXAMPLE 12

Solve the equation $14x^2 = 29x-12$.

Bring all the terms to the left-hand side:

$$14x^2-29x+12 = 0$$

$\therefore$ $$(7x-4)(2x-3) = 0$$

$\therefore$ either $7x-4 = 0$ or $2x-3 = 0$

Hence: $$x = \frac{4}{7} \quad \text{or} \quad x = \frac{3}{2}$$