

Volume 2

B. I. Bleaney and B. Bleaney

Electricity and Magnetism

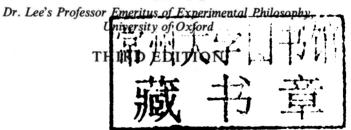
Volume 2

B. I. BLEANEY

Formerly Fellow of St. Hugh's College, Oxford

and

B. BLEANEY



OXFORD

Great Clarendon Street, Oxford, OX2 6DP,

United Kingdom

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

© Oxford University Press 1976

The moral rights of the authors have been asserted

First Edition published in 1957
Second Edition published in 1965
Third Edition published in 1976
Reissued (with corrections) as an Oxford Classic Text in 2013

Impression: 1

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, by licence or under terms agreed with the appropriate reprographics rights organization. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above

You must not circulate this work in any other form and you must impose this same condition on any acquirer

British Library Cataloguing in Publication Data

Data available

ISBN 978-0-19-964542-8 (Vol. 1) 978-0-19-964543-5 (Vol. 2) 978-0-19-965193-1 (Set)

Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY

Links to third party websites are provided by Oxford in good faith and for information only. Oxford disclaims any responsibility for the materials contained in any third party website referenced in this work.

Note on the Oxford Classic Text edition

For many years, Bleaney and Bleaney (B&B) provided an excellent foundation text in the teaching of electricity, magnetism, introductory electronics, and condensed-matter physics. In preparation for the current reissuing, I have reread the entire book from cover to cover and found nothing to change this opinion; B&B remains the most complete book of its kind. And the erudition of its authors remains very obvious; analogies with, and connections to, chemistry, engineering, molecular spectroscopy, atomic physics, geology, telecommunications, and so on are sprinkled throughout the book.

B&B is in many ways a classic, and in preparing the current edition, the intention has been to change as little as possible; anything else would have been akin to sprinkling words like 'dude' and 'dissed' through *Pride and Prejudice*. However, the present reissuing did afford the opportunity to correct a small number of misprints and to offer clarifications in a few places that students have found confusing or difficult to follow. Archaic words such as 'condenser' have (perhaps regrettably) been relegated to the index and replaced in the text by their more modern equivalents.

The earlier editions of B&B were remarkably contemporary books. Many 'cutting-edge' topics, such as the BCS theory of superconductivity, were introduced shortly after their discovery. Though such chapters still (in late 2011) constitute good introductory summaries of these subjects, the only references provided by B&B tended to be lists of early research papers. This is entirely understandable; at the time of writing, there were no textbooks on these topics! In such cases, I have introduced a few references to books that give a more recent overview of the subject. The only part of the book that has dated in a really obvious fashion is the chapter on 'Alternating current measurements'. Here, many of the techniques described have been rendered obsolete by modern digital instruments. This has been pointed out in a footnote, with the rest of the chapter left unaltered for historical interest.

The most persistent comments on the earlier editions of B&B mainly concern the index. In an age of instant informational gratification through the likes of Wikipedia and Google, the rather spare indexing of earlier editions seems to have become somewhat offputting. In response to these requests, the index has been expanded, and the complete index (to Volumes 1 and 2) is given in both volumes.

vi Note on the Oxford Classic Text edition

In making the above changes, I have been aided greatly by my former colleague Geoff Brooker. On most points we agreed. In the more obscure alterations it was good to have Geoff's support and extensive background knowledge. Guy Peskett and David Andrews have also made very useful suggestions, and numerous others have passed on comments from students, misprints etc.. I am very grateful for all of this assistance. Nevertheless, remaining mistakes and inaccuracies are my responsibility, and, in the hope of further reprints, I would be glad to be informed of them (j.singleton1 @physics.ox.ac.uk).

John Singleton November 2011 National High Magnetic Field Laboratory Los Alamos

Preface to the third edition

De manera que acordé, aunque contra mi voluntad, meter segunda vez la pluma en tan estraña lavor é tan agena de mi facultad, hurtando algunos ratos á mi principal estudio, con otras horas destinadas para recreación, puesto que no han de faltar nuevos detractores á la nueva edición.

1499

Fernando de Rojas

So I agreed, albeit unwillingly (since there cannot fail to be fresh critics of a new edition), again to exercise my pen in so strange a labour, and one so foreign to my ability, stealing some moments from my principal study, together with other hours destined for recreation.

For the third edition of this textbook the material has been completely revised and in many parts substantially rewritten. S.I. units are used throughout; references to c.g.s. units have been almost wholly eliminated, but a short conversion table is given in Appendix D. The dominance of solid-state devices in the practical world of electronics is reflected in a major change in the subject order.

Chapters 1–9 set out the macroscopic theory of electricity and magnetism, with only minor references to the atomic background, which is discussed in Chapters 10–17. A simple treatment of lattice vibrations is introduced in Chapter 10 in considering the dielectric properties of ionic solids. The discussion of conduction electrons and metals has been expanded into two chapters, and superconductivity, a topic previously excluded, is the subject of Chapter 13. Minor changes have been made in the three chapters (14–16) on magnetism. The discussion of semiconductor theory precedes new chapters on solid-state devices, but we have endeavoured to present such devices in a manner which does not presuppose a knowledge in depth of the theory. The remaining chapters, on amplifiers and oscillators, vacuum tubes, a.c. measurements, noise, and magnetic resonance, bring together the discussion of electronics and its applications.

The authors are grateful to many colleagues in Oxford and readers elsewhere for helpful comments on previous editions which have been incorporated in the present volume. In particular we are indebted to Dr. G. A. Brooker for numerous and detailed comments and suggestions; to Drs F. V. Price and J. W. Hodby, whose reading of new material on electronics in draft form resulted in substantial improvement of the presentation; to Drs F. N. H. Robinson and R. A. Stradling for several helpful suggestions; and to Messrs

C. A. Carpenter and J. Ward for the considerable trouble taken in producing Fig. 23.3. We are indebted to Professors M. Tinkham and O. V. Lounasmaa for generously sending us material in advance of publication; and to Professor L. F. Bates, F.R.S., Drs R. Dupree, and R. A. Stradling for their kindness in providing the basic diagrams for Figs 15.6, 6.15, and 17.9. We wish to thank Miss C. H. Bleaney for suggesting the quotation which appears above.

Clarendon Laboratory, Oxford February 1975 B. I. B.

B. B.

Acknowledgements

THE authors are indebted to the following for permission to use published diagrams as a basis for figures in the text: the late Sir K. S. Krishnan; G. Benedek; R. Berman; A. H. Cooke; B. R. Cooper; G. Duyckaerts, D. K. Finnemore; M. P. Garfunkel; R. V. Jones; A. F. Kip; C. Kittel; D. N. Langenberg; D. E. Mapother; K. A. G. Mendelssohn; R. W. Morse; D. E. Nagle; N. E. Phillips; H. M. Rosenberg; J. W. Stout; R. A. Stradling; W. Sucksmith; W. P. Wolf; American Institute of Physics; American Physical Society; Institution of Electrical Engineers; Institute of Physics and the Physical Society (London); Royal Society (London); Bell Telephone Laboratories; G. E. C. Hirst Research Centre.

Note added in 1989

The opportunity has been taken of dividing this textbook into two volumes.

Volume 1: Chapters 1 to 9 inclusive, covering the basic theory of electricity and magnetism.

Volume 2: Chapters 10 to 24 inclusive, covering electrical and magnetic properties of matter, including semiconductors and their applications in electronics, alternating current measurements, fluctuations and noise, magnetic resonance.

A number of minor errors have been corrected, and a section (20.8) has been added on Operational Amplifiers. We wish to thank Dr. F. N. H. Robinson for suggesting this, and Dr. J. F. Gregg, I. D. Morris, and J. C. Ward for help in its preparation. We are indebted to Dr. L. V. Morrison of the Royal Greenwich Observatory, Cambridge (Stellar Reference Frame Group), for the up-to-date plot of the variations in the length of the day, measured by the caesium clock, that now appears as Fig. 24.12. It is based on data published by the Bureau de l'Heure, Paris.

B. I. B.

B. B.

CONTENTS OF VOLUME 1

- 1. ELECTROSTATICS I
- 2. ELECTROSTATICS II
- 3. CURRENT AND VOLTAGE
- 4. THE MAGNETIC EFFECTS OF CURRENTS AND MOVING CHARGES, AND MAGNETOSTATICS
- 5. ELECTROMAGNETIC INDUCTION AND VARYING CURRENTS
- 6. MAGNETIC MATERIALS AND MAGNETIC MEASUREMENTS
- 7. ALTERNATING CURRENT THEORY
- 8. ELECTROMAGNETIC WAVES
- 9. FILTERS, TRANSMISSION LINES, AND WAVEGUIDES

APPENDIX A. VECTORS

APPENDIX B. DEPOLARIZING AND DEMAGNETIZING FACTORS

APPENDIX C. NUMERICAL VALUES OF THE FUNDAMENTAL CONSTANTS

APPENDIX D. SOME USEFUL UNIT CONVERSIONS

INDEX

Contents

CONTENTS OF VOLUME 2

10.	DIEL	ECTRICS	
	10.1.	Macroscopic quantities in an atomic medium	294
	10.2.	Macroscopic polarization and the local field	296
	10.3.	The Lorentz correction and the Clausius-Mossotti relation	298
	10.4.	Static permittivity of non-polar gases	300
	10.5.	Static permittivity of polar gases	302
	10.6.	Dispersion in gases	305
	10.7.	Static permittivity of liquids	311
	10.8.	Dispersion in polar liquids	313
	10.9.	Ionic solids	317
	10.10.	Ionized media and plasma oscillations	325
	Referen		327
	Proble	ns	327
11.	FREE	ELECTRONS IN METALS	
	11.1.	Quantum theory of free electrons in metals	330
	11.2.	Extension to finite temperatures: the Fermi-Dirac distribution function	334
	11.3.	Work function and contact potential	337
	11.4.	The photoelectric effect and secondary emission	340
	11.5.	Thermionic emission and field emission	343
	11.6.	Heat capacity of the conduction electrons	348
	11.7.	Electrical and thermal conductivity	351
	11.8.	Thermoelectricity	354
		Electron-electron interaction	360
	Referen	ices	360
	Proble	ns	361
12.	THE BAND THEORY OF METALS		
	12.1.	The wave equation for free electrons	362
	12.2.	The energy-band approximation	364
	12.3.	Electrons in a periodic potential	366
		Particle aspects	370
	12.5.	Conductors and insulators on the band theory	374
	12.6.	Electronic heat capacity	377
	12.7.	Variation of electrical conductivity with temperature	378
	12.8.	Variation of thermal conductivity with temperature	383
			385
	12.10.	The anomalous skin effect	387
	12.11.	The Hall effect	388
	12.12.	Conduction electrons in a strong magnetic field	390
	Referen		395
	Problem	ns	395

	~
v	Outoute
X	Contents

13.	SUPERCONDUCTIVITY			
	13.1.	Introduction	397	
	13.2.	The Meissner effect	399	
	13.3.	Thermodynamics of the superconducting state	402	
	13.4.	The London equations	407	
	13.5.	The Pippard coherence length	409	
	13.6.	The Bardeen-Cooper-Schrieffer (BCS) theory	411	
	13.7.	The energy gap	414	
	13.8.	Type I and Type II superconductors	418	
	13.9.	The Josephson effect	423	
		Conclusion	429	
	Referen		430	
	Proble		431	
	110010			
14.	PARAMAGNETISM			
	14.1.	A general precession theorem	432	
	14.2.	The vector model of the atom	434	
	14.3.	Magnetic moments of free atoms	442	
	14.4.	The measurement of atomic magnetic moments—the Stern-		
	2	Gerlach experiment	446	
	14.5.	Curie's law and the approach to saturation	448	
	14.6.	Susceptibility of paramagnetic solids—the 4f group	450	
	14.7.	Susceptibility of paramagnetic solids—the 3d group	456	
	14.8.	Susceptibility of paramagnetic solids—strongly bonded	150	
	14.0.	compounds	463	
	14.9.	Electronic paramagnetism—a summary	465	
		Nuclear moments and hyperfine structure	467	
		Magnetism of conduction electrons	470	
	Referen		474	
	Proble		474	
	Tiobic	mis	7/7	
15	FERROMAGNETISM			
10.	15.1.	Exchange interaction between paramagnetic ions	477	
	15.2.	The Weiss theory of spontaneous magnetization	481	
	15.3.	Ferromagnetic domains	485	
	15.4.	The gyromagnetic effect	491	
	15.5.	Thermal effects in ferromagnetism	494	
	15.6.	Measurement of the spontaneous magnetization M_0 as a function	121	
	15.0.	of temperature	497	
	15.7	Spin waves	500	
	15.8	Mechanisms of exchange interaction	503	
	Referen		509	
	Proble		510	
	1 10010	ins	310	
16	ANITT	EEDDOMACNETICM AND EEDDIMACNETICA		
10.		FERROMAGNETISM AND FERRIMAGNETISM	611	
	16.1.	Antiferromagnetism	511	
	16.2.	The molecular field and two-sublattice model	512	
	16.3.	Ferrimagnetism	519	
	16.4.	The lanthanide ('rare-earth') metals	524	
	Referen		527	
	Proble	ms	527	

		Contents	хi	
17.	7. SEMICONDUCTORS			
	17.1. Intrinsic and extrinsic conductivity		528	
	17.2. Elementary and compound semiconductors		530	
	17.3. Electron distribution and the Fermi level		534	
	17.4. Optical properties		539	
	17.5. Transport properties		544	
	17.6. Metal-semiconductor junctions		552	
	17.7. Theory of the p-n junction		557	
	References		561	
	Problems		562	
18	SOLID-STATE DIODES			
10.	18.1. Introduction		564	
	18.2. The p-n junction		565	
	18.3. The Zener diode		569	
	18.4. Rectification		571	
	18.5. Amplitude modulation and detection		573	
	18.6. Frequency changing		576	
	18.7. Frequency modulation		579	
	18.8. The point-contact diode		583	
	References		583	
	Problem		583	
19.	SOLID-STATE TRIODES			
	19.1. The junction transistor		584	
	19.2. Characteristics of the junction transistor		588	
	19.3. Equivalent circuits for the junction transistor		592	
	19.4. The field-effect transistor, or FET		596	
	19.5. The MOS field-effect transistor, or MOSFET		602	
	19.6. Summary of transistor properties		605	
	References		605	
20.	AMPLIFIERS AND OSCILLATORS			
	20.1. Negative feedback		606	
	20.2. Feedback and stabilization of a transistor amplifier		610	
	20.3. The small-signal low-frequency amplifier		613	
	20.4. Amplification at radio-frequencies		616	
	20.5. Power amplifiers		618	
	20.6. Oscillators		621	
	20.7. The multivibrator		624	
	20.8. The operational amplifier		627	
	References		630	
21.	THERMIONIC VACUUM TUBES			
	21.1. The thermionic diode		631	
	21.2. The thermionic triode		632	
	21.3. Effect of transit time on input conductance		637	
	21.4. The klystron		639	
	21.5. The magnetron		645	
	21.6. Travelling-wave tubes		650	
	References		650	
	Problems		650	

xii Contents

22.	. ALTERNATING CURRENT MEASUREMENTS		
	22.1.	Measurement of voltage, current, and power	652
	22.2.		658
	22.3.		664
	22.4.	Measurements on lines	665
	22.5.	Measurement of frequency and wavelength	669
	22.6.	Measurement of frequency and wavelength Measurement of electric permittivity	674
	22.7.	Measurement of the velocity of radio waves	676
	Refere		678
	Proble	ms	679
23.	FLU	CTUATIONS AND NOISE	
	23.1.	Brownian motion and fluctuations	681
	23.2.	Fluctuations in galvanometers	683
	23.3.	The relation between resistance noise and thermal radiation Shot noise	687
			690
	23.5.	The state of the s	
		figure)	694
	23.6.	Noise in solid-state triodes	697
	Refere		699
	Proble	ms	700
24.	MAG	NETIC RESONANCE	
	24.1.	The magnetic-resonance phenomenon	701
	24.2.		705
	24.3.		709
	24.4.		712
	24.5.		716
	24.6.	Electron magnetic resonance in atomic beams	719
	24.7.	Electron magnetic resonance in solids	722
	24.8.		729
	24.9.		733
		Azbel-Kaner resonance in metals	738
	Refere		741
	Proble	ms	741
AP	PEND	IX: NUMERICAL VALUES OF THE	
		FUNDAMENTAL CONSTANTS	744
ΑP	PEND	DIX: SOME USEFUL UNIT CONVERSIONS	745
IN	DEX		747

Volume 2

10. Dielectrics

THIS chapter is the first of several which approach electric and magnetic phenomena from an atomic viewpoint. For dielectrics the discussion is confined to linear phenomena, where the polarization is linearly proportional to the applied field. High-field effects, and effects involving spontaneous polarization (ferro- and antiferroelectricity) are excluded.

10.1. Macroscopic quantities in an atomic medium

A dielectric medium consists of an assembly of negatively charged particles (electrons) and positively charged particles (nuclei). We assume it to be electrically neutral, the total negative charge being equal to the total positive charge, and that all the charges are 'bound'. By this is meant that in zero electric field each charge occupies its equilibrium position, from which it may be slightly displaced when an electric field is applied, but that it is unable to move continuously from place to place: there is no 'free charge' and no conduction current.

In electromagnetic theory, a dielectric is a continuous medium which becomes electrically polarized under the action of an electric field. The concepts involved are the macroscopic quantities, charge density, and electric polarization, defined as continuous functions. The charge density ρ_e is the ratio of an infinitesimal charge to an infinitesimal volume, taken to the limit as the infinitesimals are reduced to zero. The electric polarization is similarly defined, with net dipole moment instead of net charge. These concepts are clearly not immediately applicable to a set of discontinuous charges, and a method of relating the electrical properties of such an assembly to the corresponding properties of a continuous medium is required.

In the electron theory of Lorentz macroscopic quantities such as charge density and polarization are regarded as averages over loosely defined volumes which are small on a macroscopic scale but large enough to contain very many elementary charges. In considering the propagation of an electromagnetic wave through the medium, such averages must be taken over regions whose linear dimensions are small compared with the wavelength. For optical waves $(\lambda \sim 5 \times 10^{-7} \text{ m})$ such dimensions can be over 100 times the atomic spacing in a solid $(\sim 2 \times 10^{-10} \text{ m})$, so that the number of charges involved in the average may be 10^6-10^9 . Nevertheless there is a conceptual difficulty. The total charge in any such volume must

10.1] Dielectrics 295

be an integral multiple of the fundamental unit of charge, but if the boundaries are moved to include or exclude just one electronic charge, an apparent change in the charge density is obtained which is clearly not meaningful. (In fact one electronic charge of 1.6×10^{-19} C in a cube of dimensions 10^{-7} m gives a charge density of 160 C m⁻³, which is enormous compared with any charge density realizable in practice.) This difficulty is overcome by taking averages over a large number of such volumes of random size, so that fluctuations in volumes of size small compared with λ^3 are smoothed out.

The averages which we require may be defined more precisely by the use of a three-dimensional Fourier analysis to represent the distribution of charge in space (Robinson 1973). The method may be illustrated by considering a simple one-dimensional example: an infinite line of charges q placed at regular intervals a_0 , for example at the points 0, $\pm a_0$, and all integral multiples of $\pm a_0$. The charge density, expressed as a Fourier series, is easily shown to be

$$\rho_{\rm e} = \frac{q}{a_0} + \frac{2q}{a_0} \sum_{n=1}^{\infty} \cos\left(\frac{2\pi nx}{a_0}\right)$$
 (10.1)

Clearly the average charge density is q/a_0 , and the spatial repetition rates of the oscillating terms are a_0^{-1} and multiples thereof. In any length l reaching from $x_0 - \frac{1}{2}l$ to $x_0 + \frac{1}{2}l$ the total charge is found by integration over this interval, and the corresponding charge density is

$$\frac{1}{l} \int \rho_{e} dx = \frac{ql}{a_{0}l} + \frac{2q}{a_{0}l} \sum_{n=1}^{\infty} \int_{x_{0}-l/2}^{x_{0}+l/2} \cos\left(\frac{2\pi nx}{a_{0}}\right) dx$$

$$= \frac{q}{a_{0}} + \sum_{n=1}^{\infty} \frac{2q}{\pi nl} \cos\left(\frac{2\pi nx_{0}}{a_{0}}\right) \sin\left(\frac{\pi nl}{a_{0}}\right) \tag{10.2}$$

For arbitrary values of x_0 and l the oscillatory terms are as often positive as negative, and the macroscopic charge density is q/a_0 . The Lorentz approach is in fact equivalent to taking an average over ranges of values of x_0 and of l which are large compared with a_0 .

This one-dimensional example illustrates the result that with a regular array of charges, as in the lattice of a crystal, Fourier analysis produces a constant component (which is the macroscopic average we require), together with components which repeat in space at the same rate as the lattice or higher. Such oscillating components cause strong scattering, the lattice acting as a three-dimensional diffraction grating for wavelengths simply related to the lattice spacing, that is, X-rays of wavelengths $\sim 2 \times 10^{-10}$ m. In contrast, much longer wavelengths are propagated freely through the lattice and can be treated using the macroscopic theory of

296 Dielectrics [10.2

Chapter 8. Imperfections on an atomic scale, such as displacements of lattice ions under the action of thermal vibrations or the irregularities of molecular position in a liquid, do not much affect this result. This is true even in a gas, since at S.T.P. the average intermolecular distance $(\sim 5 \times 10^{-9} \, \text{m})$ is still small compared with optical wavelengths. The randomness in the molecular spacing will cause some scattering, but this is small for optical waves and quite negligible for radio waves.

10.2. Macroscopic polarization and the local field

Experimentally no net charge on an atom or molecule has ever been detected, and Dylla and King (1973) have shown that the fractional difference in the numerical values of the charge on electron and proton is less than 1.3×10^{-21} . However, application of an electric field displaces the positive charges in the direction of the field and the negative charges in the opposite direction, so that every substance acquires an 'induced' electric dipole moment. It is usual to distinguish between two effects: (1) an 'electronic contribution', arising from the displacement of electrons relative to positively charged nuclei; (2) an 'ionic contribution', arising from the displacement of heavy positive ions relative to heavy negative ions, which occurs in molecules and in ionic solids containing ions of different charge.

In addition to induced moments there may be permanent electric dipole moments. In an atom or monatomic ion the distribution of charge is symmetric about the centre and, as discussed in § 2.3, no permanent electric dipole moment can be present. Some molecules also fall into this symmetry classification; simple examples are the homonuclear diatomic molecules such as H₂, N₂, O₂ and symmetric linear polyatomic molecules such as CO₂, which has the structure O—C—O. Such molecules are known as 'non-polar molecules'. Molecules which do not satisfy this symmetry requirement normally possess permanent electric dipole moments, and are called 'polar molecules'.

For a finite body, the macroscopic electric dipole moment is the vector sum of the individual dipole moments. We may distinguish between three types of contributions.

- Each atom, ion, or non-polar molecule has zero dipole moment in zero field, but acquires a dipole moment when a field is applied; such moments are known as 'induced dipoles'.
- (2) When polar molecules are present, their permanent dipole moments are completely randomly oriented in zero field, so that the vector sum is zero, at any rate on a time average; in the presence of a field, orientations in which the dipole moment is parallel to the

field have a lower energy and so are statistically more favourable, giving a resultant dipole moment in the direction of the field.

(3) In an ionic lattice, the positive ions are displaced in the direction of an applied field while the negative ions are displaced in the opposite direction, giving a resultant dipole moment to the whole body.

The techniques of Fourier analysis that were illustrated above for the charge density can also be applied to the components of the dipole moments, and under similar conditions yield a macroscopic dipole moment per unit volume. This is the polarization **P**, which appears in the macroscopic equations (cf. § 1.5)

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_r \epsilon_0 \mathbf{E} \tag{10.3}$$

and

$$\mathbf{P} = (\epsilon_{r} - 1)\epsilon_{0}\mathbf{E}.\tag{10.4}$$

In performing the Fourier analysis, we use the dipole components given by eqns (2.25). For neutral atoms and neutral molecules the result is independent of the origin of \mathbf{r} in the equation $\mathbf{p} = \int \rho_{\mathbf{e}} \mathbf{r} \, d\tau$, as pointed out in § 2.3. For charged ions this is not true; for the induced dipoles arising from the displacement of the electrons relative to the nucleus we must measure \mathbf{r} from the nucleus, while if the ion is bodily displaced in the lattice \mathbf{r} must be measured from the point occupied by the ion when no electric field is applied.

To evaluate the dipole moment, we must calculate the effect of an electric field on an electron bound in a molecule. This is a quantum-mechanical problem, whose solution is only approximate except in a simple case such as the hydrogen atom, and which we shall not pursue further. Instead we regard the medium as composed of atoms, ions, or molecules which acquire an induced dipole moment \mathbf{p}_i under the action of the electric field, and which sometimes possess also a permanent dipole moment. The size of the induced dipole \mathbf{p}_i is determined by the magnitude of the field \mathbf{E}_{local} acting on the particle; we can write

$$\mathbf{p}_{i} = \alpha \mathbf{E}_{local}, \tag{10.5}$$

where α is a quantity known as the 'polarizability' of the atom, ion, or molecule. At ordinary field strengths \mathbf{p}_i is linearly proportional to \mathbf{E}_{local} , and α is therefore a constant, independent of field strength. The polarization arising from the induced dipoles, if all are identical and the number per unit volume is n_0 , is

$$\mathbf{P} = n_0 \alpha \mathbf{E}_{local}, \tag{10.6}$$

where we have assumed that each induced dipole \mathbf{p}_i is parallel to \mathbf{E}_{local} as would be the case in the absence of any anisotropy.