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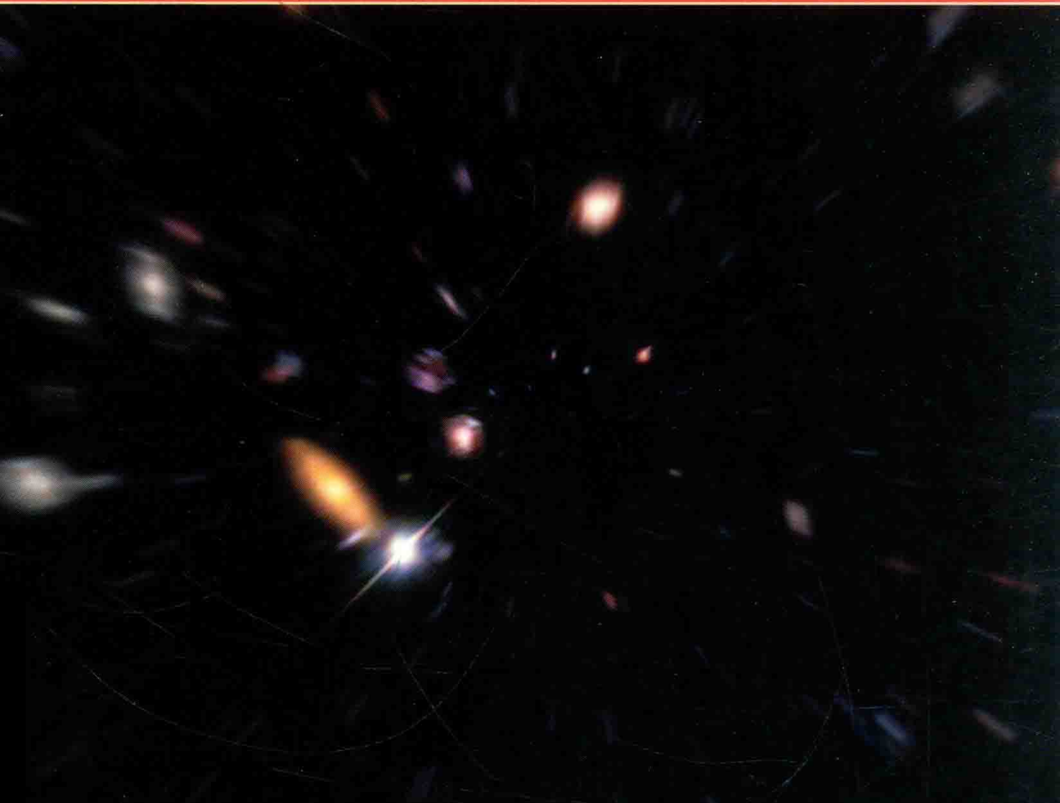
Introduction to Relativity



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Introduction to Relativity

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Preface

Overview—What Can You Expect from This Book?

Before we get to the real beginning of this book, let's look at its aims and objectives. Its intention is to teach you how to think about relativity and understand it in the simplest possible way. There are not many formulas, algebra is kept to a minimum, arguments are short and to the point, and calculus is *not* allowed until the last chapter. Vague, pseudophilosophical terms are outlawed. The inspiration for this approach is, naturally, Einstein himself. Physicists love to read Einstein's original papers because they typically included "thought experiments" that illustrated the concepts under investigation in a fashion even a child (a very smart one, anyway) could understand. For example, in this book we construct a simple clock out of mirrors and a light beam, following one of Einstein's most famous arguments, and we see how it works in a frame where it is at rest and in a frame in which it moves. In this way, we derive both time dilation (moving clocks run slower than clocks at rest) and length contraction (moving rods are shorter than rods at rest) in a transparent way.

In the course of these arguments, it will become apparent that at the heart of these unfamiliar phenomena is the fact that clocks that are separated in space and synchronized in one frame are *not* synchronized in a frame in relative motion. This fact, often called the relativity of simultaneity, is the essence of the subject. It follows simply from the basic experimental fact that there is a speed limit in nature that must be common to all observers whether they are in relative motion or not. A physical realization of the speed limit is afforded by light. We can synchronize spatially separated clocks by sending light rays between them. In this way, one of the basic postulates of relativity, that there exists a common speed limit in all inertial frames, dictates the operation and synchronization of all our clocks.

The content of relativity lies in its two basic postulates:

1. The laws of physics are the same in all inertial frames of reference.
2. There is a common finite speed limit in all inertial frames.

I hope that the novice reader is intrigued by all this. I also hope that the more seasoned physicist/teacher appreciates that we are going to build the subject up from the ground level. We are *not* going to follow a historic approach where electrodynamics played a central role in the development of the subject. That approach fails the novice. Instead, just from the idea that any inertial frame is as good as any other and the fact that there is a speed limit common to all frames, we can derive all the classic results of relativity without any specialized mathematics. The ideas of invariant intervals, even the Lorentz transformations, which are typically so prominent in relativity textbooks, are relegated to a later chapter where they are introduced as conveniences that summarize what we have already figured out through simple arguments. Visualization plays a central role in these developments. We introduce Minkowski space–time diagrams, which show the spatial and temporal coordinates of two frames in relative motion $\mathbf{v}$. Time dilation, Lorenz contraction, and, most important, the relativity of simultaneity (spatially separated clocks that are at rest and synchronized in one frame are not synchronized in a frame in relative motion) can be understood through simple pictures (cartoons, really), both qualitatively and quantitatively. From this perspective, we study the Twin Paradox and see that there is nothing really paradoxical about it—when one twin leaves the other and takes a round trip, she returns younger than her sibling. Of course, it is one thing to understand a physical phenomenon and another thing to feel comfortable with it. I know of no one who feels comfortable with the Twin Paradox, although high-energy experimentalists observe the effect daily in their experiments with high-energy elementary particles. Anyway, I hope that the reader can understand it and see that there is no contradiction involved in it.

Once we have mastered kinematics—the measurement of space and time intervals—we pass on to introductory dynamics, the study of energy, momentum, and equations of motion. In Newtonian mechanics we recall that there is momentum conservation, on the one hand, and mass conservation, on the other. In relativity, however, once we have figured out how spatial and temporal measurements are related in inertial frames, the definition and properties of momentum, energy, and mass are determined. We present another of Einstein's famous thought experiments that shows that energy and inertia (mass) are two aspects of one concept, relativistic energy, and find $E = mc^2$. The properties of relativistic momentum then follow from the properties of spatial and temporal measurements.

Although it is conventional to understand Postulate 2 (kinematics) thoroughly before delving into Postulate 1 (dynamics), the consistency of the two is central. In particular, Einstein asserted Postulate 1: If we have an inertial frame of reference in which isolated particles move at constant velocities in straight lines, then any frame moving with respect to us at velocity **v** has the same properties and there is no physical distinction between the two frames (i.e., the laws of physics are the same in both frames). This postulate constrains how objects interact. The dynamical laws of physics must then be consistent with Postulate 2, which states that there is a universal speed limit.

Newton's laws of dynamics are not consistent with the existence of a speed limit. We have to modify the notions of momentum, energy, and mass and the law of acceleration in the presence of a force to invent dynamical rules that are relativistic (i.e., that are consistent and lead to Postulate 2, the existence of the speed limit). We do this aided, again, by the original thought experiments of the masters, Einstein and Max Born. In this case, a close look at an inelastic collision in several frames of reference leads us to relativistic momenta and energy. From there we obtain the relativistic version of Newton's Second Law, that mass times acceleration equals force. We then consider high-energy collisions between particles and illustrate how energy can be converted into mass and how mass can be converted into energy in our relativistic world.

Our last topic consists of an introduction to the General Theory of Relativity, where we finally consider accelerated reference frames in Einstein's world. The key insight here is Einstein's version of the Equivalence Principle: There is no physical means to distinguish a uniform gravitational field from an accelerated reference frame.

This principle has many interesting forms and applications. Suppose you are on the surface of Earth and want to understand the influence of the gravitational field on your measuring sticks and clocks compared to those of your assistant who is at a greater height in the Empire State Building. Einstein suggests that the assistant jump out the window, because in a freely falling frame all effects of gravity are eliminated and we have a perfectly inertial environment where special relativity holds to arbitrary precision! During his descent, your assistant can make measurements of clocks and meter sticks fixed at various heights along the building and measure how their operation depends on their gravitational potential. We pursue ideas like this one in the book to derive the gravitational red-shift, the fact that clocks close to stars run more slowly than those far away from stars; the resolution of the Twin Paradox as a problem in accelerating reference frames; and the bending of light by gravitational fields.

A fascinating aspect of the Equivalence Principle is its universality, which becomes particularly clear when we calculate the bending of light as a ray glances by the Sun. Why does the light ray feel the presence of the mass of the Sun? The Equivalence Principle states that an environment with a gravitational field is equivalent to an environment in an accelerating reference frame—explicit acceleration clearly affects the trajectory of light and any other physical phenomena. So gravity becomes a problem in accelerated reference frames, which is just a problem in coordinate transformations, which is an aspect of geometry! We show that this problem in geometry must be done in the context of four-dimensional space–time, our world of Minkowski diagrams. Einstein's theory of gravity brought modern geometry, the study of curved spaces, into physics forever. We see hints of this in our work here, but our discussions do not use any mathematics beyond algebra and an elementary integral or two.

As long as we concentrate on gravitational fields of ordinary strength, we are able to use the Equivalence Principle to make reliable, accurate predictions. The Equivalence Principle reduces gravity to an apparent force, much like the centrifugal or Coriolis forces that we feel when riding on a carousel. In fact, we use relativistic turntables and rotating reference frames as an aide to studying and deriving relativistic gravitational effects. A short excursion into the world of curved surfaces helps us appreciate general relativity in applications where the gravitational field varies from point to point. We end our discussion with a look at current puzzles and unsolved problems.

Although the perspective of this book is its own, it owes much to other presentations. The influence of C. P. French's 1968 book [1] *Special Relativity* is considerable, and references throughout the text indicate where my discussions follow his. To my knowledge, this is the finest textbook written on the subject because it balances theory and experiment perfectly. The reader will find discussions of the Michelson–Morley experiment and early tests of relativity there. I do not cover those topics here because this book is aimed at students who may know little electricity and magnetism at this stage of their education. French's discussions of energy and momentum are reflected in my later chapters, and his problem sets are a significant influence on those included here. The exposition by N. D. Mermin [2] influenced several discussions of the paradoxes of relativity. This book is also recommended to the student because it shows a condensed-matter physicist learning the subject and finding a comfort level in it through thought-provoking analyses that avoid lengthy algebraic developments. The huge book by J. A. Wheeler and E. F. Taylor [3] titled *Spacetime Physics* inspired several of our discussions and problem sets. This book, a work that only the unique, creative soul of John Archibald Wheeler could produce, is recommended for its leisurely, interactive, thought-provoking character. Finally, the books by W. Rindler [4, 5], a pioneer in modern general relativity, are most highly recommended. His book *Essential Relativity* [5] is my favorite introduction to general relativity. After the student has mastered electricity and magnetism and Lagrangian mechanics, he or she could read *Essential Relativity* and see how the introductory remarks on general relativity in the second half of this book can be made quantitative.

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John B. Kogut

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Chapter 1

Physics According to Newton—A World with No Speed Limit

When you set up a problem in Newtonian mechanics, you choose a reference frame. This means that you set up a three-dimensional coordinate system, for example, so any point $\mathbf{r}$ can be labelled with an x measurement of length, a y measurement, and a z measurement, $\mathbf{r} = (x, y, z)$. Cartesian coordinates might not be the most convenient in a particular case, so you might use spherical coordinates or whatever suits the problem best.

Newton imagined carrying out experiments on a mass point m in this coordinate system. He imagined that the coordinate system was far from any external influences and under those conditions he claimed, on the basis of the experiments of Galileo and others, that the mass point could only move in a straight line at a constant velocity. Newton labeled such a frame of reference "inertial." Next, if the mass point were subject to a force, Newton claimed that its velocity would change according to his Second Law, force equals mass times acceleration,

$$\mathbf{f} = m\mathbf{a}. \tag{1.1}$$

The mass m in Eq. (1.1) is clarified in part by the Third Law, which states that if a body exerts a force $\mathbf{f}_1$ on another body, then the second body exerts a force $\mathbf{f}_2 = -\mathbf{f}_1$ on the first. This postulate is called the Law of Action–Reaction and it implies in this case that

$$m_1\mathbf{a}_1 = -m_2\mathbf{a}_2. \tag{1.2}$$

So, if the first body sets the scale for inertia, in other words, we define $m_1 \equiv 1$, then m_2 follows from Eq. (1.2).

Newton and others realized that there must be a wide class of inertial frames. If we discovered one frame that was inertial, then Newton argued that other inertial reference frames could be generated by

1. Translation—move the coordinate system to a new origin and use that system.

2. Rotation—rotate the coordinate system about some axis to a fixed, new orientation.

3. Boosts—consider a frame moving at velocity $\mathbf{v}$ with respect to the first.

Properties 1 and 2 are referred to as the uniformity and isotropy of space. Property 3, referred to as Galilean invariance, is plausible because if we have two frames in relative motion at a constant linear velocity $\mathbf{v}$, then acceleration measurements are the same in both frames. So, Eqs. (1.1) and (1.2) are unaffected—the Galilean boost has no physical impact on the dynamics and so should be a symmetry of the theory.

The reader should be aware that Newton and his colleagues argued constantly about these points. What is the origin of inertia? Why are inertial frames so special? Can't we generalize the Properties 1–3 to a wider class of reference frames? We will not discuss these issues here, but the reader might want to pursue them for a greater historical perspective. Our approach is complementary to the traditional one and carries less intellectual baggage.

Underlying Newtonian mechanics are the concepts of space and time. To measure the distance between points we imagine a measuring rod. Starting from an origin $(0, 0, 0)$, we lay down markers in the x and y and z directions so we can measure a particle's position $\mathbf{r} = (x, y, z)$ in this inertial frame. Next we need a clock. A simple device like a simple harmonic oscillator will do. Take a mass point m on the end of a spring and let it execute periodic motion back and forth. Make a convention that one unit of time passes when the mass point goes through one cycle of motion. In this way we construct a clock and measure speeds of other masses by noting how far they move in several units of time—in other words we compare the motion of our standard mass point in our simple harmonic oscillator with that of the other mass. Note that this is just what we mean by time in day-to-day situations. To make an accurate clock we need one with a sufficiently short unit of time, or period. Clocks based on the inner workings of the atom can be used in demanding, modern circumstances.

Just as it was convenient to place distance markers along the three spatial axes $\mathbf{r} = (x, y, z)$, it is convenient to place clocks on the spatial grid work. This will make it easy to measure velocities of moving particles—we just record the positions and times of the moving particle on our grid of measuring rods and clocks. When the moving particle is at the position $\mathbf{r}$ and the clock there reads a time t, we record the event. Measuring two such events allows us to calculate the particle's velocity in this frame if the particle has a uniform velocity. In setting up the grid of clocks, we must synchronize them so we can obtain meaningful time differences. This is easy to do. Place a clock at the origin and one at $\mathbf{r} = (1, 0, 0)$. Then, at the halfway point between them, place a beacon that sends out a signal in all directions. Because space is homogeneous and isotropic, the signal travels at the same speed toward both clocks. Set the clocks to zero, say, when they both receive the signal. The two clocks are synchronized and we didn't even need to know the speed of the signal emitted from the beacon. Clearly we could use this method to synchronize all the clocks on the grid.

Now we are ready to do experiments involving space-time measurements in this frame of reference. Call this frame S. It consists of the gridwork of measuring rods and clocks, all at rest, with respect to each other. Now suppose we want to compare our experimental results with those obtained by a friend of ours at rest in another fame that moves at velocity $\mathbf{v} = (v_x, 0, 0)$ with respect to us. According to our postulates, his or her measurements are as good as ours and all our physical laws can be written in his or her frame of reference without any change (Figure 1.1).

The measuring rods in S′ are identical to those in S; the clocks in S′ are also identical to those in S. Suppose that the origin of S and S′ coincide when the clock at the origin in S reads time $t = 0$ and the clock at the origin in S′ reads $t' = 0$. Now for the crucial question: Do the other grid markings and clocks at those markings also agree in the two frames S and S'? There are two distinct physics issues to consider here. The first is the operation of the clocks and rods in each frame. Following Postulate 1, all the clocks and rods in S work exactly the same as those at rest in S'. We need only

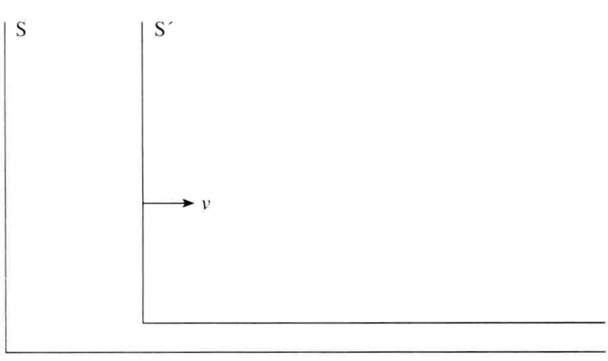

FIGURE 1.1 ▶

know that both frames are inertial—the relative velocity between the frames is physically irrelevant to the dynamics within each. The second issue is the physical mechanism by which we can transmit the information in one grid of rods and clocks to the other at relative velocity **v**. It follows from Newton's Second Law that information can be transmitted instantaneously from one frame to another. In Newton's world, objects can be accelerated to unbounded relative velocities. Accordingly, signals and information can be transmitted at unbounded velocities, essentially instantaneously. Therefore, the spatial gridwork and the times on each clock in S′ can be instantaneously broadcast to the corresponding rods and clocks in the frame S. Therefore, the gridwork of coordinates and times in both frames must be identical, even though they are in relative motion. So, the lengths of measuring rods and the rates of clocks are independent of their relative velocities. We therefore need only one measuring rod and one clock at one point in one inertial frame, and we know the positions and times of all measuring rods and clocks in any other inertial frame. For the purposes of this book, this serves as the meaning of "absolute space" and "absolute time" in Newtonian mechanics. We need not hypothesize about these notions from some philosophical basis, as was done historically, but can deduce them from the fact that in Newtonian mechanics there is no speed limit—information can be transmitted instantaneously.

We have dealt with these issues very explicitly because they will help us appreciate special relativity, where there is a speed limit, the speed of light. In all the rules of Newtonian mechanics, the rules of how things work, the Second and Third Laws, do not distinguish between inertial frames. The dynamics do satisfy a principle of relativity. The difference between the two theories comes from the fact that one has a speed limit and one does not. This affects how information is shared between frames. It also affects the dynamics within each frame—Einstein's form of Newton's force equals mass times acceleration is different because it must not permit velocities greater than the speed limit. Each theory is consistent within its own rules. For example, the rule by which times and position measurements are compared between the two frames, S and S′, in relative motion in Newton's world reads

$$\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t', \end{aligned} \tag{1.3}$$

where x, y, z, and t are measurements in S and x', y', z', and t' are the corresponding measurements in S′. For example, if we measure the position of a particle in S′ to be x', y', and z', at time t', then the coordinates of this event (measurement) in frame S are given by Eq. (1.3). These relations,

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