FINITE MATHEMATICS APPLIED TO THE REAL WORLD

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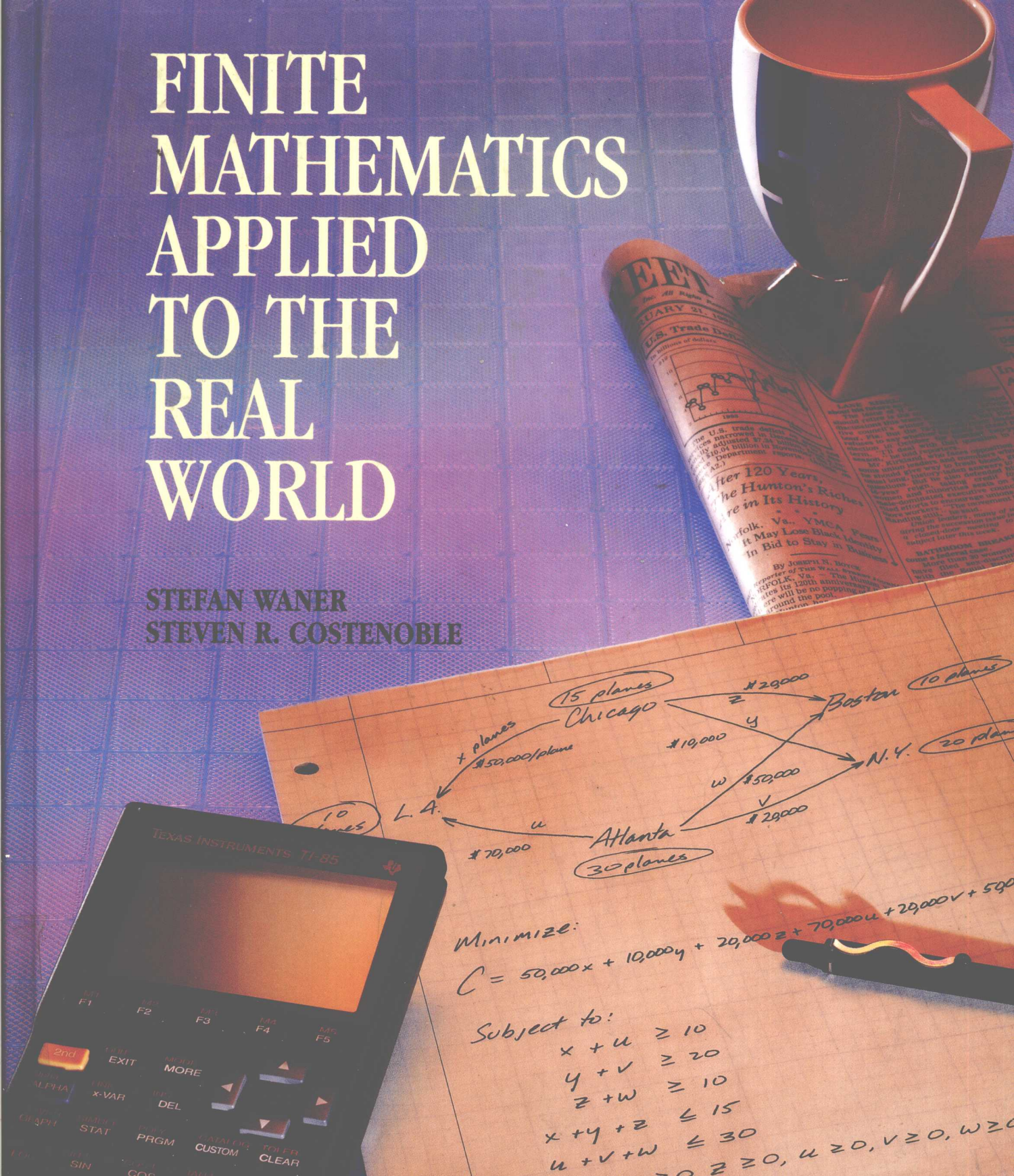

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Finite Mathematics Applied to the Real World

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Preface

This book is intended for a course in finite mathematics for students majoring in business, the social sciences, or the liberal arts. It is designed to address the considerable challenge of generating enthusiasm and developing mathematical sophistication in an audience that is often ill-prepared for and disaffected by the traditional finite mathematics courses offered on many college campuses.

This text is ambitious; we take the positive view that interested and motivated students can overcome any deficiencies in their mathematical background and attain a surprising degree of mathematical sophistication. The strong emphasis in this book on developing mathematical concepts and the abundance of relevant applications are dictated by this view. No less importantly, this book is one that a student whose primary interest is not mathematics can relate to and enjoy reading.

OUR APPROACH AND PHILOSOPHY

Our approach has been influenced by the current reform movement in college and pre-college mathematics. Within a framework of fairly traditional topics, we incorporate important features of the various reform projects, including a thorough integration of graphing technology, a focus on real applications, and an emphasis on mathematical concepts through the extensive use of conceptual exercises and pedagogical techniques such as the Rule of Three (numerical, geometric, and algebraic approach to concepts). In fact, we include a fourth rule: verbal communication of mathematical concepts. We implement this element through verbalizing mathematical concepts, rephrasing sentences into forms that easily go over to mathematical statements, and writing exercises.

At the same time, we retain the strong features of some of the more traditional texts, including an abundance of practice and drill exercises where appropriate, large numbers of applications to choose from, and inclusion of the standard topics in applied finite mathematics.

In addition to combining what we view as the strongest features of reform and traditional texts, we have worked to create a unique and fresh approach to pedagogy and style.

EMPHASIS ON CONCEPTS

As we develop each mathematical concept, we steer the student directly toward the most important ideas with as few obstacles as possible. We do this by equipping the student with relevant skills and a working knowledge of a

topic before considering its abstract foundations in detail. We avoid prematurely side-tracking the student with rigor and abstract subtleties; the appropriate time for abstraction and rigor is later, once the student has a firm grasp of the underlying concepts. Our approach enables students to learn mathematics in the most natural way—by example. Just as one learns to speak before mastering grammar or to play a musical instrument before studying harmony, so the understanding of new mathematical concepts is best established on a solid foundation of relevant skills rather than upon abstraction devoid of context.

So that learning of concepts by example will be effective, discussion and explanation of key concepts within each example must not obscure the mathematical simplicity of the solution. We address this issue in many examples with a separate *Before we go on* discussion following the solution. Here we explain, discuss, and explore the example's solution. We sometimes remind the student that an example is not always finished when a solution is found—we should check the solution and examine its implications.

More globally, the organization of material within each chapter and section has also been planned with conceptual development in mind. We present a new concept or an application of an old one as directly as possible, with many worked examples and references to actual data, even if this means postponement of some of the underlying theory. Once the concept has progressed to the point where the student is sufficiently comfortable with the material and can relate it to the real world, we return for a retroactive in-depth look at the foundations of what we have been doing and then develop the concept further if need be. In this way, the theory reinforces the student's knowledge and view of the world and is not seen as meaningless abstraction.

THOROUGH INTEGRATION OF GRAPHING TECHNOLOGY

The use of graphing calculators and computer software has been thoroughly integrated throughout the discussion, examples, and exercise sets, beginning with the first example of the graph of an equation in Chapter 1. In many examples we discuss how to use either graphing technology or computer spreadsheet software to solve the example; the sections are marked with the  symbol. Groups of exercises for which the use of graphing calculators or computers is suggested or required are also fully integrated into the exercise sets and carry the  symbol. This focus on graphing technology plays an important conceptual and pedagogical role in the presentation of many topics. For example, in Chapter 1, we include a discussion of using a spreadsheet program to do linear regression. In Chapter 2, our discussion of row reduction includes examples and exercises written specifically for the use of technology. We have also included (in Appendix B: Using a Graphing Calcu-

lator) a graphing calculator program that pivots on any selected non-zero entry of a matrix.

Some of the real power of technology is seen in the chapter on the mathematics of finance, where we guide the student in the use of a graphing calculator to answer questions that would normally require either logarithms or the use of tables.

Appendix B: Using a Graphing Calculator introduces the student to a graphing calculator, provides a detailed discussion on its use in all the major topics of the text, and also provides programs and interesting applications. It also includes several programs referred to in the text. Discussion and exercises on proper entry of functions into graphing calculators and computers are included in the first section of Appendix A: Algebra Review.

Although we emphasize graphing technology throughout the text, we are mindful of the varying degree of emphasis on and use of graphing calculators in college courses. The text is not dependent on this technology. Students who are equipped with nothing more than a scientific calculator will not find themselves at a disadvantage.

FOCUS ON REAL APPLICATIONS

We are particularly proud of the diversity, breadth, and sheer abundance of examples and exercises that are based on real, referenced data from business, economics, the life sciences, and the social sciences. This focus on real data has helped create a text that students in diverse fields can relate to and that instructors can use to demonstrate the importance and relevance of calculus in the real world.

Our coverage of real applications begins with the very first example of Chapter 1, where the Dow Jones Average is used to introduce the discussion on coordinates and graphs. It continues uniformly throughout the text, which includes innumerable examples and exercises based on real data.

At the same time, we have been careful to strike a pedagogically sound balance between applications based on real data and more traditional generic applications. The density and selection of real data-based applications have been tailored to the pedagogical goals and appropriate difficulty level for each section.

STYLE

It is a common complaint that many students do not actually read mathematics texts but simply search through them for examples that match the assigned exercises. We would like students to read this book—we would like students to *enjoy* reading this book. Thus we have written this book in a conversational and student-oriented style. We make frequent use of a question-and-answer dialogue format (indicated with the ***Q/A*** symbol) in order to encourage the development of the student's mathematical curiosity and intuition. We hope that this text will give the student insight into how a mathematician develops and thinks about mathematical ideas and their applications.

EXERCISE SETS

The strength of our exercise sets is one of the best features of this text. Our collection of more than 2,800 exercises provides a wealth of material that can be used to challenge students at almost every level of preparation. The exercises include everything from straightforward drills to interesting and rather challenging applications. We also include, in virtually every section of every chapter, applications based on real data (including data from more than 100 major corporations and government agencies); conceptual and discussion exercises useful for writing assignments; graphing calculator exercises; and what we hope are amusing exercises. In addition, every chapter contains a collection of chapter review exercises. Communication and Reasoning Exercises appear at appropriate places in the text.

Many of the scenarios used in application examples and exercises are revisited several times throughout the book. Thus, for instance, students will find themselves using different techniques, such as solving systems of equations and then using linear programming to analyze a transportation problem, or first using matrix multiplication and then Markov processes to analyze population shifts in the United States. Reusing scenarios and important functions this way provides unifying threads and shows students the complex texture of real-life problems.

PEDAGOGICAL FEATURES

- ***Q/A*** An important pedagogical tool we employ is the frequent use of informal question-and-answer dialogues. These often anticipate the kind of questions that may occur to the student and also guide the student through the development of new concepts.
- **You're the Expert** The *You're the Expert* feature at the end of each chapter is an extended application using and illustrating the central ideas of the chapter. The themes of these applications are varied, and they are designed to be as unintimidating as possible. For example, we do not pull complicated formulas out of thin air but focus instead on the development of mathematical models appropriate to the topics. Among the more notable of these applications are a careful analysis of the "Monty Hall problem" and a discussion of the Japanese economy based on actual input-output data. These applications are also ideal for assignments as projects, and it is to this end that we have included groups of exercises at the end of each.
- **Before we go on** Most examples are followed by supplementary discussions under the heading *Before we go on*. These discussions may include a check on the solution to a problem, a discussion of the feasibility and significance of a solution, or an in-depth look at what the solution means.

- **Systematic and Careful Treatment of Word Problems** We take an organized approach to the analysis and discussion of word problems, where the student is shown how to "zero in" on the unknown(s) in a word problem, and how to reword pertinent phrases into forms readily translatable into mathematical expressions. For example, the student is shown how to transform a potentially confusing phrase such as "there are at least twice as many goats as sheep" into a clearer form, such as: "the number of goats is at least twice the number of sheep" (which translates directly into $x \geq 2y$).
- **Communication and Reasoning Exercises** These exercises are designed to broaden the student's grasp of the mathematical concepts. In some, the student is asked to provide examples, to illustrate a point or design an application with a given solution; others are "fill in the blank" type exercises or exercises that invite discussion and debate. These exercises often have no single correct answer.
- **Conceptual and Computational Devices** The text features a wide variety of novel devices to assist the student in overcoming hurdles. These include a careful guide to the use of counting arguments (equipped with an "acid test" that enables the student to check the validity of a particular approach to a problem), numerous rewording guides for setting up systems of linear equations, linear programming problems and applications in conditional probability, and Bayes' theorem. We also use computational techniques for matrix reduction and simplex method pivoting that are easiest for hand calculation, including a technique for avoiding manipulating fractions.
- **Cautions, Hints, and Notes** Most sections include suggestions designed to assist the student in avoiding common errors and in tracking down their source when they do occur in a calculation.
- **Footnotes** Footnotes throughout the text provide sources, interesting background, extended discussion, and various asides.

MORE DISTINGUISHING FEATURES

- **Mathematical Rigor** Rather than present a sequence of formal theorems without proof, we have taken considerable care to motivate and present informal proofs of virtually *all* stated results.
- **Markov Chains** The treatment of Markov Chains includes discussions of both regular and absorbing chains, including expected time to absorption and relative strengths of different absorbing states. We also include exercises and examples involving random walks with reflecting and absorbing barriers as well as simple diffusion models.

- **Functions** Our treatment of functions was planned with the diverse needs of different course requirements in mind: Instructors preferring a concise treatment of functions can cover the material in the first section of Chapter 1 and simply skip the second (optional) section. When combined with the first, the second section provides the same coverage as most applied mathematics texts that include calculus and thus prepares students to go on to calculus.
- **Counting Arguments** We focus on counting procedures rather than formulas. It is our experience that students have difficulties in formulating systematic counting procedures. Most texts suppress this aspect of attacking a counting problem and focus on formulas, but without the proper counting procedures students become confused about which formulas to use when.

RELATED BOOKS IN THE SERIES

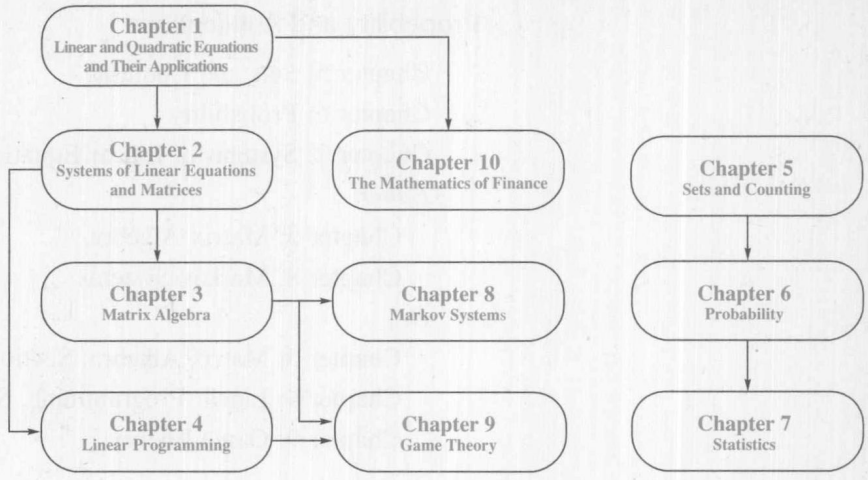
This is one of three books by the authors on mathematics for business, the social sciences, and the liberal arts. *Calculus Applied to the Real World*, which can be used as a sequel to this book, covers the topics of basic calculus. These topics include functions, limits, derivatives, integrals, and functions of several variables. *Finite Mathematics and Calculus Applied to the Real World* combines most of *Calculus Applied to the Real World* with this book to provide a book suitable for use in a two-semester sequence in finite mathematics and calculus.

ORGANIZATION AND COURSE OPTIONS

Care has been taken in the format of this text to provide the greatest possible flexibility in course design.

- The chapter on linear programming does not require the material in Chapter 3 (Matrix Algebra).
- The Mathematics of Finance chapter can be covered at any time after Chapter 1, as it is independent of the remaining chapters.
- The material on counting arguments (Sections 3 and 4 of Chapter 5) is not required for the chapter on probability theory (optional Sections 4 and 7 excluded).
- The chapter on probability is not a prerequisite for our discussion of Markov chains and game theory, so that, for instance, Markov chains could be taught as an application of matrix algebra in a course that does not include probability theory.

The following chart shows the logical dependence of the chapters. Notice again that Chapter 10 (Mathematics of Finance) can be covered at any time after Chapter 1, and the complete independence of the sequence Chapters 5 through 7.



Following are suggested chapter orders that illustrate the flexibility of this text for one-semester courses in finite mathematics. Coverage of specific sections within each chapter could vary with the emphasis of the particular course.

Finite Mathematics with Probability

Chapter 1: Linear and Quadratic Equations and Their Applications
Chapter 2: Systems of Linear Equations and Matrices
Chapter 3: Matrix Algebra (Section 4 optional)
Chapter 4: Linear Programming
Chapter 5: Sets and Counting (Sections 3 and 4 optional)
Chapter 6: Probability (Sections 4 and 7 optional)
Chapter 7: Statistics

Linear Mathematics with Applications

Chapter 1: Linear and Quadratic Equations and Their Applications
Chapter 2: Systems of Linear Equations and Matrices
Chapter 3: Matrix Algebra
Chapter 4: Linear Programming
Any or all of the following Systems:
Chapter 8: Markov Systems
Chapter 9: Game Theory
Chapter 10: The Mathematics of Finance

Probability and Applications

Chapter 5: Sets and Counting
Chapter 6: Probability
Chapter 2: Systems of Linear Equations and Matrices
Either:
Chapter 3: Matrix Algebra
Chapter 8: Markov Systems
Or:
Chapter 3: Matrix Algebra, Sections 1 & 2
Chapter 4: Linear Programming, Sections 1–3
Chapter 9: Game Theory

SUPPLEMENTS

For the Instructor

HarperCollins Test Generator/Editor for Mathematics with QuizMaster is fully networkable and is available in both IBM and Macintosh versions. The system features printed graphics and accurate mathematical symbols. The program allows the instructor to choose problems either randomly from a section or problem type or manually while viewing them on the screen, with the options to regenerate variables or scramble the order of questions while printing if desired. The editing feature allows instructors to customize the chapter data disks by adding their own problems. The Test Generator comes free to adopters.

Instructor's Resource Manual provides detailed discussion of the material in each section, complete solutions to the *Communications and Reasoning Exercises* and the *You're the Expert* exercise sets, teaching tips, and a large collection of sample test questions.

Instructor's Complete Solution Manual contains solutions to every exercise in the texts.

Steven's Software is a very user-friendly set of tools for the Macintosh that can be used either by students for independent exploration or for instructors for demonstration of: the Gauss-Jordan Method, Matrix Algebra, and the Simplex Method.

For the Student

Interactive Tutorial Software with Management System is available in IBM and Macintosh versions and is fully networkable. As with Test Generator/Editor, this innovative software is algorithm-driven, automatically regenerating constants so that a student will not see values repeated if he or she

revisits any particular problem for additional practice. The tutorial is self-paced and provides unlimited opportunities to review lessons and to practice problem solving. If a student gives a wrong answer, he or she can request to see the problem worked out and get a textbook page reference. The program is menu-driven for ease of use, and on-screen help can be obtained at any time with a single keystroke. Students' scores are automatically recorded and can be printed for a permanent record. The optional **Management System** lets instructors record student scores on disk and print diagnostic reports for individual students or classes. This software is free to adopters, but may also be purchased by students for home use. (Macintosh version ISBN 0-06-502589-X; IBM version 0-06-502588-1)

Steven's Software described above, is very useful to the student.

Explorations in Finite Mathematics (IBM format only), by David Schneider (ISBN 0-673-46932-8), University of Maryland, contains on one disk a wider selection of routines than in any similar software supplement. Included are utilities for Gaussian elimination, matrix operations, graphical and simplex methods for linear programming problems, probability, binomial distribution, simple and compound interest, loan and annuity analysis, finance table, difference equations, and more. Refined monitor display for fractions, color capabilities, choice of exact or approximate calculations with matrices, and refined printing capabilities further set this apart from other programs.

Matrix with Linear Programming (IBM format only), by Maylin Dittmore (ISBN 0-06-501266-6) is designed to assist the student in any course of study that involves the use of matrices. MATRIX was not only created to help the student with tedious calculations associated with matrices, but also to help them gain an understanding of and appreciation for real-world problems that can be analyzed and solved using matrices.

Student's Resource Manual (ISBN 0-06-501809-5) provides complete, worked-out solutions to the odd-numbered exercises in the text. The manual also includes comprehensive chapter summaries and true/false quizzes for each chapter that help students both review and test their understanding.

Topics in Finite Mathematics: An Introduction to the Electronic Spreadsheet, by Sam Spero, Cuyahoga Community College (ISBN 0-06-500300-4), is a user-friendly guide designed to introduce students to the various ways one can approach problem solving with spreadsheets. Knowledge of spreadsheets is not assumed, and the approach is adaptable to all spreadsheet programs.

Graphing Calculator Lessons for Finite Mathematics by Paula G. Young (ISBN 0-06-501330-1) contains activities for utilizing the graphing calculator in the course.

CLASS TESTED

This book was used in manuscript form by many of our colleagues at Hofstra University. Their reactions and input were crucial to the development of this book. We thank them for all of their help; their names appear in the Acknowledgments section on the following pages.

ACCURACY

Accuracy checking was carried out at every stage of the production process. In particular, each chapter was checked by at least two different mathematicians. We gratefully acknowledge their help in this critical aspect of the project: Steven Blasberg, West Valley College; Patricia Blus, National Lewis; Susan Boyer, University of Maryland—College Park; Richard L. Conlon, University of Wisconsin—Stevens Point; Vivian Freund; Kathleen R. Pirtle; Franklin Pirtle; James Wooland, Florida State University; Earl J. Zwick, University of Terre Haute.

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