K A STROUD

ENGINEERING MATHEMATICS

PROGRAMMES AND PROBLEMS
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SECOND EDITION

Engineering Mathematics

Programmes and Problems

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ENGINEERING MATHEMATICS

PREFACE TO THE FIRST EDITION

The purpose of this book is to provide a complete year's course in mathematics for those studying in the engineering, technical and scientific fields. The material has been specially written for courses leading to

(i) Part I of B.Sc. Engineering Degrees,

(ii) Higher National Diploma and Higher National Certificate in technological subjects, and for other courses of a comparable level. While formal proofs are included where necessary to promote understanding, the emphasis throughout is on providing the student with sound mathematical skills and with a working knowledge and appreciation of the basic concepts involved. The programmed structure ensures that the book is highly suited for general class use and for individual self-study, and also provides a ready means for remedial work or subsequent revision.

The book is the outcome of some eight years' work undertaken in the development of programmed learning techniques in the Department of Mathematics at the Lanchester College of Technology, Coventry. For the past four years, the whole of the mathematics of the first year of various Engineering Degree courses has been presented in programmed form, in conjunction with seminar and tutorial periods. The results obtained have proved to be highly satisfactory, and further extension and development of these learning techniques are being pursued.

Each programme has been extensively validated before being produced in its final form and has consistently reached a success level above 80/80, i.e. at least 80% of the students have obtained at least 80% of the possible marks in carefully structured criterion tests. In a research programme, carried out against control groups receiving the normal lectures, students working from programmes have attained significantly higher mean scores than those in the control groups and the spread of marks has been considerably reduced. The general pattern has also been reflected in the results of the sessional examinations.

The advantages of working at one's own rate, the intensity of the student involvement, and the immediate assessment of responses, are well known to those already acquainted with programmed learning activities. Programmed learning in the first year of a student's course at a college or university provides the additional advantage of bridging the gap between the rather highly organised aspect of school life and the freer environment and greater personal responsibility for his own progress which face every student on entry to the realms of higher education.

Acknowledgement and thanks are due to all those who have assisted in any way in the development of the work, including those who have been actively engaged in validation processes. I especially wish to record my sincere thanks for the continued encouragement and support which I received from my present Head of Department at the College,

Mr. J. E. Sellars, M.Sc., A.F.R.Ae.S., F.I.M.A., and also from
Mr. R. Wooldridge, M.C., B.Sc., F.I.M.A., formerly Head of Department, now Principal of Derby College of Technology. Acknowledgement is also made of the many sources, too numerous to list, from which the selected examples quoted in the programmes have been gleaned over the years. Their inclusion contributes in no small way to the success of the work.

K. A. Stroud

PREFACE TO THE SECOND EDITION

The continued success of *Engineering Mathematics* since its first publication has been reflected in the numbers of courses for which it has been adopted as the official class text and also in the correspondence from numerous individuals who have welcomed the self-instructional aspects of the work.

Over the years, however, syllabuses of existing courses have undergone some modification and new courses have been established. As a result, suggestions have been received from time to time requesting the inclusion of further programme topics in addition to those already provided as core material for relevant undergraduate and comparable courses. Unlimited expansion of the book to accommodate all the topics requested is hardly feasible on account of the physical size of the book and the commercial aspects of production. However, in the light of these representations and as a result of further research undertaken by the author and the publishers, it has now been found possible to provide a new edition of *Engineering Mathematics* incorporating three of the topics for which there is clearly a wide demand.

The additional programmes cover the following topics:

(a) *Matrices*: Definitions; types of matrices; operations; transpose; inverse; solution of linear equations; eigenvalues and eigenvectors.

(b) *Curves and curve fitting*: Standard curves; asymptotes; systematic curve sketching; curve recognition; curve fitting; method of least squares.

(c) *Statistics*: Discrete and continuous data; grouped data; frequency and relative frequency; histograms; central tendency – mean, mode and median; coding; frequency polygons and frequency curves; dispersion – range, variance and standard deviation; normal distribution and standardised normal curve.

The three new programmes follow the structure of the previous material and each is provided with numerous worked examples and exercises. As before, each programme concludes with a short Test Exercise for self-assessment and a set of Further Problems provides valuable extra practice. A complete set of answers is available at the end of the book.

Advantage has also been taken during the revision of the book to amend a small number of minor points in other parts of the text and it is anticipated that, in its new up-dated form, the book will have an even greater appeal and continue to provide a worthwhile service.

K. A. Stroud
1982

HINTS ON USING THE BOOK

This book contains twenty-seven lessons, each of which has been written in such a way as to make learning more effective and more interesting. It is almost like having a personal tutor, for you proceed at your own rate of learning and any difficulties you may have are cleared before you have the chance to practise incorrect ideas or techniques.

You will find that each programme is divided into sections called frames, each of which normally occupies half a page. When you start a programme, begin at frame 1. Read each frame carefully and carry out any instructions or exercise which you are asked to do. In almost every frame, you are required to make a response of some kind, testing your understanding of the information in the frame, and you can immediately compare your answer with the correct answer given in the next frame. To obtain the greatest benefit, you are strongly advised to cover up the following frame until you have made your response. When a series of dots occurs, you are expected to supply the missing word, phrase, or number. At every stage, you will be guided along the right path. There is no need to hurry: read the frames carefully and follow the directions exactly. In this way, you must learn.

At the end of each programme, you will find a short Test Exercise. This is set directly on what you have learned in the lesson: the questions are straightforward and contain no tricks. To provide you with the necessary practice, a set of Further Problems is also included: do as many of these problems as you can. Remember that in mathematics, as in many other situations, practice makes perfect – or more nearly so.

Even if you feel you have done some of the topics before, work steadily through each programme: it will serve as useful revision and fill in any gaps in your knowledge that you may have.

USEFUL BACKGROUND INFORMATION

I. Algebraic Identities

$(a+b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^2 = a^2 - 2ab + b^2$ $\qquad$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$a^2 - b^2 = (a-b)(a+b).$ $\qquad$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

II. Trigonometrical Identities

(1) $\sin^2\theta + \cos^2\theta = 1$; $\quad \sec^2\theta = 1 + \tan^2\theta$; $\quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(2) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(3) Let $A = B = \theta$. $\quad \therefore \sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

(4) Let $\theta = \frac{\emptyset}{2}$ $\quad \therefore \sin \emptyset = 2 \sin \frac{\emptyset}{2} \cos \frac{\emptyset}{2}$

$$\cos \emptyset = \cos^2 \frac{\emptyset}{2} - \sin^2 \frac{\emptyset}{2}$$

$$= 1 - 2 \sin^2 \frac{\emptyset}{2}$$

$$= 2 \cos^2 \frac{\emptyset}{2} - 1$$

$$\tan \emptyset = \frac{2 \tan \frac{\emptyset}{2}}{1 - \tan^2 \frac{\emptyset}{2}}$$

(5) $\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$

$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$

$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$

$\cos D - \cos C = 2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$

(6) $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$
$2 \cos A \sin B = \sin (A + B) - \sin (A - B)$
$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$
$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

(7) Negative angles: $\sin (-\theta) = -\sin \theta$
$\cos (-\theta) = \cos \theta$
$\tan (-\theta) = -\tan \theta$

(8) Angles having the same trig. ratios:

(i) Same sine: θ and $(180^\circ - \theta)$
(ii) Same cosine: θ and $(360^\circ - \theta)$, i.e. $(-\theta)$
(iii) Same tangent: θ and $(180^\circ + \theta)$

(9) $a \sin \theta + b \cos \theta = A \sin (\theta + \alpha)$
$a \sin \theta - b \cos \theta = A \sin (\theta - \alpha)$
$a \cos \theta + b \sin \theta = A \cos (\theta - \alpha)$
$a \cos \theta - b \sin \theta = A \cos (\theta + \alpha)$

$$\text{where: } \begin{cases} A = \sqrt{(a^2 + b^2)} \\ \alpha = \tan^{-1} \dfrac{b}{a} \qquad (0° < \alpha < 90°) \end{cases}$$

III. Standard Curves

(1) *Straight line:*

Slope, $m = \dfrac{dy}{dx} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Angle between two lines, $\tan \theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

For parallel lines, $m_2 = m_1$

For perpendicular lines, $m_1 m_2 = -1$

Equation of a straight line (slope = m)

(i) Intercept c on real y-axis: $y = mx + c$
(ii) Passing through (x_1, y_1): $y - y_1 = m(x - x_1)$
(iii) Joining (x_1, y_1) and (x_2, y_2): $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

(2) *Circle:*

Centre at origin, radius r: $x^2 + y^2 = r^2$
Centre (h, k), radius r: $(x - h)^2 + (y - k)^2 = r^2$
General equation: $x^2 + y^2 + 2gx + 2fy + c = 0$
with centre $(-g, -f)$; radius $= \sqrt{(g^2 + f^2 - c)}$
Parametric equations: $x = r \cos \theta$, $y = r \sin \theta$

(3) *Parabola:*

Vertex at origin, focus $(a, 0)$: $y^2 = 4ax$
Parametric equations: $x = at^2$, $y = 2at$

(4) *Ellipse:*

Centre at origin, foci $(\pm\sqrt{[a^2 - b^2]}, 0)$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where a = semi major axis, b = semi minor axis

Parametric equations: $x = a\cos\theta$, $y = b\sin\theta$

(5) *Hyperbola:*

Centre at origin, foci $(\pm\sqrt{[a^2 + b^2]}, 0)$: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Parametric equations: $x = a\sec\theta$, $y = b\tan\theta$

Rectangular hyperbola:

Centre at origin, vertex $\pm\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$: $xy = \frac{a^2}{2} = c^2$ where $c = \frac{a}{\sqrt{2}}$

i.e. $xy = c^2$

Parametric equations: $x = ct$, $y = c/t$

IV. Laws of Mathematics

(1) *Associative laws* – for addition and multiplication

$$a + (b + c) = (a + b) + c$$
$$a(bc) = (ab)c$$

(2) *Commutative laws* – for addition and multiplication

$$a + b = b + a$$
$$ab = ba$$

(3) *Distributive laws* – for multiplication and division

$$a(b + c) = ab + ac$$
$$\frac{b + c}{a} = \frac{b}{a} + \frac{c}{a} \text{ (provided } a \neq 0)$$

Programme 1

COMPLEX NUMBERS

PART 1

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