EIGHTH EDI

## **SWOKOWSKI**

## COLE



# Fundamentals of TRIGONOMETRY

Includes
POCKET REFERENCE CARD

### EIGHTH EDITION

**FUNDAMENTALS** 

OF

**TRIGONOMETRY** 

**EARL W. SWOKOWSKI** 

**JEFFERY A. COLE** 

Anoka-Ramsey Community College



PWS PUBLISHING COMPANY
Boston

Copyright © 1993, 1989, 1986, 1981 by PWS Publishing Company.

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transcribed, in any form or by any means-electronic, mechanical, photocopying, recording, or otherwise—without the prior written permission of the publisher.

PWS Publishing Company is a division of Wadsworth, Inc.

#### LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Swokowski, Earl William.

Fundamentals of trigonometry / Earl W. Swokowski, Jeffery A. Cole.

— 8th ed.

p. cm.

Includes index.

ISBN 0-534-93211-8

1. Trigonometry, Plane. I. Cole, Jeffery A. (Jeffery Alan),

1955- . II. Title.

QA533.S95 1993

516.24'2-dc20

92-45810

CIP

Chapter opening photos appear courtesy of *The Stock Market*; Ch. 1, page 2 © Tom Sanders; Ch. 2, page 54 © Peter Wilson; Ch. 3, page 156 © William E. Whitehurst; Ch. 4, page 218 © John Madere; Ch. 5, page 282 © Ted Horowitz; Ch. 6, page 338 © Ken Straiton.

Printed in the United States of America. 93 94 95 96 97 - 10 9 8 7 6 5 4 3

Sponsoring Editor Timothy L. Anderson Developmental Editor Barbara Lovenvirth Production Editor Elise S. Kaiser Manufacturing Coordinator Marcia A. Locke Production Lifland et al., Bookmakers

Composition Syntax International/Monotype Composition Company, Inc.

Technical Artwork Scientific Illustrators

Interior/Cover Design Elise S. Kaiser

Cover Printer New England Book Components

Text Printer/Binder Arcata Graphics/Hawkins

Cover Image Steven Hunt © 1992

**FUNDAMENTALS** 

OF

TRIGONOMETRY



#### THE PRINDLE, WEBER & SCHMIDT SERIES IN MATHEMATICS

Althoen and Bumcrot, Introduction to Discrete Mathematics

Bean, Sharp, and Sharp, Precalculus

Boye, Kavanaugh, and Williams, Elementary Algebra

Boye, Kavanaugh, and Williams, Intermediate Algebra

Burden and Faires, Numerical Analysis, Fifth Edition

Cass and O'Connor, Fundamentals with Elements of Algebra

Cullen, Linear Algebra and Differential Equations, Second Edition

Dick and Patton, Calculus, Volume I and Volume II

Dick and Patton, Technology in Calculus: A Sourcebook of Activities

Eves, In Mathematical Circles

Eves, Mathematical Circles Squared

Eves, Return to Mathematical Circles

Faires and Burden, Numerical Methods

Fletcher, Hoyle, and Patty, Foundations of Discrete Mathematics

Fletcher and Patty, Foundations of Higher Mathematics, Second Edition

Fraser, Intermediate Algebra: An Early Functions Approach

Gantner and Gantner, Trigonometry

Geltner and Peterson, Geometry for College Students, Second Edition

Gilbert and Gilbert, Elements of Modern Algebra, Third Edition

Gobran, Beginning Algebra, Fifth Edition

Gobran, Intermediate Algebra, Fourth Edition

Gordon, Calculus and the Computer

Hall, Beginning Algebra

Hall, Intermediate Algebra

Hall, Algebra for College Students

Hall, College Algebra with Applications, Third Edition

Hartfiel and Hobbs, Elementary Linear Algebra

Huff and Peterson, College Algebra Activities for the TI-81 Graphics Calculator

Humi and Miller, Boundary-Value Problems and Partial Differential Equations

Kaufmann, Elementary Algebra for College Students, Fourth Edition

Kaufmann, Intermediate Algebra for College Students, Fourth Edition

Kaufmann, Elementary and Intermediate Algebra: A Combined Approach

Kaufmann, Algebra for College Students, Fourth Edition

Kaufmann, Algebra with Trigonometry for College Students, Third Edition

Kaufmann, College Algebra, Second Edition

Kaufmann, Trigonometry

Kaufmann, College Algebra and Trigonometry, Second Edition

Kaufmann, Precalculus, Second Edition

Kennedy and Green, Prealgebra for College Students

Laufer, Discrete Mathematics and Applied Modern Algebra

Lavoie, Discovering Mathematics

Nicholson, Elementary Linear Algebra with Applications, Second Edition

Nicholson, Introduction to Abstract Algebra

Pence, Calculus Activities for Graphic Calculators

Pence, Calculus Activities for the TI-81 Graphic Calculator

Pence, Calculus Activities for the HP-48X and the TI-85 Calculators, Second Edition Plybon, An Introduction to Applied Numerical Analysis

Powers, Elementary Differential Equations

Powers, Elementary Differential Equations with Boundary-Value Problems

Proga, Arithmetic and Algebra, Third Edition

Proga, Basic Mathematics, Third Edition

Rice and Strange, Plane Trigonometry, Sixth Edition

Rogers, Haney, and Laird, Fundamentals of Business Mathematics

Schelin and Bange, Mathematical Analysis for Business and Economics, Second Edition

Sgroi and Sgroi, Mathematics for Elementary School Teachers

Swokowski and Cole, Fundamentals of College Algebra, Eighth Edition

Swokowski and Cole, Fundamentals of Algebra and Trigonometry, Eighth Edition

Swokowski and Cole, Fundamentals of Trigonometry, Eighth Edition

Swokowski and Cole, Algebra and Trigonometry with Analytic Geometry, Eighth Edition

Swokowski, Precalculus: Functions and Graphs, Sixth Edition

Swokowski, Calculus, Fifth Edition

Swokowski, Calculus, Fifth Edition, Late Trigonometry Version

Swokowski, Calculus of a Single Variable

Tan, Applied Finite Mathematics, Third Edition

Tan, Calculus for the Managerial, Life, and Social Sciences, Second Edition

Tan, Applied Calculus, Second Edition

Tan, College Mathematics, Second Edition

Trim, Applied Partial Differential Equations

Venit and Bishop, Elementary Linear Algebra, Alternate Second Edition

Venit and Bishop, Elementary Linear Algebra, Third Edition

Wiggins, Problem Solver for Finite Mathematics and Calculus

Willard, Calculus and Its Applications, Second Edition

Wood and Capell, Arithmetic

Wood and Capell, Intermediate Algebra

Wood, Capell, and Hall, Developmental Mathematics, Fourth Edition

Zill, Calculus, Third Edition

Zill, A First Course in Differential Equations, Fifth Edition

Zill and Cullen, Differential Equations with Boundary-Value Problems, Third Edition

Zill and Cullen, Advanced Engineering Mathematics



#### THE PRINDLE, WEBER & SCHMIDT SERIES IN ADVANCED MATHEMATICS

Brabenec, Introduction to Real Analysis

Ehrlich, Fundamental Concepts of Abstract Algebra

Eves, Foundations and Fundamental Concepts of Mathematics, Third Edition

Keisler, Elementary Calculus: An Infinitesimal Approach, Second Edition

Kirkwood, An Introduction to Real Analysis

Patty, Foundations of Topology

Ruckle, Modern Analysis: Measure Theory and Functional Analysis with Applications

Sieradski, An Introduction to Topology and Homotopy

#### **DEDICATION**

3 · Y · · ·

This Eighth Edition of Fundamentals of Trigonometry is dedicated to the memory of Earl W. Swokowski.

I am grateful for having been given the opportunity to work with a great mentor.

#### A NOTE FROM THE PUBLISHER

Earl W. Swokowski died on June 2, 1992. We at PWS-KENT are grateful to have been associated with one of the premier mathematics authors of the last twenty-five years, and we will miss him.

Earl has left us a rich legacy of extraordinary work and a tradition of excellence. Jeffery A. Cole has worked with Earl over the course of several editions of this precalculus series, and we are confident that he will carry on this tradition of uncompromising quality.

### PREFACE

One of our main goals in preparing the eighth edition of *Fundamentals of Trigonometry* was to enhance the clarity of the discussions. We wanted to enable the student to more easily understand the concepts presented, but we did not want to sacrifice the mathematical soundness that has been paramount to the success of this text. By improving the explanations of the more difficult concepts and including step-by-step comments on the solutions of the examples, we have furthered our goal of helping the student while maintaining mathematical rigor.

Treatment of various topics has been updated to include the use of a graphics calculator. Graphics calculator exercises and examples are found throughout the text, wherever they are appropriate to the material being presented.

Suggestions provided by reviewers and instructors for improvements in the arrangement of topics and the writing have resulted in a great deal of reorganization for this edition. The principal changes made to each chapter are highlighted below.

#### CHANGES FOR THE EIGHTH EDITION

**Chapter I** Graphics calculator examples and exercises first appear in Section 1.2. The topic of horizontally stretching or compressing graphs has been added to Section 1.4, providing a sound foundation for the study of periods of trigonometric functions. A brief survey of exponential and logarithmic functions has been added as Section 1.5 to acquaint the student with these functions, which will be used in some of the new calculator exercises in Chapters 2–4. A thorough treatment of these functions appears in Chapter 5.

**Chapter 2** The formula for the area of a circular sector has been added to Section 2.1. In Section 2.2, the discussion of calculator solutions for equations requiring inverse trigonometric functions has been enhanced. Verification of simple trigonometric identities has been added to Sections 2.2 and 2.3.

Chapters 3 and 4 The inclusion of calculator usage and exercises has strengthened these chapters, and general rewriting has made all sections easier to understand. Topics dealing with complex numbers are now included in Chapter 4.

Chapter 5 Two new theorems highlight the fact that exponential and logarithmic functions are one-to-one. Greater emphasis has been placed on changing exponential forms to equivalent logarithmic forms (and vice versa) in the solving of exponential and logarithmic equations.

Chapter 6 A new relationship for finding the distance from the vertex of a parabola to its focus is emphasized in Section 6.2. The graphing of half-parabolas, half-ellipses, and portions of a hyperbola has been added to this chapter, as have exercises dealing with properties of paraboloids and ellipsoids. The orientation of parametrized curves is thoroughly discussed in Section 6.8.

#### **FEATURES**

Illustrations Brief demonstrations of the use of definitions, laws, and theorems are provided in the form of illustrations.

**Examples** All examples, well structured and graded by difficulty, have been titled for this edition. Many examples contain graphs, charts, or tables to help the student understand procedures and solutions.

**Step-by-Step Explanations** In order to help students follow them more easily, step-by-step explanations have been added to the solutions in examples.

Checks The solutions to some examples are explicitly checked, to remind students to verify that their solutions satisfy the conditions of the problems.

Calculator Examples Wherever appropriate, examples requiring the use of a graphics calculator have been added to the text. These are designated by a green graphics calculator icon and illustrated with a figure reproduced from a graphics calculator screen.

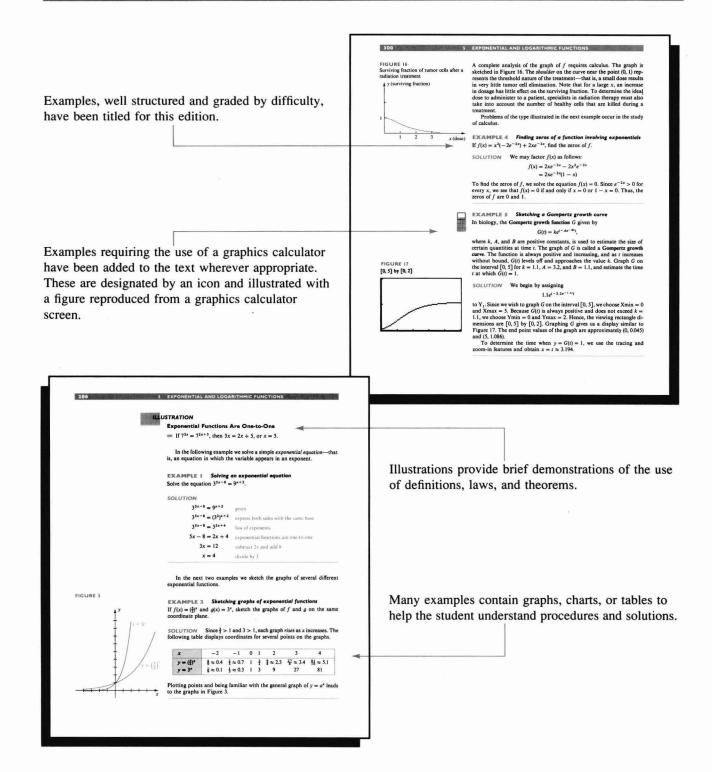
Calculator Exercises Exercises specifically designed to be solved with a graphics calculator or computer software have been included in appropriate sections. Over 100 of these exercises, designated with a , have been added to this edition.

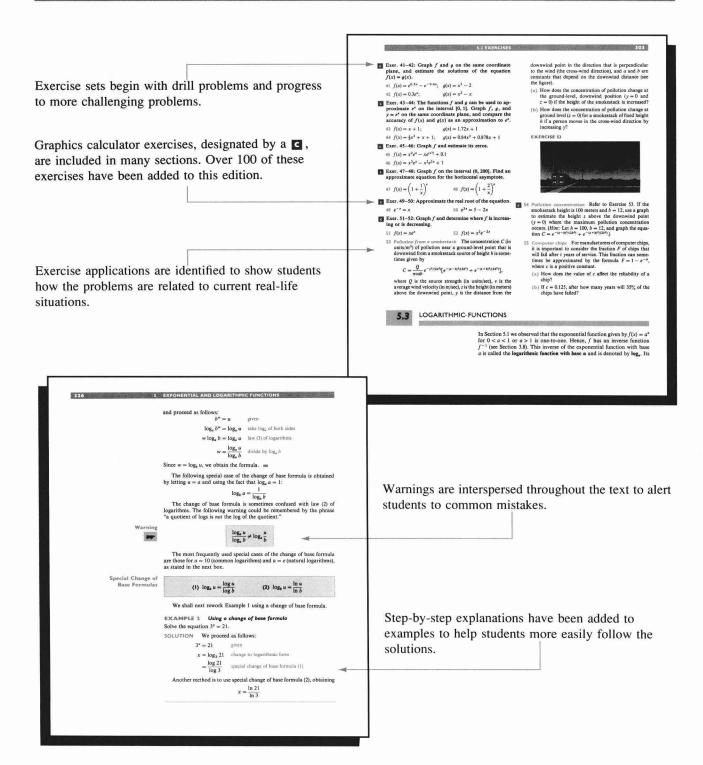
**Exercise Applications** Applications in the exercise sets are identified with titles that show students how the problems are related to practical situations.

**Exercise Sets** After beginning with drill problems, exercise sets progress to more challenging problems. Many exercises containing graphs have been added.

Warnings Interspersed throughout the text are warnings to alert students to common mistakes.

Text Art Each figure has been redrawn and relabeled, and graphs have been computer-generated for accuracy, using the latest technology. Additional colors are used to distinguish between different parts of figures. For example,





PREFACE

the graph of one function may be shown in blue and that of a second function in red. Labels are the same color as the parts of the figure they identify.

**Text Design** The text has been completely redesigned to ensure that discussions are easy to follow and important concepts are highlighted. Color is employed pedagogically to clarify complex graphs and to help students visualize applied problems.

Answer Section The answer section at the end of the text provides answers for most of the odd-numbered exercises, as well as answers for all chapter review exercises. Considerable thought and effort were devoted to making this section a learning device for the student. For instance, verifications are given of trigonometric identities, and numerical answers for many exercises are stated in both exact and approximate form. Graphs, proofs, and hints are included wherever appropriate.

**Flexibility** Syllabi from schools using previous editions attest to the flexibility of this text. Sections and chapters can be arranged in many different ways, depending on objectives and the length of the course.

#### TEACHING TOOLS FOR THE INSTRUCTOR

**Instructor's Solutions Manual** Included in the *Instructor's Solutions Manual* are answers to all exercises and reasonably detailed solutions to most exercises. The manual has been thoroughly reviewed for accuracy.

**EXPTest** This computerized test bank for IBM and compatibles contains over 700 problems, many new to this edition. Questions are multiple choice, true-false, and open-ended. Instructors can interact with the program by adding to existing questions and producing individual tests.

**EXPTest Sampler with Demonstration Disk** The sampler demonstrates the computerized test bank's capabilities and contains sample tests.

**ExamBuilder** This computerized test bank for the Macintosh has features and questions similar to those of EXPTest, described above. A demo is available.

**Test Bank with Chapter Tests** All the questions found in EXPTest (and ExamBuilder), along with their answers, appear in print form in the *Test Bank*. Two sample tests are also included for each chapter.

**Transparencies** Full-color acetates provide enlarged versions of illustrations found in the text. This teaching tool is available to adopters only.

#### LEARNING TOOLS FOR THE STUDENT

Quick Reference Card Packaged with this edition of the text is a new tool for solving exercises—a formula card. This perforated card, found in the back of the book, will aid students in mastering the key formulas, equations, and graphs in the course. By serving as a quick reference and minimizing the need for page turning, the formula card will reduce the time spent on tedious tasks so that the student can focus on the central concepts and principles of the course.

Student's Solutions Manual (Cole) Solutions are given for approximately one-third of the exercises, as well as strategies for solving additional exercises. Many helpful hints and warnings are also included.

**INVESTIGATE** This text-specific tutorial software package for the Macintosh, IBM, and compatibles helps students review material as needed.

College Algebra: In Simplest Terms A series of videotapes, produced by the Annenberg/CPB Collection, is available to qualified adopters. This lively series shows college algebra at work in everyday situations. Symbols, charts, pictures, and state-of-the-art computer graphics illustrate basic algebraic techniques.

Precalculus in Context: Functioning in the Real World (Davis/Moran/Murphy) This lab manual consists of 12 projects that encourage students to explore precalculus concepts. Graphics calculators and computer graphing software are used to solve each experiment and its corresponding exercises.

College Algebra Activities for the TI-81 Graphics Calculator (*Huff/Peterson*) Designed to supplement any college algebra course, this manual offers concise instructions for using the TI-81 graphics calculator, provides examples of how to apply the instructions, and demonstrates techniques for using the calculator as a problem-solving tool. Exercises provide students with additional experience.

TrueBasic for College Algebra (Kemeny/Kurtz) This graphing software package is ideal for classroom demonstrations, individual study, and problem-solving.

Student Edition of Theorist This Macintosh-compatible software combines powerful algebra and graphics capabilities with an intuitive, user-friendly interface.

#### **ACKNOWLEDGMENTS**

Special thanks go to Gary Rockswold, of Mankato State University, for supplying most of the fine assortment of new applied problems and calculator exercises. We also wish to thank Thomas Vanden Eynden, of Thomas More College, for doing a detailed accuracy check of the exercises; Joan Cole, for additional proofreading of the page proofs; and George Morris, of Scientific Illustrators, for creating the mathematically precise art package.

PREFACE

Many changes for this edition are due to the following individuals, who reviewed the manuscript. Their comments about pedagogy and their specific recommendations about the content of precalculus courses helped us to improve the book:

Norma M. Agras

Miami-Dade Community College, South Campus

Daniel D. Anderson *University of Iowa* 

Duane E. Deal Ball State University

Marcy Diles

University of Arkansas at Little Rock

David E. Dobbs

University of Tennessee

John P. Edwards, Jr.

Anne Arundel Community College

Norman Eric Ellis

Essex Community College

Lucinda Gallagher

Florida State University

Albert A. Grasser

Oakland Community College

Sarita Gupta

Northern Illinois University

Keith Kuchar

Northern Illinois University

Nancy Long

Trinity Valley Community College

Carl C. Maneri

Wright State University

Eldon L. Miller

University of Mississippi

Roger B. Nelsen

Lewis and Clark College

Bonny J. Peters

St. Petersburg Junior College

John M. Plachy

Metropolitan State College

Richard Quint

Ventura College

Gerald E. Rubin

Marshall University

Jean E. Rubin

Purdue University

Fred Safier

City College of San Francisco

David Fred Snyder

Southwest Texas State University

Shirley C. Sorenson

University of Maryland, College Park

Marvel D. Townsend

University of Florida

Jan Vandever

South Dakota State University

Arnold L. Villone

San Diego State University

Bruce Williamson

University of Wisconsin-River Falls

Mary Woods

University of Louisville

We are thankful for the excellent cooperation of the staff of PWS-KENT. Several people in the company deserve special mention. Editors David Geggis and Tim Anderson supervised the project and were continual sources of information and advice. Elise Kaiser coordinated the production of the text and is responsible for the exquisite design. Sally Lifland, Denise Throckmorton, and Gladys Moore, all of Lifland et al., Bookmakers, took exceptional care in seeing that no inconsistencies occurred and offered many helpful suggestions.

In addition to all the persons named here, we express our sincere gratitude to the many students and teachers who have helped shape our views on mathematics education. Please feel free to write to us about any aspect of this text—we value your opinion.

EARL W. SWOKOWSKI JEFFERY A. COLE

# CONTENTS

Topics from Algebra		3
1.1	REAL NUMBERS	4
1.2	RECTANGULAR COORDINATE SYSTEMS AND GRAPHS	
1.3	FUNCTIONS	23
1.4	GRAPHS OF FUNCTIONS	33
1.5	EXPONENTIAL AND LOGARITHMIC FUNCTIONS	44
	Chapter I Review Exercises	51
The Trigonomet	ANGLES	55
Silverine and a silverine and		
2.2	THE TRIGONOMETRIC FUNCTIONS	
2.3	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS	65 79
2.3		65
	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS	65 79
2.4	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC FUNCTIONS OF ANGLES	65 79 91
2.4	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC FUNCTIONS OF ANGLES VALUES OF THE TRIGONOMETRIC FUNCTIONS	65 79 91
2.4 2.5 2.6	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC FUNCTIONS OF ANGLES VALUES OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC GRAPHS	65 79 91 100
2.4 2.5 2.6 2.7	GRAPHS OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC FUNCTIONS OF ANGLES VALUES OF THE TRIGONOMETRIC FUNCTIONS TRIGONOMETRIC GRAPHS ADDITIONAL TRIGONOMETRIC GRAPHS	65 79 91 100 107
	1.1 1.2 1.3 1.4 1.5	1.2 RECTANGULAR COORDINATE SYSTEMS AND GRAPHS 1.3 FUNCTIONS 1.4 GRAPHS OF FUNCTIONS 1.5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS Chapter 1 Review Exercises  The Trigonometric Functions

3	Analytic Trigono	ometry	157
	rinary are ringene	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	
	3.1	VERIFYING TRIGONOMETRIC IDENTITIES	158
	3.2	TRIGONOMETRIC EQUATIONS	162
	3.3	THE ADDITION AND SUBTRACTION FORMULAS	174
	3.4	MULTIPLE-ANGLE FORMULAS	184
	3.5	PRODUCT-TO-SUM AND SUM-TO-PRODUCT FORMULAS	194
	3.6	THE INVERSE TRIGONOMETRIC FUNCTIONS	199
		Chapter 3 Review Exercises	214
4	Applications of	Trigonometry	219
	4.1	THE LAW OF SINES	220
	4.2	THE LAW OF COSINES	230
	4.3	COMPLEX NUMBERS	240
	4.4	TRIGONOMETRIC FORM FOR COMPLEX NUMBERS	246
	4.5	DE MOIVRE'S THEOREM AND nTH ROOTS OF COMPLEX NUMBERS	25
	4.6	VECTORS	256
	4.7	THE DOT PRODUCT	269

**Chapter 4 Review Exercises** 

278