Roger Penrose
Collected Works

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1981–1989

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ROGER PENROSE
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PREFACE

The topics addressed, in these *Collected Works*, vary over quite an extensive range, the boundaries between their areas being sometimes quite blurred. It would have been hard to classify all these articles and arrange them according to individual topics. Moreover, my own mode of working has been, on the whole, to keep most of my different scientific or mathematical interests running concurrently. Accordingly, it has seemed appropriate not to attempt to group the papers together into separate tidy categories. To do so would have provided a somewhat misleading picture of my overall mode of thinking. Instead, the ordering of articles that has been adopted is essentially chronological, at least with regard to the date of publication. The resulting somewhat higgledy-piggledy appearance that we find in the sometimes abrupt juxtaposition of different topics is, perhaps, fairly illustrative of my actual mode of working.

There are two main kinds of drive that underlie these works. On the one hand, some of the articles have arisen mainly from purely mathematical considerations, which can be ultimately aesthetic or just matters of simple curiosity. On the other hand, motivations for many others of the articles came primarily from a strong desire to discover more about the underlying principles governing the workings of the physical world. Yet, one of the deep mysteries about what has been discovered about these principles is the great breadth of intrinsically appealing mathematics that is found to underlie the actual operation of the physics of the universe, so these two drives are found to relate closely to one another. The reader may perhaps note the distinct geometrical flavour of much of my own mathematics, evidencing the strong appeal that geometry exerts on me, an appeal that is not infrequently intertwined with desire for algebraic elegance. Indeed, I have found it remarkable how much of the workings of our universe appear to depend upon sophisticated and powerful geometry and algebra.

My own very geometrical perspectives, as opposed to purely computational ones, are perhaps made more obvious in the later volumes of this series than in the early ones, but this appearance is deceptive. Very early in my career as a Cambridge graduate student, I had developed a graphical notation which enabled me to carry out many kinds of calculation in an entirely diagrammatic way. For a long time I was rather secretive of this procedure, and in my earlier writings would take care to remove all trace of these visual origins. I do not doubt that my predilections for such geometric or algebraically elegant forms of understanding have influenced the specific choices of direction in which my endeavours towards furthering our understandings of the physics of the world have taken.

The focus of these directions has become more clarified as time has moved on, and has narrowed down to such a degree that there has emerged a somewhat disturbing repetitiveness in many of the later articles. A good number of these provide rather non-technical accounts. An example of such repetitiveness occurs with my descriptions of gravitational collapse to a black hole. There is, however, a gradually developing outlook on this issue, leading to an emphasis on the fundamental contrast in character between the space-time singularities that inevitably appear within black holes and the space-time singularity that constitutes the big bang, this contrast being central to the existence of a second law of thermodynamics in our universe. The same kind of repetitiveness applies to my frequently repeated claim of a need for a fundamental change in the basic laws of quantum theory, where only towards the end do my suggestions become particularly specific. Likewise, many mildly differing accounts are provided outlining the formalism and the basic ideas that underlie twistor theory. The later ones lead up to various viewpoints on the 'googly' problem, which has constituted the main obstruction to the finding of a full understanding of general relativity in twistor terms. Despite such repetition, it is my hope that there is a sufficient variety of emphasis and of historical development in such articles for it to have been worth while to include them all here.

Several colleagues have collaborated with me in writing the works presented here, and I can mention but only a few. Foremost, I have collaborated long and particularly fruitfully with Ezra T. Newman, especially with regard to the asymptotics of general relativity, and also with Wolfgang Rindler in the development of the formalism of 2-spinor calculus. Other collaborators who have played important roles in helping to develop the mathematics of discrete physics and twistor theory have been Erwin Kronheimer, Michael Eastwood, Ronny Wells, and Denny Hill. Among my collaborators, providing very significant inputs, who were former students in my research group, are Andrew Hodges, Paul Tod, Lane Hughston, George Sparling, Richard Ward, Nick Woodhouse, and Lionel Mason, but there were many others. Although not playing roles as actual collaborators, fundamental influences in my research have come from Dennis Sciama and Michael Atiyah.

Roger Penrose
April 2010

BIOGRAPHY

Early years and influences

I was born in Colchester, Essex, UK on 8 August 1931. My father, Lionel Sharples Penrose, FRS, was a highly talented and intelligent man, with many interests. He, himself, had come from a Quaker family, his own father (James Doyle Penrose) having been a professional artist and his mother (Josephine Peckover) was from a well-to-do Quaker banking family. Lionel's primary professional interest was in the causes of hereditary mental disease. He was Galton Professor of Human Genetics at University College London from 1945 and, following retirement, Director of the Kennedy–Galton Institue, St Albans. He had a very considerable artistic talent and a keen interest in music, especially Mozart, but also Bach and other classical composers, dabbling a little in composition himself, particularly when there was a puzzle aspect in what was involved (where he once wrote a piece that could be played equally well upside down as right-way up). He liked making puzzles of various kinds in wood, and spent much of his later life designing wooden models illustrating self-reproduction. He was recognized as a fine composer of chess problems. He had a definite fondness for mathematics and was an innovative statistician. He loved explaining matters of science and mathematics. On one occasion, when I told him that my mathematics teacher at school had informed us that he would be explaining calculus the next day, Lionel immediately took me aside so as to be sure to be the first to teach me about the beauties of calculus. This made a great impression on me. Yet he was an extremely difficult person to communicate with on personal matters.

My mother, born Margaret Leathes, was a charming and caring woman with a very subtle mind. She had been Head Girl at Bedales school, and became medically qualified, but found she was not able to practice after her marriage. After Lionel died, in 1972, she married Max Newman FRS, a fascinating man, who had been Professor of Mathematics in Manchester and, as we later learned, a key figure at Bletchley Park during the war. Margaret's father, John Beresford Leathes, FRS, had been a distinguished professor of physiology, and her mother, a concert pianist, born Sara Mara Natanson from a Jewish family in Latvia, had disowned her family (then in St Petersberg), becoming completely secretive about her origins, and even about her maiden name. Margaret enjoyed geometry at school, but did not

communicate much on this topic with me. My older brother Oliver was very precocious, and I learnt a lot of physics and mathematics from him as I was growing up. He later became a distinguished mathematical physicist (statistical mechanics) and FRS. My younger brother Jonathan excelled at games, particularly chess, and later became British Chess Champion a record 10 times, and then a top correspondence chess player. My much younger sister Shirley Victoria, now Shirley Hodgson, became a highly regarded Professor of Cancer Genetics.

This atmosphere of science, mathematics, art, and music, and of puzzles and games, had been an important part of my upbringing. During the war years, the family lived in London, Ontario, Canada, where Lionel became Director of Psychiatric Research at the Ontario Hospital. We returned to England in 1945, where I attended University College School, London. As an undergraduate, I studied Mathematics at University College London, from 1951 to 1954; then I went to St John's College Cambridge to do research in algebraic geometry, initially under William Hodge and subsequently John Todd. In my same year, under Hodge, was Micheal Atiyah, who was an inspirational, though somewhat intimidating colleague. Frank Adams and Christopher Zeemen (see Chapter 13) were contemporaries who also influenced me, and I became good friends with Hans Liebeck and Douglas Munn (see Chapter 3), who were close research-student colleagues at St John's.

Yet, the colleague who had the greatest influence on the development of my research during this period, and for many years later, was the cosmologist Dennis Sciama. Even before studying at Cambridge, I had a chance meeting with him during a visit to my brother Oliver in Cambridge in which I raised a question that had worried me in relation to the inspiring series of BBC radio talks given by Fred Hoyle in 1951 (The nature of the universe. A series of broadcast lectures). On the strength of this encounter (at which I used a space-time diagram with tilted light cones to make my point), Dennis had evidently formed the opinion that it would be worth while to interest me more in physics, even trying, later, to persuade me to switch my topic of research to cosmology. I was then too committed to my many pure-mathematical activities, but I learnt a great deal of physics from Dennis, and of the excitements involved in striving to uncover secrets of the universe. Another influential colleague was Felix Pirani, from whom I learnt much about developments in general relativity.

Pivotal were the superb lecture courses of Bondi, on general relativity, and Dirac, on quantum mechanics. On the pure-mathematical side, I recall particularly an elegant information-packed algebra course by Philip Hall. Lectures by Steen on mathematical logic turned out to influence me strongly towards my later viewpoint concerning the relation between computation and conscious understanding (e.g. Chapters 169, 184, 186).

Degrees

1952 BSc Mathematics (Spec) (1st Cl), University College, London

1957 PhD (Cantab), St John's College, Cambridge

Employment

1956	NRDC London (research on numerical analysis with computers); later consultant
1956–1957	Assistant Lecturer, Bedford College, London
1957–1960	Research Fellow, St John's College, Cambridge
1959–1961	NATO Research Fellow, Princeton, Syracuse, Cornell Universities
1961–1963	Research Associate, King's College, London
1963–1964	Visiting Associate Professor, University of Texas, Austin, Texas
1964–1966	Reader (Applied Mathematics), Birkbeck College, London
1966–1967	Visiting appointments: Yeshiva, Princeton, Cornell, Chicago Universities; lecturer Battelle Inst., Seattle
1967–1973	Professor (Applied Mathematics), Birkbeck College, London
1969–1970	Visiting professor, Yeshiva New York, Princeton, and Cornell Universities
1973–1998	Rouse Ball Professor (Mathematics), Oxford; 1998– Emeritus
1978	Miller Fellow, Univ. of California, Berkeley
1982–1987	Lovett Professor (Mathematics), Rice Univ., Houston, Texas, USA (Part-time)
1987–1993	Distinguished Professor (Physics, Mathematics), Syracuse Univ. USA (Part-time)
1993–	Francis and Helen Pentz Distinguished Professor (Physics, Mathematics; part-time)
1998–2001	Gresham Professor (Geometry), Gresham College, London

Honours Prizes include 1971 Heinemann Prize; 1972 Fellow of Royal Society; 1978 Plenary Lecture, International Congress of Mathematicians, Helsinki; 1985 Royal Medal (Royal Society);1988 Wolf Prize (physics, with Hawking); 1990 Albert Einstein Medal; 1994 knighted (services to science); 1994 Polish Academy of Sciences; 1998 US National Academy of Sciences; 2000 Order of Merit; 2004 de Morgan Medal (London Mathematical Society); 2001 Royal Irish Academy (Science); 2008 Copley Medal (Royal Society).

Marriages

1958	Joan Isabel Wedge; divorced 1980
1988	Vanessa Dee Thomas

Children

1963	Christopher Shaun
1964	Toby Nicholas
1966	Eric Alexander
2000	Maxwell Sebastian

Honorary degrees

1992	University of New Brunswick, Canada
1993	University of Surrey
1994	University of Bath
1995	University of London
1996	University of Glasgow
1996	University of Essex
1997	University of St Andrews
1998	University of Santiniketan, India
1998	Open University
2002	University of Southampton
2004	University of Waterloo, Canada
2005	University of Leuven, Belgium
2005	Warsaw University
2005	University of Athens
2006	University of York

Books

1972	Techniques of Differential Topology in Relativity (NSF and SIAM)
1984 and 1986	Spinors and Space-Time (Volumes 1 and 2; Cambridge University Press; with Rindler)
1989	The Emperor's New Mind (Oxford University Press 1989)
1994	Shadows of the Mind (Oxford University Press)
1996	The Nature of Space and Time (Princeton University Press; with Hawking)
1997	The Large, the Small and the Human Mind (Cambridge University Press)
2004	The Road to Reality (Jonathan Cape)

INTRODUCTION

Among the new developments that occurred during the period covered by this Volume was the introduction of a particular notion of 'quasi-local mass-momentum and angular momentum'. This was the topic of my Royal Society paper that constitutes Chapter 132, for which the *Twistor Newsletter* article (TN13) in Chapter 130 provides an outline. For this purpose, a new notion of twistor is introduced that applies to a general spacelike 2-surface $\mathcal{S}$ which has the topology of a sphere S^2. Such a *2-surface twistor*—sometimes referred to as a 'superficial twistor'—is obtained by writing down the twistor equation $\nabla^{(A}_{\ A'}\ \omega^{B)} = 0$ and then selecting only those components of this equation for which the derivative acts purely tangentially to $\mathcal{S}$. This enables us to avoid the consistency conditions that forbid the twistor equation having non-trivial solutions in general curved spacetimes. We obtain two complex equations for two complex quantities, and the Atiyah–Singer theorem can be used to tell us that, in the general case, there is a complex 4-dimensional vector space of solutions, this being the 2-surface twistor space $\mathbb{T}(\mathcal{S})$ assigned to $\mathcal{S}$. These 2-surface twistors act as (spin-$\frac{1}{2}$) helicity-lowering operators, and by combining a pair of them with the Weyl curvature tensor, its spin-2 nature is reduced to spin 1, like electromagnetism, and a Gauss-type charge integral provides a suggested measure of the gravitational source surrounded by $\mathcal{S}$. The different choices of 2-surface twistors are intended to provide the needed 10 components of energy-momentum and angular momentum. Many encouraging results were initially obtained by various students and colleagues—particularly Paul Tod—but then difficulties and anomalies began to emerge, which were not all removed by natural-looking modifications of the definition, indicating that our understanding of what is going on lacked some essential insights—a situation that remains with us today.

An extensive paper with Michael Eastwood and Ronny Wells gave a thorough account of the relation between twistor cohomology and massless fields (Chapter 129). Toby Bailey found how to describe charged sources in twistor theory in terms of relative cohomology and bundles over non-Hausdorff twistor spaces (Chapters 134 and 135), following an earlier observation that George Sparling and I had made concerning the twistor description of the Coulomb field (Chapter 108). I subsequently explored a relation between relative 2nd cohomology and non-Hausdorffness later (Chapters 162 and 169).

Other issues began to take my serious attention during this period, the most important being my increasing conviction that the puzzling issue of quantum measurement will have to be resolved by the appropriate unification of quantum mechanics with general relativity, where this union must involve an actual change in the rules of quantum mechanics, not only in space-time structure. In Chapter 121, I used an argument employing 'Hawking's box' to argue that quantum state reduction should indeed be a gravitational effect, for consistency with the information loss in black holes. I proposed a specific criterion for when a gravitationally induced state-reduction ought to take place (Chapter 154—though later I became convinced that a different proposal is more appropriate). This Volume also contains my first excursions into my proposals that physical basis of consciousness might be related to the issue of quantum state reduction (Chapters 170, 185, and 187).

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