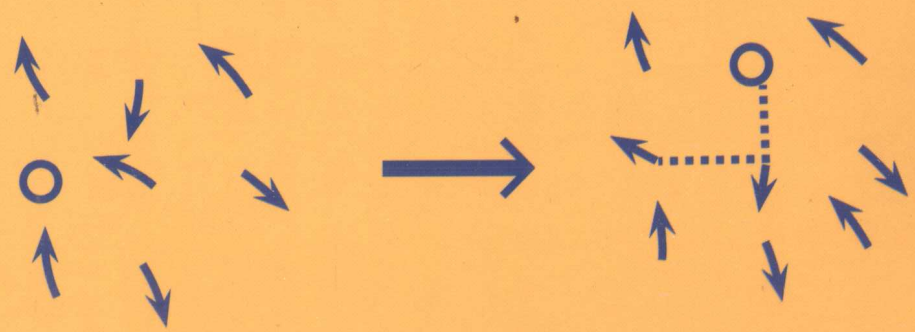
Assa Auerbach

Interacting Electrons and Quantum Magnetism

相互作用电子和量子磁性



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To my parents, Ruth and Israel.

Preface

In the excitement and rapid pace of developments, writing pedagogical texts has low priority for most researchers. However, in transforming my lecture notes[1] into this book, I found a personal benefit: the organization of what I understand in a (hopefully simple) logical sequence. Very little in this text is my original contribution. Most of the knowledge was collected from the research literature. Some was acquired by conversations with colleagues; a kind of physics oral tradition passed between disciples of a similar faith.

For many years, diagramatic perturbation theory has been the major theoretical tool for treating interactions in metals, semiconductors, itinerant magnets, and superconductors. It is in essence a weak coupling expansion about free quasiparticles. Many experimental discoveries during the last decade, including heavy fermions, fractional quantum Hall effect, high-temperature superconductivity, and quantum spin chains, are not readily accessible from the weak coupling point of view. Therefore, recent years have seen vigorous development of alternative, nonperturbative tools for handling *strong* electron–electron interactions.

I concentrate on two basic paradigms of strongly interacting (or constrained) quantum systems: the Hubbard model and the Heisenberg model. These models are vehicles for fundamental concepts, such as effective Hamiltonians, variational ground states, spontaneous symmetry breaking, and quantum disorder. In addition, they are used as test grounds for various nonperturbative approximation schemes that have found applications in diverse areas of theoretical physics.

The level of the text should be appropriate for a graduate student with some background in solid state physics (single electron theory) and familiarity with second quantization. The exercises vary in difficulty and complement the text with specific examples and corollaries. Some of the mathematical background material is relegated to the appendices.

I owe most to the relentless efforts of Maxim Raykin, Moshe Havilio and Ziad Musslimani, whose careful proofreading weeded out inconsistencies and helped clarify numerous points. I am also heavily indebted to Duncan Haldane, who introduced me to quantum magnetism, and to my friend

[1] for a graduate course on Quantum Many Particle Systems given at Boston University and at the Technion during 1990–1993.

and colleague, Dan Arovas, who taught me about parent Hamiltonians, the single mode approximation, and many other things,[2] and for his critical comments. I am grateful for the support of the Alfred P. Sloan Foundation, which enabled me to complete this book.

Assa Auerbach
Haifa, 1997

[2]including the use of *phantom daggers*.

Contents

Part I

Basic Models

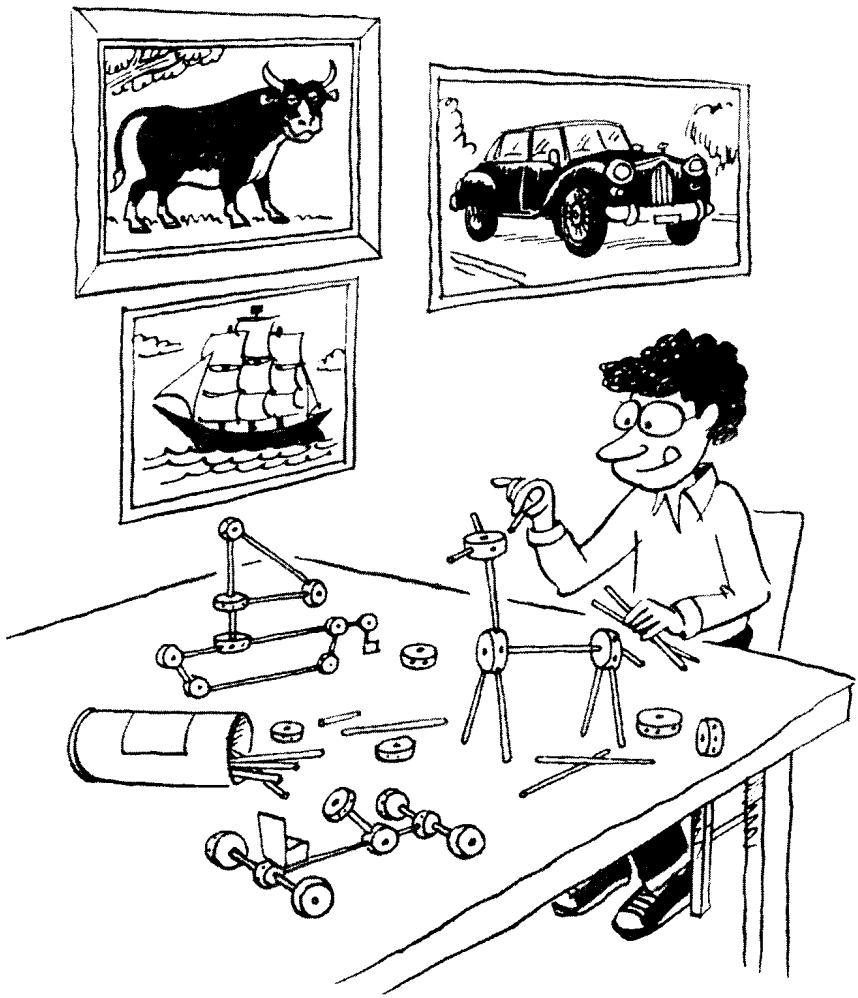

Illustration by Dick Codor.

1

Electron Interactions in Solids

1.1 Single Electron Theory

A single electron moving in a periodic potential is described by the band structure equation

$$H^0\phi_{\mathbf{k}s}(\mathbf{x}) \equiv \left[-\frac{\hbar^2}{2m}\nabla^2 + V^{ion}(\mathbf{x})\right]\phi_{\mathbf{k}s}(\mathbf{x}) = \epsilon_{\mathbf{k}}\phi_{\mathbf{k}s}(\mathbf{x}), \tag{1.1}$$

where $\phi_{\mathbf{k}s}$ and $\epsilon_{\mathbf{k}}$ are the Bloch wave function and band energy, respectively. $\mathbf{k}$ is the electron's lattice momentum, and $s = \uparrow, \downarrow$ is its spin in the S^z direction. Here we supress the band index and ignore spin orbit coupling.

For N_e electrons, the Schrödinger equation can be reduced to a set of band structure equations (1.1) only if the Hamiltonian is a separable sum of single particle Hamiltonians,

$$\mathcal{H}^0 = \sum_{i=1}^{N_e} H^0[\nabla_i, \mathbf{x}_i]. \tag{1.2}$$

The eigenstates of (1.2) are *Fock states* (see Appendix A) constructed by Slater determinants:

$$\Psi^{Fock}_{[\mathbf{k},s]}(\mathbf{x}_1, \ldots, \mathbf{x}_{N_e}) = \det_{ij}\left[\phi_{\mathbf{k}_i,s_i}(\mathbf{x}_j)\right] \tag{1.3}$$

and the eigenenergies are

$$E_{[\mathbf{k}]} = \sum_{i=1}^{N_e} \epsilon_{\mathbf{k}_i}. \tag{1.4}$$

In the ground state the lowest N_e states are occupied, and the uppermost energy is called the *Fermi energy*,

$$\max \epsilon_{\mathbf{k}} = \epsilon_F. \tag{1.5}$$

Equation (1.5) defines the *Fermi surface* in $\mathbf{k}$ space.

The Hamiltonian that includes interactions between electrons is

$$\mathcal{H} = \mathcal{H}^0 + \frac{1}{2}\sum_{i\neq j} v^{el-el}(\mathbf{x}_i, \mathbf{x}_j). \tag{1.6}$$