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数学奥林匹克
英文版系列

Mathematical Olympiad in China

数学奥林匹克在中国

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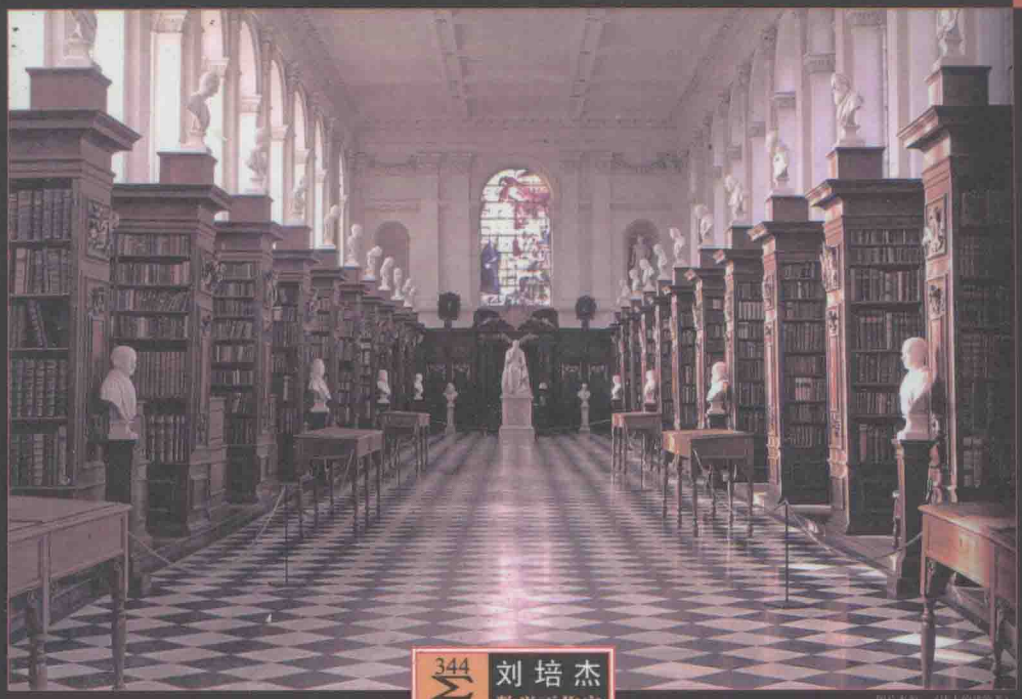

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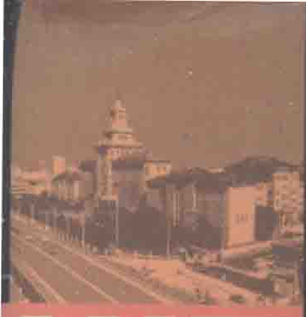

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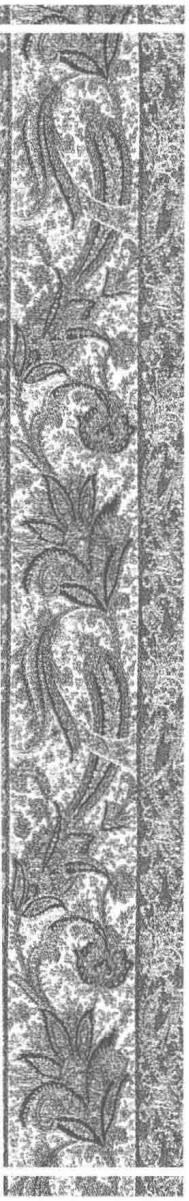

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内容简介

本书介绍了从1986年至2013年的国际数学奥林匹克竞赛在中国的发展情况,并着重介绍了从1986年以来历届国际数学奥林匹克竞赛的试题及解答技巧,最后介绍了历届中国数学奥林匹克竞赛试题.

本书适合准备参加高中数学奥林匹克竞赛的学生及辅导教师和广大数学爱好者参考阅读.

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Mathematical Olympiad in China

Chapter 1

1.1 International Mathematical Olympiad(IMO) and China Mathematical Contest —Written before the 31st IMO[①]

In 1986, China applied for holding the International Mathematical Olympiad (IMO). The year after, the IMO Permanent Committee resolved that the 31st IMO would be held by China in 1990. To hold the 31st IMO is a major event in China educational and mathematical circles.

IMO is the greatest disciplinary contest of middle school students in scale and influence throughout the world. If Olympic Games is said to be a great match of human physical power, the Mathematical Olympiad would be a competition of world's youth in intelligence. This traditional contest activity has pushed forward the exchange in mathematical education of various countries, promoted mathematical education level, enhanced mutual understanding of young students in countries throughout the world. Simultaneously, it has stimulated the interest of masses of middle school students in mathematics, and has been helpful to finding and training young talented personnel. Such an activity provides an appropriate stage for various nations to express their own intelligence and wisdom. Therefore, it attracts great attention of more and more nations. Many countries strive to hold IMO actively, at present the host countries before 2000 have been fully arranged. China is the first Asian country to hold this activity. As a great Oriental country, it is an international duty we should carry out, and also a contribution made by us to the mankind.

---

① Wang Shouren, Institute of Mathematics, Academia Sinica of China.

1.1.1 A Brief Introduction to IMO

Competition in solving hard problems have existed since the ancient time. There were records of competition in solving hard geometric problems in ancient Greece. In the early 16th century, there was a vivid description of a competition in solving cubic roots. In the 18th century, France ever did conduct an independent mathematical contest. In the 19th century, scholars put forward a few significant mathematical problems, through an competition for awards some important discoveries were often obtained. The scientific giant F. Gauss was just a winner of the competition.

The mathematical contest in the modern sense started in 1894. At that time, in order to commemorate the appointment of the President of Hungarian Mathematical Society, Professor Eötvös as the Minister of Education, Hungarians conducted a mathematical contest named after Eötvös. It was Russians who mentioned the earliest mathematical contest in the same breath with sports competition In 1934, they named the middle school students' mathematical contest in Moscow and Leningrad with Mathematical Olympiad. In the early 1950's a pyramid-like system of mathematical contest was formed in some East-European countries. That is, hierarchical contests: to the effect that the contest took place in schools first, then in an area, at last throughout the country. The winners of each level will participate in the contest of the next higher level. An even much higher level is, of course, the international contest.

In 1956, Professor Roman of Rumania sponsored the International Mathematical Olympiad. In 1959, the first IMO was held in Belasov, Rumania. The Soviet Union and six East-European countries participated. Since then, it takes place annually. The participants in first several IMO's were still those countries. In 1963 and 1964, Yugoslavia and Mongolia participated in succession. In 1965, Finland participated. In 1967, France, England, Italy and Sweden joined. In 1974, along with the participation of the United States and other countries the IMO was extended from Europe to other Continents. In 1977, the number of participant countries was 21. In 1983, it increased to 32. In 1987 it exceeded 40. In 1988, the participant countries and areas reached 54. The latest one is the 30th IMO held in the Federal Republic of Germany in 1989 with 50 participant teams. The number of contestant

students has increased from early 40 ~ 50 to now neraly 300. IMO is taking place annually since its start, except in 1980. After the 1979 IMO held by England, Mongolia was determined to conduct the next IMO. But, owing to shortage of funds Mongolia was powerless to do it, while IMO lacked an international coordinating organ to seek for substitute host country. Thus, the 1980 IMO was cancelled. In 1980, the International Mathematical Edcation Committee(IMEC) established under it the IMO Subcommittee which is responsible for arranging annual organizers. Therefore, since 1981 the IMO tradition has not been interrupted, we believe it will not be interrupted in future.

Now, the operation mode of IMO has been systematized. It is expressed mainly in the following:

1. The participant countries (or areas) serve as the host country in turn. The entire activities are led by the Organizing Committee set up by the host country. The vocational activity is held by the Jury(a committee in charge of the examination) consisting of all Team Leaders. The funds for activities are borne by the host country;

2. Members of each team must not exceed 8, in which the contestants do not exceed 6, Leader and Deputy Leader are 1 respectively;

3. Examination problems are provided by all participant teams, from which the Jury selects out 6 problems. 7 points for each problems, the full score is 42 points. Examination is carried out in 2 days, and 3 problems for each day, 4.5 hours for one-day examination;

4. Examination papers is judged with score by the Coordinating Committee set up by the host country, with the help of all Team Leaders. The Jury decides the awarded name list;

5. The number of awarded contestants is about a half of the total of participants. The proportion of contestants whom the first, second, third grade awards are conferred on is about 1:2:3. If examination papers present a very elegant or mathematically significant solution to a certain problems, they can be offered the special award.

In principle, IMO is not a contest between teams, all contestants are solving problems independently. So, the prize is only awarded to the individual. But since the number of contestants in each team is the same, it will inevitably give rest to comparing all teams by the total score of each team. The results of the comparison are often published on the material of IMO and various mathematical publications. Here let us also make analysis on total scores in the past IMO'S.

Since the Soviet Union, Rumania, Hungary, German Democratic Republic have tradition of mathematical contest, they are in the rank of IMO Powers. The Soviet Union has acquired the first place for 12 times, Hungary acquired for 6 times, and Rumania acquired for 3 times. In the late-participated countries, the United States, Federal Republic of Germany, China have outstanding records. These three countries joined the contest in 1974, 1977 and 1985 respectively. USA and FRG have won the first place two times respectively. China won the first place one time. An astonishing coincidence is the IMO Powers would be almost the same with the Olympic Games Powers. No wonder mathematicians have named mathematical contest as the Mathematical Olympiad. We might as well list the first 8 places of the total scores as Table 1.1 in the last four years as an evidence:

Table 1.1

place / year	1	2	3	4	5	6	7	8
1986	USSR USA		FRG	China	GDR	Rumania	Bulgaria	Czecho-slovakia
1987	Rumania	FRG	USSR	GDR	USA	Hungary	Bulgaria	China
1988	USSR	Rumania China		FRG	Vietnam	USA	GDR	Bulgaria
1989	China	Rumania	USSR	GDR	USA	Czecho-slovakia	Bulgaria	FRG

1.1.2 A Historic Review of China Mathematical Contest

In 1956, initiated by China older-generation mathematicians Hua Luogeng, Su Buqing, Jiang Zehan, Ke Zhao, Wu Daren and Li Guoping, the middle school students Mathematical Contests were held separately in Beijing, Shanghai, Tianjin, Wuhan, etc.; Hereafter, mathematical contest activities were developed in succession throughout the country. At that time, there were active mathematical extracurricular activity groups in many middle schools. On the eve of contest, famous mathematicians Hua Luogeng, Duan Xuefu, Wu Wenjun, Min Sihe, Zhao Cigeng, Wang Shouren, Yue Minyi, Yu Jiarong, Gong Sheng, etc., either gave a lecture on a special topic to middle school students, or directly took part in problems assignning work. At that time, there were no one-after-one selection examinations,

and no precontest concentrated trains, as well. Mathematical contest activity and normal mathematics teaching supplemented each other, it was welcomed by teachers and students of middle schools. However, it is a pity such beneficial mathematical extracurricular activities were carried out only for two years, then compelled to stop.

In 1978, the Ministry of Education, China Association of Science and Technology(CAST), and Central Committee of the Communist Youth League jointly conducted the middle school students Mathematatical Olympiad of eight provinces and municipalities. In the next year, the first nationwide Mathematical Olympiad was held, and all 29 provinces, municipalities and autonomous regions throughout the country participated in this contest activity. Professor Hua Luogeng and Wang Shouren took charge of vocational activities of the contest. After the 1979 Mathematical Contest activity closed, people found mathematical contest at that time seemed to get something essentially different from those in the 1950's. Aimed at the good place in competition, lots of schools and local departments concerned increased charge successively, selected contestants in each level, concentrated manpower and did a rush job of training, all these added to students load, even upset the normal teaching order. Therefore, it was resolved that the nationwide Mathematical Olympiad would not be conducted by the State.

In 1980, the Popularizing Work Committee of the Chinese Mathematical Society (CMS) decided that league mathematical contest would be held by Mathematical Societies of various provinces(municipalities, autonomous regions) in turn, in a pattern of "being run by the local people and subsidized by the state", and clearly stipulated the principles and aim of the contest, required that mathematical contest should supplement normal teaching with each other. In 1981, the first league mathematical contest was run by the CMS. The later ones were run by Shanghai, Anhui, Guizhou, Tianjin(Hubei for middle school), Sichuan(Shanxi for junior), Henan (Heilongjiang for junior), Jiangxi(Yunnan for junior), Shandong(hunan for junior), in succession. At present, senior middle school students participating in league contests have reached 50 000, and junior middle school students participating are nearly 100 000. Professors Sun Shuben, Mei Xiangming and Qiu Zonghu, Directors of three sessions of Popularizing Work Committee, CMS, took charge of the league contest work.

In order to prepare the team participating in the IMO, since 1986 CMS has cooperated with related universities and colleges in holding the mathematical winter camp annually, carrying out pre-IMO warming-up match to select the contestants

participating in the IMO. Since this year, the Chern Sing-Shen(S. S. Chern) Cup is set up to be awarded to the provincial(municipal, autonomous regional) team which gets the first place in total score.

1.1.3 Activities of China in the IMO and the 31st IMO

In 1985, CMS appointed Wang Shouren and Qiu Zonghu to lead two students participating in the 26th IMO which held in Finland. It was the first time for China to join this activity. The principal aim was to investigate and study, and to establish connection with the IMO. Since then, through domestic mathematical contest, CMS selected contestants, concentrated them in training, and formally organized the representative team to participate in the 27th, 28th, 29th and 30th IMO's held in Poland, Cuba, Australia and Federal Germany respectively. The success on which the world attention is focused has been attained by the Chinese Team as Table 1.2:

Table 1.2

year	Medals			Total Score
	Gold	Silver	Bronze	
1986	3	1	1	the fourth place
1987	2	2	2	the eighth place
1988	2	4		the second place
1989	4	2		the first place

From the records listed, above it can be seen that in the four years since formally participating in the IMO China has not only ranked among the world's eight Powers in this activity, but also acquired a champion for Asian countries for the first time. It is a result of hardworking and diligent cultivation by educators and mathematical workers of China. It has also shown China has young people of talent coming forth in large numbers, and great potential in intelligence. In particular, it should be mentioned that coaches of Chinese Representative Team Qiu Zonghu, Shan Zun, Du Xilu, Shu Wuchang, Ma Xiwen and Chang Gengzhe have done a remarkable work.

The 31st IMO will take place in Beijing from July 8th to July 19th, 1990. The CPC Central Committee and the State Council have paid great attention to this activity, and instructed the State Education Commission (SEC), CAST, Beijing

Municipal Government, National Natural Science Foundation (NNSF), and CMS jointly to hold it. The leading organizations and working groups have been formed. Preparatory jobs are actively going on. The Chairman of the Organizing Committee is Mr. Li Tieying (Minister of the SEC), Vice-Chairmen are Wang Yuan (Mathematician, President of the CMS), Liu Bin (Deputy Minister of the SEC), Lu Yucheng (Deputy Mayor of Beijing), Cao Lingzhong (Secretary of the Secretariat of the CAST) and Wang Ren (Vice-Chairman of the NNSF). Wang Shouren holds the post of the Secretary-General, Qiu Zonghu is the Standing Deputy Secretary-General, Wang Minyang and Xie Kening are the Deputy Secretary-General. Luo Shengxiong is the Chief of the Office.

Besides working organizations, there is an Advisory Committee consisting of famous mathematicians. The Chairman of the Advisory Committee is Professor Chern Shing-Shen, and Members include Su Buqing, Jiang Zehan, Ke Zhao, Wu Daren, Wu Wenjun, Duan Xuefu, Gu Chaohao, Chen Jingren, Hsiang Wu-Yi, Yang Le, etc.

The 31st IMO intends to invite 60 countries and areas to organize teams to participate in this activity. The numbers of participant countries and teams members will exceed the records of allpast IMO's. The IMO Permanent Committee and the United Nations Educational, Scientific, and Cultural Organization (UNESCO) will send important officials to superintend.

We believe, through holding the 31st IMO it will raise China's position in the international educational and scientific circles, and enhance our nation's self-confidence.

Olympiad's Mathematics

Chapter 2

2.1 The Application of Projective Geometry Methods to Problem Proving in Geometry[①]

There are usually two different ways of solving a mathematics problem. One is making full use of the knowledge and methods having been learned to solve the problem. The other is deducing certain more general conclusions by introducing some new (of course, not entirely irrelevant to the stated problem) concepts and methods, disregarding the problem to be solved for the moment while the problem to be solved originally is included in these conclusions as a special case, or promptly obtained as a simple inference of these conclusions.

Several geometry problems are now give as follows:

Problem 1 In $\triangle ABC$, H is any point on AD, the height of the triangle. Lines BH, CH intersect AC and AB at E and F respectively. Connect ED, FD.

Prove that $\angle FDH = \angle HDE$. (Fig. 2.1)

Problem 2 In the square $ABCD$, let $EF // BC$, and EF interests CD at F. Choose any point G on EF, so that line AG intersects BF at H, and DH intersects BC at I. (abbreviated as $AG \times BF = H$, $DH \times BC = I$, in this article, this notation will be adopted later to denote the intersection of two straight lines) Prove that $GI // AB$. (Fig. 2.2)

① Shan Zun, Nanjing Normal University.