SCHAUM'S A-Z

A-Z MATHEMATICS

All you need to know for study success, from *Absolute error* to *Zero vector*

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JOHN BERRY, TED GRAHAM, JENNY SHARP, AND ELIZABETH BERRY

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MATHEMATICS

HOW TO USE THIS BOOK

The *Schaum's A–Z* is an alphabetical textbook designed for ease of use. Each entry begins with a short definition or explanation. Many entries have worked examples showing typical short examination type questions.

In writing each entry we have kept three important questions in mind. "What does the entry mean? Why do I need to know it? How is it used?" This follows the familiar view of mathematics knowledge consisting of "concept, context and skill."

The *Schaum's A–Z* is not designed to replace your textbook or teacher! We hope that you will see it as a helpful complementary part of your studies. It is designed for you to use as you study new topics in mathematics to provide a dictionary of words and techniques that you become familiar with. When you meet a new topic in mathematics check its meaning in the *Schaum's A–Z* to get an initial overview. Return to the handbook from time to time to review keywords, phrases or ideas and have it by your side during your exam period.

We hope that the *Schaum's A–Z* proves an invaluable resource, fully relevant from the first day of your mathematics, mechanics, statistics or decision and discrete mathematics course and then on after school, college or university.

John Berry, Ted Graham, Jenny Sharp and Elizabeth Berry

ACKNOWLEDGMENTS

Researching, writing and editing a book of this size requires teamwork and hard work. It also requires the occasional willingness to sacrifice technical accuracy in favor of clarity. A considerable amount of our time went into checking the entries, but, if any mistakes have slipped through, the authors accept full responsibility.

The authors are pleased to acknowledge the support of our colleagues in the Centre for Teaching Mathematics, Wendy, Stuart, Roger, Karen and Raymond for helping to clarify our understanding of some of the entries! In particular we thank Karen Eccles for helping to prepare the manuscript. The other people to be thanked are Tim Sipka of Alma College, Michigan and the team at Hodder Headline.

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Dr Ted Graham for the photo on page 61 and Pictor International Ltd., London for the photos on pages 28, 78, 80 and 92.

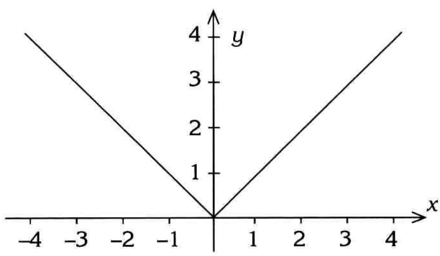
A

absolute and relative error: absolute error is the actual error that exists when an estimate or approximation is made. For example, if a rod is measured as having a length of 199.5 cm when its length is in fact exactly 200 cm, then the absolute error is 0.5 cm.

The relative error is given by the absolute error divided by the true value and is often given as a percentage. For the example above the relative error is:

$$\frac{0.5}{200} = 0.0025 = 0.25\%$$

absolute value: the absolute value of a number, x, is written as $|x|$, the modulus of x. The modulus of a number is the size of the number, that is the number without its sign. For example $|5| = 5$, but $|-7| = 7$. This means that $|x|$ is never negative. The graph below shows $y = |x|$.



acceleration: defined as the rate of change of velocity. It is a vector quantity, having both magnitude and direction.

If an object is traveling in a straight line with constant speed, its velocity is constant and its acceleration is zero.

If an object is traveling along a curve with constant speed then the acceleration is variable because the direction of motion, and hence the velocity, are changing. So constant speed does not always mean zero acceleration.

The table shows the various situations that can occur.

velocity		acceleration
magnitude (speed)	direction	
constant	constant	zero
variable	constant	nonzero
constant	variable	nonzero
variable	variable	nonzero

We often denote the acceleration by the symbol **a**; if the velocity of the object is **v** then:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

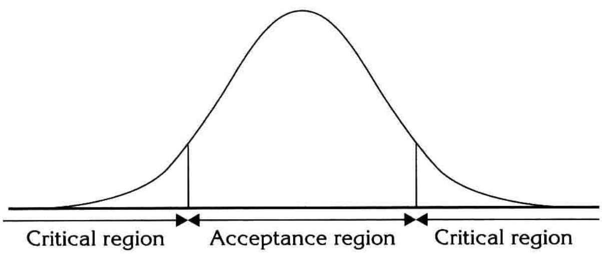
For an object moving in a straight line with position $x(t)$ and velocity $v(t)$ then:

$$a = \frac{dv}{dt} \quad \text{or} \quad a = v\frac{dv}{dx} \quad \text{or} \quad a = \frac{d^2x}{dt^2}$$

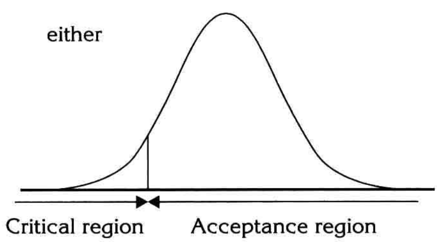
(See also *angular acceleration, constant acceleration equations.*)

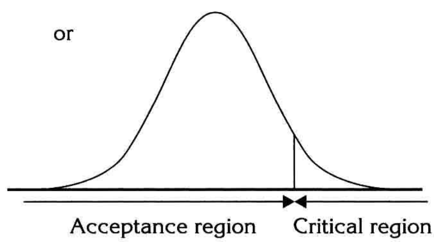
acceptance region: if the *test statistic* falls in the acceptance region the *null hypothesis*, H_0, is accepted. The acceptance region is defined by the *critical values* of the test and can be illustrated diagramatically.

For a *two-tailed* test:



and a *one-tailed* test:





accuracy: when a number is quoted to a certain degree of accuracy we can place certain bounds on the actual value of the number. For example if $x = 16.7$ to 1 decimal place, then $16.65 \leq x < 16.75$. (See also *absolute and relative errors.*)

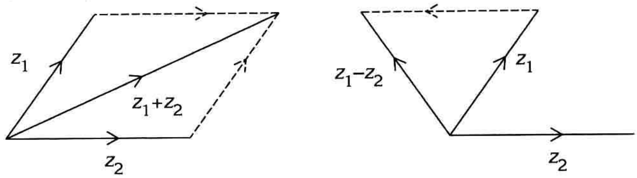
activity network: see *precedence network.*

activity on arc network: a precedence network in which the *arcs* represent the activities, while the *vertices* represent *events*. (See *precedence network.*)

activity on vertex network: a precedence network in which the activities are represented by the *vertices* of a *network*, and *edges* show the order of precedence. (See *precedence network.*)

acute: an acute angle is an angle smaller than 90°. In an acute triangle all the angles are less than 90°.

addition and subtraction of complex numbers: when complex numbers are added the real parts of the numbers must be added together and the imaginary parts added together separately. The addition and subtraction of complex numbers can be treated geometrically as shown below.



Addition Subtraction

Example:

Three complex numbers are

$z_1 = 4 + 2i$, $z_2 = -5 + 6i$ $z_3 = 4 - 8i$.

Find

(a) $z_1 + z_2$ (b) $z_1 - z_3$ (c) $z_1 + z_2 - z_3$

Solution:

(a)

$$\begin{aligned} z_1 + z_2 &= (4 + 2i) + (-5 + 6i) \\ &= (4 + (-5)) + (2 + 6)i \\ &= -1 + 8i \end{aligned}$$

(b)

$$\begin{aligned} z_1 - z_3 &= (4 + 2i) - (4 - 8i) \\ &= (4 - 4) + (2 + 8)i \\ &= 10i \end{aligned}$$

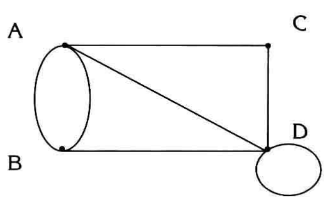
(c)

$$\begin{aligned} z_1 + z_2 - z_3 &= (4 + 2i) + (-5 + 6i) - (4 - 8i) \\ &= (4 + (-5) - 4) + (2 + 6 - (-8))i \\ &= -5 + 16i \end{aligned}$$

addition of vectors: see *vectors*.

adjacency matrix: a matrix representing the *vertices* and *edges* of a *graph*. Each row and column of the matrix represents a *vertex* of a graph and the numbers give the number of edges joining each pair or vertices.

	A	B	C	D
A	0	2	1	1
B	2	0	0	1
C	1	0	0	1
D	1	1	1	2



algorithm: a systematic process for finding a solution to a problem.

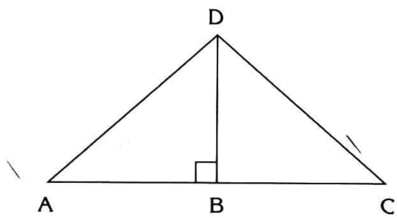
alternating path: a path in a *bipartite graph* that joins *vertices* from one subset to vertices in the other, in such a way that alternate *edges* only are in the initial matching, and initial and final vertices are not incident with an edge in the matching.

alternative hypothesis (H_1): this is the hypothesis that is accepted if the *null hypothesis* H_0 is rejected when performing a hypothesis test. The alternative hypothesis depends on the type of test being performed: *one-tailed* or *two-tailed*. A one-tailed hypothesis test considers strictly an increase or a decrease but not both, whereas a two-tailed hypothesis test considers any change in the parameter.

For example, consider a sample of peas from a species which was known to have a mean mass of 0.1 g. We wish to test if the mean mass of the peas in the sample differs from 0.1 g. Here the null hypothesis is $H_0 : \mu = 0.1$ and the alternative hypothesis is $H_1 : \mu \neq 0.1$. This is a two-tailed test.

If we wished to test if the mean mass of the peas in the sample had increased, the alternative hypothesis would be $H_1 : \mu > 0.1$. If testing for a decrease then it would be $H_1 : \mu < 0.1$. These would be one-tailed hypotheses.

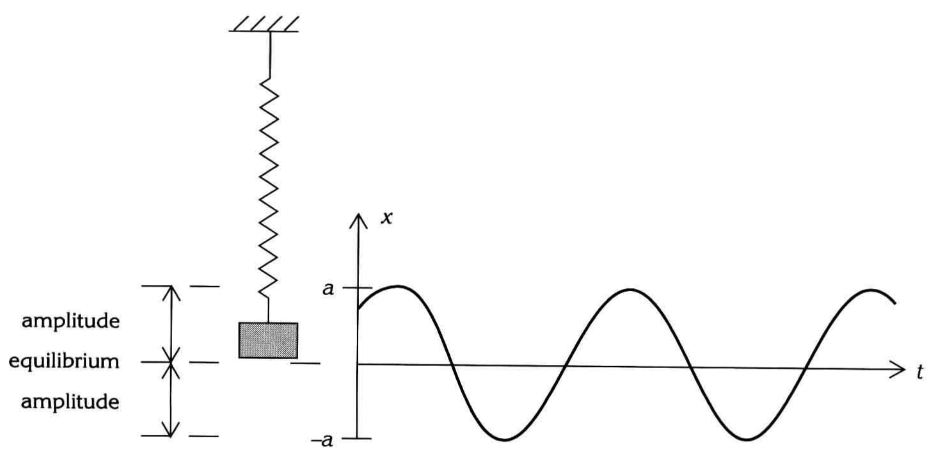
altitude of a triangle: an altitude of a triangle is a line that is perpendicular to one side and passes through the opposite vertex.



In the diagram BD is an altitude of the triangle ACD.

amplitude: the amplitude of oscillating motion of an object is the largest displacement from the equilibrium position of that object.

In the figure the object is moving up and down on the end of a spring.



For an ideal situation as the object moves, the displacement from equilibrium x will form a trigonometric curve between limits $x = -a$ and $x = +a$. This is called *simple harmonic motion*.

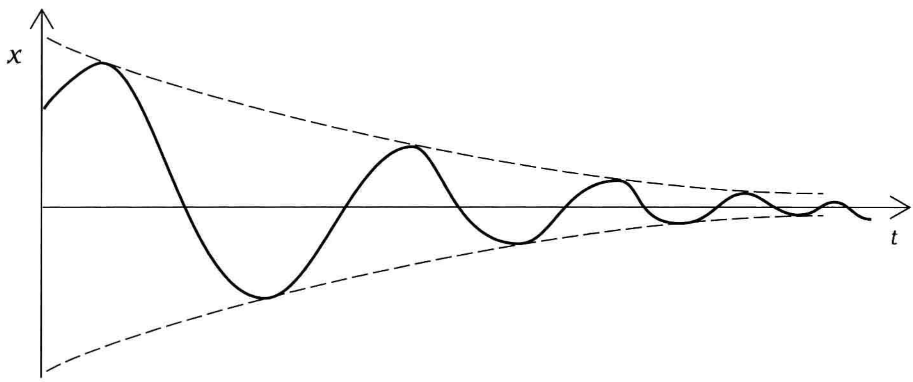
The equation of the curve is:

$$x = a\cos(\omega t + \varepsilon)$$

and a is the amplitude of the motion.

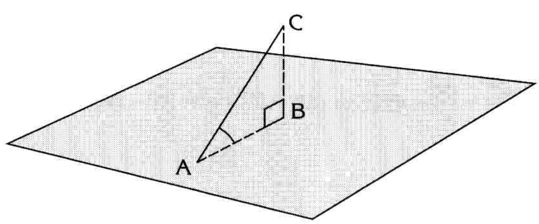
In a real system the oscillation will gradually reduce in size. We say that the oscillation is *damped*.

For such a system the graph of x against t is shown below.

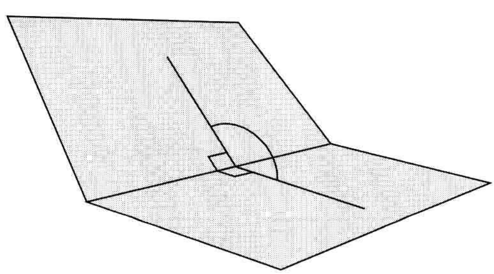


The bounding curve describes the amplitude, which is a decreasing function of time.

angle between a line and a plane: the angle between the line and its projection onto the plane. In the figure below, the line AB is the projection of the line AC onto the plane and is such that the line BC is perpendicular to the plane.



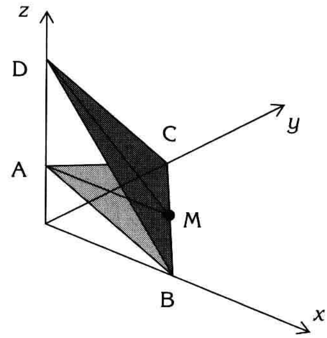
angle between two planes: the angle between lines in each plane that are both perpendicular to the line of intersection of the planes.



Example:

The points A, B, C and D have coordinates (0, 0, 2), (6, 0, 0), (0, 6, 0) and (0, 0, 8), respectively. Find the angle between the planes ABC and BCD.

Solution:



The diagram shows the planes and the midpoint of the line BC. The required angle is the angle between the lines AM and DM. The coordinates of M are:

$$\left(\frac{6+0}{2}, \frac{0+6}{2}, \frac{0+0}{2}\right) = (3, 3, 0)$$

The length of AM is:

$$\sqrt{(3-0)^2 + (3-0)^2 + (0-2)^2} = \sqrt{22}$$

The length of DM is:

$$\sqrt{(3-0)^2 + (3-0)^2 + (0-8)^2} = \sqrt{82}$$

The length of AD is 6.

Using the cosine rule in the form

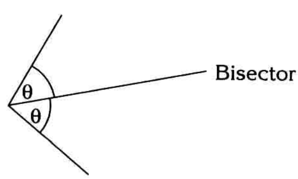
$$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}, \text{ with angle AMD} = \theta,\ b = \sqrt{22},\ c = \sqrt{82}$$

gives:

$$\cos\theta = \frac{22 + 82 - 6^2}{2 \times \sqrt{22} \times \sqrt{82}}$$

$$= 0.8005 \quad \text{so} \quad \theta = 36.82°$$

angle bisector: an angle bisector cuts an angle into two equal parts, as shown in the diagram below.



angle between two lines: the angle between the two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is θ, where

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

Note that if $m_1 = m_2$ the lines are parallel and that if $m_1 \times m_2 = -1$ the lines are perpendicular.

Example:

Find the angle between the lines $y = 4x - 3$ and $y = 5x + 1$.

Solution:

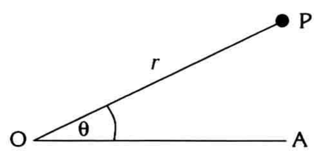
The lines have gradients 4 and 5, so the angle θ between the lines is given by:

$$\tan\theta = \frac{5 - 4}{1 + 5 \times 4}$$

$$= \frac{1}{21}$$

$$\theta = 2.73°$$

angular speed: suppose that a point P moves in a plane so that its position in polar coordinates is (r,θ) then the angular speed of the point is the rate of change of the angle θ.



In the figure the point O and the line OA are fixed. In symbols the angular speed is often denoted by ω, so that

$$\omega = \frac{d\theta}{dt}$$

The units of angular speed are usually radians per second; however in many applications it makes more sense to use revolutions per minute (rev/min).

Example:

A compact disc on a hi-fi system rotates at 33⅓ rev/min. Convert this angular speed to radians per second.

Solution:

1 revolution = 2π radians and 1 minute= 60 seconds.

So 33⅓ revolutions per minute $= \dfrac{33.33 \times 2\pi}{60}$

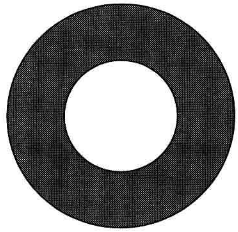
$= 3.49$ radians per second

(See also *circular motion*.)

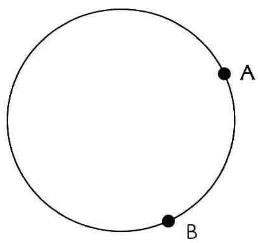
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

angular acceleration: defined as the rate of change of angular velocity.

annulus: an annulus is the region between two concentric circles. This is the shaded region shown in the diagram below.



arc: an arc is a continuous part of a curve. For example a part of a circle is usually referred to as an arc. The diagram below shows a circle split into two arcs by the points A and B.



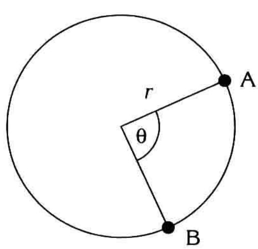
arc (or edge): a line connecting two *vertices* in a *graph*. For a *directed network* each *edge* has a direction of "flow" associated with it.

arccos: a term used for the inverse of the *cosine* function, more usually written $\cos^{-1}$; see *$\cos^{-1}$* for the definition.

arc length: the term arc length is used to describe the distance along a curve between two specified points. The length of an arc can be calculated easily for an arc that is part of a circle. The figure shows a circle and the arc length between the points A and B can be calculated using one of the two formulas.

$$\text{arc length} = \frac{2\pi r\theta}{360} \quad \text{if } \theta \text{ is in degrees.}$$

$$\text{arc length} = r\theta \quad \text{if } \theta \text{ is in radians.}$$



arcsin: a term used for the inverse of the *sine* function. It is more common to use $\sin^{-1}$, and readers should consult the *$\sin^{-1}$* entry.

arctan: term used for the inverse of the *tangent* function. It is more common to use $\tan^{-1}$, and readers should consult the *tan^{-1}* entry.

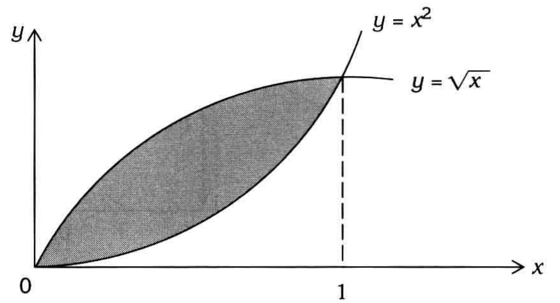
area: an area is a region enclosed within a boundary. For example, the area of a rectangle of sides length a and b is ab. The area between the graphs of two functions can be evaluated using integration.

Example:

Find the area enclosed by the two curves $y = x^2$ and $y = \sqrt{x}$.

Solution:

First we sketch the functions to see where the area lies in the x–y plane.



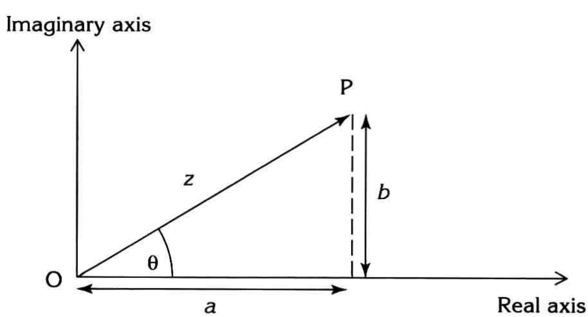
The graphs intersect at $x = 0$ and $x = 1$ and the area required is shown shaded in the diagram.

$$\text{Area} = \int_0^1 \sqrt{x} - x^2\,dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

The area between the curves is $\frac{1}{3}$.

argand diagram: an argand diagram is used to give a geometrical interpretation to a complex number. The diagram below shows how the complex number $z = a + bi$ can be represented.

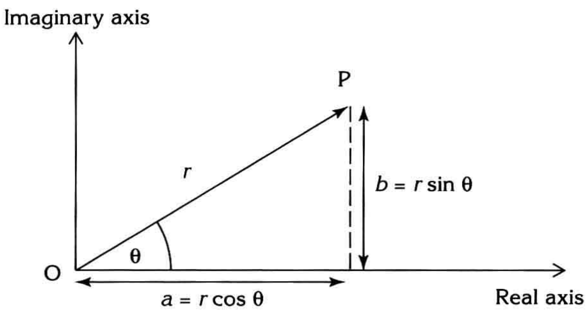


argument of a complex number

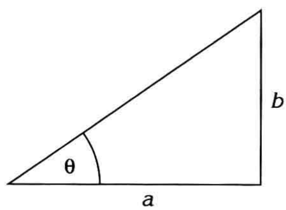
The complex number z is represented by the point P or the line OP. The modulus of the complex number z is written as $|z|$ and is the length of OP. It is calculated using $|z| = \sqrt{(a^2 + b^2)}$. The argument of the number z, written as arg z, is the angle between the line OP and the real axis, shown as θ on the diagram. It can be found using

$$\arg z = \tan^{-1}\left(\frac{b}{a}\right) \text{ where } -\pi < \arg(z) \leq \pi$$

If the modulus and argument of a complex number are r and θ, respectively, the number can be expressed in the form $z = a + bi$. The diagram shows how a and b can be expressed in terms of r and θ, so that $z = r\cos\theta + r\sin\theta i$.



argument of a complex number: this is the angle between the complex number, when it is represented as a line, and the real axis. The argument of the complex number $z = a + bi$ is written as arg(z) and is the angle θ in the triangle above.



For the figure shown above:

$$\arg(z) = \tan^{-1}\left(\frac{b}{a}\right) \quad \text{where} \quad -\pi < \arg(z) \leq \pi$$

arithmetic mean: the arithmetic mean or simply the *mean* of a set of numbers (x_1, x_2, ... x_n) is denoted by $\bar{x}$ (pronounced x bar) and is a measure of the central location or *average* of the set of numbers. It is given by the formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The arithmetic mean is a good *measure of location for symmetric data*. Most calculators will calculate the mean of a set of data.

Example 1:

Find the mean of the following numbers: 3, 2, 5, 6, 4, 2, 3, 6, 7, 1, 2, 4.

Solution:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$= \frac{1}{12}(3 + 2 + 5 + 6 + 4 + 2 + 3 + 6 + 7 + 1 + 2 + 4)$$

$$= \frac{45}{12} = 3.75$$

For a *discrete frequency distribution* where the values (x_1, x_2, ... x_n) have corresponding frequencies of (f_1, f_2, ... f_n), the arithmetic mean is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Example 2:

Find the mean of the following data.

x	1	2	3	4	5	6	7
frequency f	6	4	6	7	2	8	3

Solution:

Compile a *frequency table*:

x	f	fx
1	6	6
2	4	8
3	6	18
4	7	28
5	2	10
6	8	48
7	3	21
	$\sum_{i=1}^{n} f_i = 36$	$\sum_{i=1}^{n} f_i x_i = 139$

The arithmetic mean of this data is:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{139}{36} = 3.86 \quad \text{(to 2 d.p.)}$$

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z