TOPICS IN CONTEMPORARY MATHEMATICS

SIXTH EDITION

BELLO | BRITTON

Topics in Contemporary Mathematics

SIXTH EDITION

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Preface

In this sixth edition we have continued with our goal of introducing the student to the many interesting mathematical concepts that are used in our contemporary world. We have tried to bring out the basic ideas and techniques as simply and clearly as possible and have related these ideas to other areas—such as sociology, psychology, and business—that will be attractive to the reader. Whenever feasible, elementary applications are given; these can be found in a section feature called "Getting Started," throughout the text's discussion and examples, and in the lesson problem sets. As suggested by the "Standards for Introductory College Mathematics" of AMATYC, the more abstract and theoretical aspects of the subject matter have been deemphasized. Instead, emphasis has been placed on the understanding and use of the various concepts that are introduced. An important aid to this goal will be found in the exercises, which include more than 4100 problems ranging from some necessary routine drill to challenges for the better students. The reader will find considerable support and explanation in the more than 500 worked-out examples.

Features of the Sixth Edition

We have followed the valuable suggestions of users of previous editions and the many reviewers contributing to this sixth edition to clarify the exposition, expand the coverage, and, in general, improve the book.

- We have completely redesigned the format of the book and provided many new examples and exercises.
- As in the previous edition, each chapter begins with a Preview feature detailing the material to be covered in the chapter and the ways in which the topics are related to each other.
- Each section begins with a "Getting Started" feature. These applications offer a motivating introduction for the techniques and ideas to be covered and are drawn from a vast array of fields.
- A problem-solving section has been added to Chapter 1, and many examples have been added in pertinent sections to emphasize problem-solving methods throughout the text. These special examples use a unique two-column format to describe the general problem-solving method and then demonstrate a specific use.
- We have retained our successful feature "In Other Words" to give students the opportunity to use writing to clarify and express ideas, concepts, and procedures. Students will think, talk, and write mathematics when they work these problems, which are included in every exercise set.
- A set of Research Questions is included at the end of each chapter to help students master research and library techniques as well as to explore how the topics under discussion were developed. These questions can be assigned to individual students or as group projects. An expanded Research Bibliography detailing sources for researching these questions is provided at the end of the book.
- In Chapter 2 we have combined, condensed, and clarified the treatment of truth tables and the relationship between conditional statements and implication.
- The *Numeration Systems* chapter has been condensed and now includes Octals and Hexadecimal Systems in Section 3.3.
- Chapter 4 has been renamed *Number Theory and the Real Numbers* and now features a section dealing with the natural numbers, cardinals, ordinals, the Fundamental Theorem of Arithmetic, divisibility rules, and the GCF and LCM and its applications.
- Chapter 5, *Equations, Inequalities, and Problem Solving*, now starts with a section discussing the solution of algebraic sentences and progressing to graphing these sentences. A section dealing with modeling and emphasizing verbal models and the techniques used to translate them into algebraic models has been added.

- Chapter 6, *Functions and Graphs*, now starts by discussing relations and functions, emphasizing the algebraic and graphic aspects of linear functions and inequalities.
- The *Geometry* chapter has been extensively revised. Traditional topics such as points, lines, planes, and angles have been condensed and new ones such as non-Euclidean geometry, topology and networks, and chaos and fractals have been introduced.
- Based on our reviewers' and users' suggestions, we have clarified the exposition and development in the *Counting* and *Probability* chapters.
- The *Statistics* chapter now starts with a section dealing with sampling.
- Chapter 12 has been revised; its new title, *Your Money and Your Math*, reflects the emphasis of applying the concepts students have learned to their daily life.
- Finally, the explanation of the metric system now appears as an Appendix.
- In writing this sixth edition we have paid careful attention to the "Standards for Introductory College Mathematics" set forth by AMATYC. Accordingly, we have included numeracy (Chapters 3 and 4), symbolism and algebra (Chapter 5), geometry and measurement (Chapter 7), functions (Chapter 6), probability and statistics (Chapters 10 and 11), and deductive proof (Chapters 1 and 2) among the topics covered in the book.
- We have made many significant efforts to address the NCTM curriculum recommendations regarding communication (In Other Words), reasoning (Chapter 2), connections (Discovery features and Mathematical Systems), algebra (Chapters 5 and 6), geometry (Chapter 7), statistics (Chapter 11), probability (Chapters 9 and 10), mathematical structures (Chapter 8), and problem solving (throughout the book).

Suggested Courses Using This Book

The book is quite flexible, with a large selection of topics available to suit various courses. The entire book can be covered easily in a full year's course, while many alternative choices can be made for a two-quarter or a one-semester course. Here are some of the courses for which the book is suggested:

- General education or liberal arts mathematics (the text follows most of the CUPM [Committee on the Undergraduate Program in Mathematics] and AMATYC recommendations for liberal arts mathematics)
- Topics in contemporary mathematics courses
- College mathematics or survey of mathematics courses
- Introduction to mathematics or applications of mathematics courses

There are a few more advanced topics that may be included or omitted at the instructor's discretion. These choices will not affect the continuity of any chapter presentation or syllabus as a whole. The topics include the following sections: 1.6 Infinite Sets; 2.7 Switching Networks; 6.6 Linear Programming; 7.6 Networks, Non-Euclidean Geometry, and Topology; 7.7 Chaos and Fractals; and 8.5 Game Theory.

Supporting Material and Supplements

This text has an extensive support package that includes:

- An *Instructor's Guide* containing commentary and teaching suggestions for each chapter, suggested course syllabi, answers to even-numbered problems in the exercises, a variety of five test forms per chapter, and answers to all test questions in the guide.
- A *Student's Solutions and Study Guide* containing complete solutions to all odd-numbered problems in the exercises and to all the problems in the chapter Practice Tests.
- Algorithmic testing from ips Publishing offering a total of 200 algorithms for creating a nearly unlimited variety of test forms for each chapter of the text. Available in IBM PC and Macintosh versions.
- ESA computerized testing that allows instructors to edit and print fixed-item tests. The full range of user features includes pull-down menus, dialog boxes, random or manual item selection, and

export/import capability. Available in IBM PC and Macintosh versions.

- Preparing for the CLAST—Mathematics, which is a competency-based study guide that reviews and offers preparatory material for the CLAST (College Level Academic Skills Test) objectives required by the State of Florida for mathematics.
- Algorithmic CLAST software, also from ips Publishing, for preparing practice test forms covering every CLAST mathematics objective.
- Videotapes reviewing key topics in the text.

A Word About Problem Solving

Problem solving has become a fixture in mathematics textbooks. Led by the teachings of George Polya, and following the recommendations of the NCTM and the MAA, most mathematics books at this level cover the topic. Many texts, however, front-load much of their presentation in the first chapter; all of the techniques, procedures, and pedagogy are paraded in Chapter 1 and then promptly forgotten. We have chosen to integrate problem solving where it is needed and, consequently, where it can be taught and learned most effectively.

For example, a few of the strategies suggested by Polya himself call for making a table, writing an equation, making a diagram, and accounting for all possibilities. Why not wait to present these techniques in the chapters dealing with truth tables, algebra, geometry, and counting, respectively, where the pertinent methods can be effectively displayed, rather than laboriously creating artificial solutions only to demonstrate strategies by solving artificial problems?

The artificial approach, after all, is not problem solving; it is problem making. It will make problems for the instructor, for the student, and (one might argue) for society at large. As Professor Beberman put it, "I think in some cases we have tried to answer questions that students never raise and to resolve doubts they never had. . . ." Therefore, we have worked very hard to dispel the unfortunate notion gained by many students that mathematics is an artificial subject riddled with uninteresting and contrived problems. As an ongoing theme of this text, problem solving is presented purposefully in meaningful and appropriate contexts where students can best understand and appreciate its methods. Above all, we hope that this integrated approach will help students learn how to apply problem-solving techniques in the real world once the course is over.

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I. B.
J. R. B.

A User's Guide to Features

This new edition of *Topics in Contemporary Mathematics* contains a wide variety of features designed to help build the reader's understanding of mathematics by placing the work to be done **in context.** A student who utilizes the features in this book will gain a better understanding of the history behind each topic, how the topic relates to everyday life, how different topics in the course interrelate, and—most importantly—how to think about solving problems in the real world once the course is over.

The **features** in *Topics* have been carefully written to ensure that, while many are optional, they are **interrelated.** In addition, the text utilizes a **four-color design** to highlight pedagogical features, emphasize each feature's function, and make them visually interesting.

Putting Material in Context

Chapter Preview

Each chapter begins with a list of topics for quick reference. The introduction that follows provides an overview of the material being studied and explains the ways in which the topics are related.

The Human Side of Mathematics

Who devised this material or contributed to its development? Placed at the beginning of each chapter to serve as motivation and offer historical perspective, this feature helps to communicate the message that mathematics is a growing body of knowledge, that it is a human endeavor, and that every topic studied began as part of a problem-solving process. Remarks on **Looking Ahead** link the biography to upcoming material.

8

Mathematical Systems and Matrices

In this chapter we will study different mathematical systems. First, we consider rectangular arrays of numbers called **matrices** and the operations that can be performed with these matrices, as well as their applications. As it turns out, matrices can be used to solve systems of linear equations by using *elementary row operations*. They can also be used to code and decode messages, a method employed by intelligence services around the world.

We then start the study of mathematical systems by considering *clock arithmetic* in a 12- or 5-hour clock and generalize the idea to *modular* arithmetic. We continue with a discussion of abstract mathematical systems, including two mathematical structures called **groups** and **fields,** and end the chapter with a study of game theory, a subject with numerous applications to business, games of chance, and military science.

The Human Side of Mathematics

Sophie Germain was born in Paris in 1776. As a child she spent many hours in her father's library. There, she came across the legend of Archimedes, who was so engrossed in the study of a geometric figure he had drawn that he failed to respond to a Roman soldier. "The soldier flew into a passion, unsheathed his glorious sword, and dispatched the unarmed veteran geometer of seventy-five." How fascinating the problem must have been, Germain reasoned, to command such intense concentration! From that point she was determined to learn all she could about mathematics.

In 1794 the Ecole Polytechnic was founded, but women were not admitted as regular students. Germain obtained Joseph-Louis Lagrange's lectures from her friends and, at the end of the course, handed her observations to the professor under the pen name M. Le Blanc. So impressed was Lagrange with the originality of the notes that he

Sophie Germain
1776–1831

429

Getting Started

Appearing at the beginning of every section, **Getting Started** is an application that demonstrates how the material relates to the real world. Hundreds of applications are used to introduce some of the techniques and ideas to be covered in each new section.

238 5 Equations, Inequalities, and Problem Solving

already invented the method of least squares. He entered the University of Göttingen, where he spent 3 years completing his *Disquisitiones Arithmeticae* (*Arithmetical Researches*). In 1798, he went to the University of Helmstedt, where he was awarded his Ph.D. His doctoral thesis gave the first proof of the fundamental theorem of algebra, that every algebraic equation has at least one root among the complex numbers.

His *Disquisitiones*, published in 1801, is regarded as the basic work in the theory of numbers. During his life he also made great contributions to astronomy, geodesy (the measurement of the Earth), geometry, theoretical physics, and complex numbers and functions. Along with his masterful theoretical research, he was also a well-known inventor; among other things, he made significant contributions to the invention of the electric telegraph in the early 1830s.

Archimedes, Newton, and Gauss, these three are in a class by themselves among the great mathematicians, and it is not for ordinary mortals to attempt to rate them in the order of merit.
—E. T. Bell

Looking Ahead: Much of Gauss's work in pure mathematics dealt with number theory, the concept of complex numbers, and the solutions to algebraic equations, which is the focus of this chapter.

5.1 Solutions of First-Degree Sentences

GETTING STARTED **Crickets, Ants, and Temperatures**

Does temperature affect animal behavior? You must know about bears hibernating in the winter and the languid nature of students in the spring. But what about the behavior of crickets and ants? Can you tell whether crickets will stop chirping before ants stop crawling? In the Discovery section, you will find that the number N of chirps a cricket makes per minute *satisfies* the *equation* $N = 4(F - 40)$, where F is the temperature in degrees Fahrenheit. What happens as the temperature increases? In problem 93, Exercise 5.1, you will find that the speed S (in cm/sec) for certain types of ants is $S = \frac{1}{6}(C - 4)$, where C is the temperature in degrees Celsius (see photo). What happens as the temperature decreases? The relationship between Fahrenheit and Celsius temperature is given by $F = \frac{9}{5}C + 32$. Armed with this information, can you tell whether crickets stop chirping before ants stop crawling?

entary algebra was first treated in a systematic fashion by the Arabs during eriod before the Renaissance, when Europe was almost at a standstill ectually. By the early 1600s, algebra had become a fairly well-developed h of mathematics, and mathematicians were beginning to discover that a iage of algebra and geometry could be highly beneficial to both subjects. has been said that algebra is arithmetic made simple, and it is true that a amount of elementary algebra enables us to solve many problems that d be quite difficult by purely arithmetic means. In this chapter we shall der some of the simpler algebraic techniques that are used in problem ng.

e have already made frequent use of various symbols, usually letters of the bet, as placeholders for the elements of a set of numbers. For example, we

$a + b = b + a$ a, b real numbers

594 11 Statistics

2. Sort or tally the data into the appropriate classes.
3. Count the number of items in each class.
4. Display the results in a table.
5. If desired, make a histogram and/or frequency polygon of the distribution.

By following this procedure in the next two examples, we will see that it is really not very difficult to tabulate a frequency distribution and construct a histogram or frequency polygon.

Problem Solving

Making Histograms

1. **Read** the problem.

Make a frequency distribution with three classes and construct the corresponding histogram for the men's jeans prices shown in the Getting Started section.

2. **Select** the unknown.

We want to make a frequency distribution and then a histogram for the men's jeans' prices.

3. **Think** of a plan.

We need to create three classes and determine their frequencies.

4. **Use** the procedure we have studied to carry out the plan. What is the class width? What are the class limits? What are the class boundaries? Are these boundaries convenient for this problem?

Since the highest price is 60 and the lowest 14, the class width is

$$\frac{60 - 14}{3} = 15.3$$

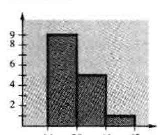
which is rounded *up* to 16. The lower limits for our classes are 14, 30, and 46, making the upper limits 29, 45, and 61. Thus, the class boundaries are the halfway points between 29 and 30 (29.5), 45 and 46 (45.5), and $45.5 + 16 = 61.5$. However, these boundaries are not convenient or natural, so we choose to make our class limits 14 to 30, 30 to 46, and 46 to 62. The classes can be described by the inequalities shown in Table 11.6, where p represents the price. The tallies and frequencies are shown in Table 11.6 and the histogram in Figure 11.5.

Make the frequency distribution. Draw the histogram.

TABLE 11.6

Class	*Tally*	*Frequency*
$14 \le p < 30$	𝍸 IIII	9
$30 \le p < 46$	𝍸	5
$46 \le p < 62$	I	1

FIGURE 11.5



5. **Verify** the solution.

TRY EXAMPLE 3 NOW   Cover the solution, write your own, and then check your work.

Problem-Solving Skills

Problem solving is introduced in Chapter 1 and presented as an ongoing theme throughout *Topics*. Specific **Problem Solving** examples are clearly formatted using two columns. The left column uses the RSTUV method (Read, Select, Think, Use, and Verify) to guide the reader through the problem. In the right column, the solution is carefully developed. Similar standard examples follow and provide additional reinforcement.

Using Your Knowledge

Interesting application problems help students generalize material they have learned and apply it immediately to similar real-life situations. They are included as an answer to that often-asked question, "Why do I have to learn this, and what good is it?"

Discovery

More challenging problems are provided to further develop critical thinking and problem-solving skills. These are brief excursions into related topics, extensions, and generalizations.

Using Your Knowledge

Gepetto Scissore, a barber in the small town of Sevilla, who was naturally called the Barber of Sevilla, decided that as a public service he would shave all those men and only those men of the village who did not shave themselves. Let $S = \{x \mid x \text{ is a man of the village who shaves himself}\}$ and $D = \{x \mid x \text{ is a man of the village who does not shave himself}\}$.

77. If g represents Gepetto,
a. is $g \in S$? **b.** is $g \in D$?

The preceding problem is a popularization of the *Russell paradox*, named after its discoverer, Bertrand Russell. In studying sets, it seems that one can classify sets as those which are members of themselves and those which are not members of themselves. Suppose that we consider the two sets of sets

$M = \{X \mid X \in X, X \text{ is a set}\}$ and
$N = \{X \mid X \notin X, X \text{ is a set}\}$

78. Answer the following questions:
a. Is $N \in M$? **b.** Is $N \in N$?
Think about the consequences of your answers!

Discovery

You should find the following paradox amusing, puzzling, and perhaps even thought provoking. Define a *self-descriptive word* to be a word that makes good sense when put into both blanks of the sentence "____ is a(n) ____ word." Two simple examples of self-descriptive words are "English" and "short." Just try them out!

Now define a *non-self-descriptive word* to be a word that is not self-descriptive. Most words will fit into this category. Try it out again. Now consider the following question.

79. Let S be the set of self-descriptive words, and let S' be the set of non-self-descriptive words. How would you classify the word *non-self-descriptive*? Is it an element of S? Or is it an element of S'? You should get into difficulty no matter how you answer these questions. Think about it!

1.3 Set Operations

GETTING STARTED **Diagnosis and Set Operations**

In diagnosing diseases, it is extremely important to recognize the symptoms that ...nguish one disease from another (usually called the differential diagnosis). ...example, some symptoms for hypoglycemia (too little sugar in the blood) ...hyperglycemia (too much sugar in the blood) are identical. Short of a blood ... how does a doctor determine whether a person is hypoglycemic or ...rglycemic? First, doctors are aware that a certain universal set of symptoms ...indicate the presence of either hypoglycemia or hyperglycemia. Next, they ...rd symptoms that are common to both conditions. Table 1.2 shows that ...ea and headaches are associated with both diseases. Thus, those two ...ptoms will not help make a diagnosis and should be disregarded. The ...asis should be on visual disturbances, trembling, stomach cramps, and ...l breathing. Do you know which diagnosis to make when presented with the ...ptoms in Table 1.2? You will encounter similar questions in Exercise 1.3, ...lems 40–54, and in Using Your Knowledge, problems 67–69.

...ften important to ascertain which elements two given sets have in common. ...xample, the sets of symptoms exhibited by patients with too little sugar in

Roller bladers benefit from a low-impact, highly aerobic workout.

EXAMPLE 6 The lower limit L (heartbeats per minute) of your target zone is a *function* of your age a (in years) and is given by

$$L(a) = -\tfrac{2}{3}a + 150$$

Find the value of L for people who are the following ages:

(a) 30 years old (b) 45 years old

Solution

(a) We need to find $L(30)$, and because

$$\begin{aligned} L(a) &= -\tfrac{2}{3}a + 150 \\ L(30) &= -\tfrac{2}{3}(30) + 150 \\ &= -20 + 150 = 130 \end{aligned}$$

This result means that a 30-year-old person should try to attain at least 130 heartbeats per minute while exercising.

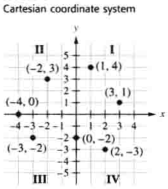
(b) Here, we want to find $L(45)$. Proceeding as before, we obtain

$$\begin{aligned} L(45) &= -\tfrac{2}{3}(45) + 150 \\ &= -30 + 150 = 120 \end{aligned}$$

(Find the value of L for your own age.) ■

C. Graphing Relations and Functions

FIGURE 6.1
Cartesian coordinate system



We shall now study a method for drawing pictures of relations and functions. Figure 6.1 shows two number lines drawn perpendicular to each other. The horizontal line is labeled x and is called the **x axis.** The vertical line is labeled y and is called the **y axis.** The intersection of the two axes is the **origin.** We mark a number scale with the 0 point at the origin on each of the axes. The four regions into which the plane is divided by these axes are called **quadrants** and are numbered I, II, III, and IV, as shown in Figure 6.1. This diagram forms a **Cartesian coordinate system** (named after René Descartes). We can make pictures of relations on such a coordinate system.

Figure 6.1 shows the usual way in which the positive directions along the axes are chosen. On the x axis, to the right of the origin is positive and to the left is negative. On the y axis, up from the origin is positive and down is negative. To

Examples, Exercise Sets, and Applications

The more than 500 examples in *Topics* include a wide range of computational, drill, and applied problems selected to build confidence, competency, skill, and understanding.

More than 4100 carefully developed exercises provide extensive practice with drill and applied problems included in each section. Each exercise set is carefully graded to build student confidence in solving problems. Answers to odd-numbered exercises appear in the back of the text.

Extensive applications, a key strength of this text, are integrated throughout the examples and exercises.

Calculator Corners

Another problem-solving skill students should develop is the ability to understand how and when to utilize technology. These features provide essential background on how to solve problems using a calculator.

In Other Words

Useful as a writing exercise or for class discussion, these brief questions provide the opportunity to think about and clarify ideas, concepts, and procedures.

because this would make $y = 0$ (no triangle!). However, if x is any odd integer greater than 1, there is a right triangle with the desired relationship between the sides. If you choose $x = 3$, and solve $x^2 = 2y + 1$ for y, you find $y = 4$. This gives you the well-known 3-4-5 right triangle.

b. Make a table of the next four of these triangles.

In Other Words

The solutions of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

61. What type of solution do you get if $b^2 - 4ac = 0$?

62. What type of solution do you get if $b^2 - 4ac > 0$?

63. What type of solution do you get if $b^2 - 4ac < 0$?

Using Your Knowledge

The distance h (in feet) traveled in t seconds by an object dropped from a point above the surface of the Earth is given by the formula

$$h = 16t^2$$

op of a 64-ft
e object to hit

65. A man jumps from a height of 28 ft into a tank of water. How long does it take him to hit the water?

66. An object is dropped from a height of 144 ft. How long does it take for the object to hit the ground?

Discovery

Charlie Brown received a chain letter. Several days later, after receiving more letters, he found that the number he had received was a *perfect square*. (The numbers 1^2, 2^2, 3^2, and so on, are perfect squares.) Charlie decided to throw away some of the letters, and being very superstitious, he threw away 13^2 of them. To his surprise, he found that the number he had left was still a perfect square.

67. What is the maximum number of letters Charlie could have received before throwing any away? (*Hint:* Let x^2 be the initial number of letters and let y^2 be the number he had left. Then $x^2 = y^2 + 13^2$. Remember that x and y are integers, and you will want to make y as large as possible relative to x because you want x to be as large as possible.)

Calculator Corner

Your calculator can be extremely helpful in finding the roots of a quadratic equation by using the quadratic formula. Of course, the roots you obtain are being approximated by decimals. It is most convenient to start with the radical part in the solution of the quadratic equation and then store this value so you can


The Transamerica Corporation

The photograph shows the Transamerica pyramid in San Francisco. At 853 ft in height (48 stories) it is the tallest building on the city's skyline. The pyramid portion of the structure is built on a square base and is thus an example of a *square pyramid*. Pyramids are one example of solids bounded by polygons.

A. Three-Dimensional Figures

A solid bounded by plane polygons is called a **polyhedron.** The polygons are the **faces** of the polyhedron, the sides of the polygons are the **edges,** and the vertices of the polygons are the **vertices** of the polyhedron. The Egyptian pyramids are polyhedrons with five faces, one of which is a square (the base) and the others are triangles. A square pyramid has eight edges and five vertices. The Transamerica pyramid is a striking example of the use of a square pyramid in the design of a modern building.

A **convex polyhedron** is one that lies entirely to one side of the plane of each of its faces. A polyhedron that is not convex is called **concave,** or **reentrant.** The polyhedrons shown in Figure 7.59 are a **cube,** a **rectangular parallelepiped,** a six-sided polyhedron with triangular faces (panel C), and a seven-sided polyhedron (panel D). The first three polyhedrons are convex, and the fourth is concave (reentrant).

If two faces of a polyhedron lie in parallel planes and if the edges that are not in these planes are all parallel to each other, then the polyhedron is called a

Exercise 7.5

A

1. Refer to Figure 7.59C, and name the following:
a. The vertices **b.** The edges

2. Refer to Figure 7.60 and repeat problem 1.

3. Refer to Figure 7.59D, and name the bottom face.

4. Refer to Figure 7.59D, and name the left-hand back face.

In each of problems 5–8, make a sketch of the figure.

5. A triangular pyramid

6. A triangular prism surmounted by a triangular pyramid with the top base of the prism as the base of the pyramid

7. A six-sided polyhedron that is convex and is not a parallelepiped

8. An eight-sided polyhedron with triangular faces

9. a. If the edges of a cube are doubled in length, what happens to the volume?
b. What if the lengths are tripled?

B

10. A pyramid has a rectangular base. Suppose that the edges of the base and the height of the pyramid are all doubled in length. What happens to the volume?

11. For a rectangular solid that is 20 in. long, 10 in. wide, and 8 in. high, find the following:
a. The volume V
b. The total surface area S

Subsections Correlated to Exercise Sets

Before students can make decisions about how to solve a problem, they must first master the basic skills within each topic. To help the reader identify the different skills within a topic, each subsection is clearly titled and marked with an A, B, C, etc. These subsections are then correlated with the exercise sets to help students draw connections between subsection presentations and problems.

End-of-Chapter Study Aids

Summary

The Chapter Summary provides brief definitions and examples for key topics within a given chapter. Importantly, it also contains section references to encourage students to reread sections, rather than memorizing a definition out of context.

52. What is the first term of the sequence? What is the fourth term of the sequence and what does it represent?

53. Write the nth term of the sequence.

54. On the basis of the pattern found in problems 52 and 53, estimate what the average daily room cost would be in the year 2000 and in the year 2010.

Chapter 4 SUMMARY

Section	*Item*	*Meaning*	*Example*
4.1	$N = \{1, 2, 3, \ldots\}$	The natural numbers	All counting numbers such as 10, 27, 38, and so on
4.1A	$n(A)$	The cardinal number of A	If $A = \{a, b\}$, then $n(A) = 2$.
4.1A	1st, 2nd, 3rd, . . .	Ordinal numbers	This is the *first* one.
4.1A	123-45-6789	Number used for identification	A Social Security number
4.1B	Prime number	A number with exactly two divisors, 1 and itself	2, 3, 5, 7, 11, . . .
4.1B	Composite number	A number with more than two divisors	4, 33, 50, . . .
4.1C	$12 = 2^2 \cdot 3$	Prime factorization of 12	
4.1E	GCF	Greatest common factor	18 is the GCF of 216 and 234.
4.1E	LCM	Least common multiple	252 is the LCM of 18, 21, and 28.
4.2A	$a \cdot 1 = 1 \cdot a = a$	Identity property for multiplication	$1 \cdot 97 = 97$ and $83 \cdot 1 = 83$
4.2A	$W = \{0, 1, 2, \ldots\}$	The set of whole numbers	
4.2A	$0 + a = a + 0 = a$	Identity property for addition	$0 + 13 = 13$ and $84 + 0 = 84$
		The set of integers	
		The number line	
		Additive inverse property	$3 + (-3) = 0$
		Definition of subtraction	$3 - 7 = 3 + (-7)$
		A set is closed under a given operation if, when the operation is performed on elements of the set, the result is also an element of the set.	The natural numbers are closed under multiplication. The integers are closed under subtraction, but the natural numbers are *not*. ($3 - 5 = -2$ is *not* a natural number.)

Research Questions

Sources of information for these questions can be found in the Bibliography at the end of the book.

1. Write a report on the Egyptian numeration system.
2. Trace the development of the Babylonian numeration system, with special emphasis on the base used.
3. Write a report on the Sumerian numeration system.
4. Write a report on the Mayan numeration system and find out how 0s were used in that system.
5. Write a report on the life and works of Mohammed al-Khowarizmi, with special emphasis on the books he wrote.
6. Write a report on A. Henry Rhind and the Rhind papyrus.
7. Write a report on the uses of binary arithmetic in computers.
8. Write a report on Leonardo Fibonacci and his book the *Liber Abaci*.
9. Write a report about the development and use of ASCII.

Chapter 3 PRACTICE TEST

1. Write the following in Egyptian numerals:
   a. 63 **b.** 735
2. Write the following in decimal notation:
   a. ∩∩||| **b.** 9∩∩|
3. Write the following in Babylonian numerals:
   a. 63 **b.** 735
4. Write the following in decimal notation:
   a. ▼ <<▼▼ **b.** ▼▼ <▼
5. Do the multiplication 23×21 using the following:
   a. The Egyptian method of successive duplication
   b. The Egyptian method of mediation and duplation
6. Write the following in Roman numerals:
   a. 53 **b.** 42 **c.** 22,000
7. Write the following in decimal notation:
   a. LXVII **b.** $\overline{\mathrm{XLVIII}}$
8. Write the following in expanded form:
   a. 2507 **b.** 189
9. Write the following in decimal notation:
   a. $(3 \times 10^3) + (7 \times 10^2) + (2 \times 10^0)$
   b. $(5 \times 10^4) + (9 \times 10^3) + (4 \times 10)$

Research Questions

These questions provide an additional opportunity to explore how various mathematical topics were developed. **Research Questions** combine with **In Other Words** to provide a strong, interesting writing component to the course and are an excellent opportunity for group learning as well.

Practice Test

These tests are designed to help students check their comprehension. Practice Tests can help to further develop problem-solving and test-taking skills. Answers to all Practice Test items appear at the back of the book.

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