OPTIMIZATION AND PROBABILITY IN SYSTEMS ENGINEERING

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PREFACE

This book is intended for engineers with interests in certain operations research topics. It is a usable reference for those operations research analysts, economists, management scientists, and applied mathematicians whose work brings them into contact with its subject matter. More specifically, this book is intended for those systems engineers and other technical workers who either have an interest in, or are working in, areas of such designations as systems analysis, operations analysis, system design, reliability, maintainability, system effectiveness, and system cost-effectiveness. Emphasis in the text is upon optimization techniques and probabilistic concepts usable or encountered in the systems engineering process, that is, which are applicable during the design, development, and evaluation of systems.

This text is an outgrowth of notes used in a one-semester systems engineering course that was offered as part of the U.C.L.A. engineering extension program and is now given at U.C.I. as a one-year course. It is intended to provide adequate coverage for a one-year course in the mathematical aspects of operations research as applied to engineering problems, in addition to serving as a reference book for the practicing systems engineer. It is expected that any course based on the material presented here should have as basic prerequisites the usual elementary courses in integral and differential calculus, linear algebra and matrices, differential equations, and probability theory.

As a result, any such course would normally be taken at the senior or first-year graduate level.

Chapter 1 provides an introduction to the elements of the systems engineering process, the mathematical subprocesses, and the mathematical approach to modeling these subprocesses. The use of both optimization techniques and probability theory in the selection of the optimal system configuration from among various system alternatives is discussed. Types of system measures and cost and effectiveness tradeoff criteria are presented.

Chapters 2 through 4 are oriented toward optimization techniques and their applications to systems engineering problems. Chapter 2 reviews the essentials of the differential calculus useful in finding maxima and minima of functions. The Lagrange multiplier method is presented and optimization subject to nonlinear constraints discussed. A brief treatment is also given of maximizing and minimizing functions of discrete variables using elementary difference equations. In Chapter 3 the linear programming technique is discussed, including both the simplex and the two-phase simplex methods, and illustrated with numerous examples. Chapter 4 is a collection of optimization techniques basically recursive in nature, including the use of generating functions to solve difference equations and the method of dynamic programming.

Chapters 5 through 10 are concerned with the development of probabilistic models to analyze the behavior of systems. Chapter 5 deals specifically with the elementary aspects of reliability theory, multi-component structures, and the properties of the system failure rate function. Chapter 6 is an extension of certain aspects of reliability theory in which failed components are allowed to be repaired. The act of repair can be regarded as a renewal process and so elementary renewal theory is discussed. In Chapter 7 are introduced the measures of system dependability and availability for systems in which repair of failures is allowed. In Chapter 8 are presented elementary results on spares provisioning and spares allocation for selected criteria. The topics of this chapter find application in the area of integrated logistics support, a field of special interest to military and industrial logisticians. Replacement theory is also discussed as a sparing process. Chapter 9 provides a summary of the elementary results on Markov chains and illustrates the close connection between repair and spares provisioning processes and Markov processes. Chapter 10 is concerned with queuing systems of the type that arise most frequently in systems engineering analyses. Basic results and models of queuing systems are discussed.

The emphasis is on mathematics, but only to the extent of presenting the tools and techniques, together with their limitations, which are of practical value to the systems engineer. This book is not designed to be a mathematics text as the mathematician may be accustomed to, but is intended to be a mathematics text for systems engineers in the most general sense. It is not unreasonable to demand that an engineer have at least some understanding of why certain mathematical techniques work, when they are valid, and what their limitations are. As a result, theorems are not in general proven, but an attempt is made in some cases to provide motivation to determine why the results are true. Derivations of formulas are sometimes given, if not particularly difficult and lengthy, to provide the reader with some feeling as to where they come from and the level of difficulty required to obtain them. By limiting the formula development, a broader range of topics can be covered.

I am deeply indebted to the numerous students who have taken the time to express their constructive comments, and I have benefited considerably from discussions with colleagues at North American Rockwell Corporation (NR) and in the academic community. I am particularly indebted to Dr. Robert Jezik of the Autonetics Division of NR, who read most of the manuscript, and to Professor Klaus Ritter of the Mathematics Research Center at the University of Wisconsin, who reviewed the optimization material. My special thanks go to Mrs. Jo White who, even though I had assured her that each revision was the last, painstakingly and patiently typed the original manuscript and its numerous revisions.

JOHN G. RAU

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MATHEMATICAL ASPECTS OF THE SYSTEMS ENGINEERING PROCESS

1 THE SYSTEMS ENGINEERING PROCESS

Systems Engineering, or what we will refer to as the Systems Engineering Process, consists of the application of scientific methods in integrating the definition, design, planning, development, manufacture, and evaluation of systems. It encompasses such terms as systems approach, system analysis, system integration, system functional analysis, system requirements analysis, reliability analysis, task analysis, maintenance analysis, and operations analysis. It is fundamentally concerned with deriving a coherent total system to achieve a stated set of objectives subject to physical, environmental, state-of-the-art, and economical constraints.

The Systems Engineering Process (Figure 1.1) can be described in simplest terms as the sequential process of system definition, system development, and system evaluation. *System definition* consists of defining what the system is required to do, what its subsystems are, its operating environment, its attributes, the functional relationships between its components, and its basic design. *System*

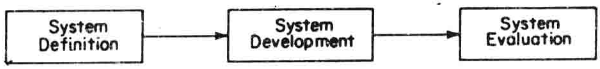

FIGURE 1.1 SYSTEMS ENGINEERING PROCESS.

development consists of finalizing the system design, planning and implementing its development process, and actualsystem production. *System evaluation* consists of placing the system in operation, observing its behavior, comparing its observed behavior with that which is required, recommending means for extending both system life and usefulness, and evaluating over-all system performance.

The main task of the system definition phase once the initial system objectives and requirements are established is to explore the design problem and ~~to~~ identify its elements such as parameters, constraints, and criteria. Consideration must be given to the maintenance implications on system design. Maintenance planning should be considered in this phase to preclude the possibility of developing a technically feasible system from an operations viewpoint before it is determined that the maintenance implications will prove costly or impractical, or prevent the system from performing as much as desired. Consideration must also be given to system reliability (and perhaps even availability) requirements. Preliminary cost versus effectiveness tradeoffs are sometimes performed in the selection of the optimal design alternative.

If the system must be designed for high reliability, the system designer may consider component redundancy (duplication or paralleling of components) as a means for increasing the system reliability. He may find that redundancy is

(1) the quickest solution, if time is of prime importance;
(2) the easiest solution, if the component is already designed;
(3) the cheapest solution, if the component is economical in comparison with the cost of redesign;
(4) the only solution, if the reliability requirement is beyond the state of the art.

On the other hand, in weighing its disadvantages, the designer may find that redundancy will

(1) prove too expensive, if the components are costly;
(2) exceed the limitations on size and weight, particularly in missiles and satellites;
(3) exceed the power limitations, particularly in active redundancy;
(4) attenuate the input signal, requiring additional amplifiers (which increase complexity);
(5) require sensing and switching circuitry so complex as to offset the advantage of redundancy.

Obviously, components of low or critical reliability are the ones to which redundancy can be the most beneficially applied.

Referring to Figure 1.2, the number of controllers in a computer system usually determines the number of simultaneous operations possible. In the simplest case, (a), the allocation of controllers to devices is restricted and full use of controllers *A* and *B* is possible only if one of devices 1 and 2 and one of devices 3 and 4 are working. In (b) is illustrated a more practical system design in which both controllers can be used if any two devices work. Clearly, the design of (b) would be more preferable to the design of (a) because of the increased (i.e., there are less restrictions) number of ways one could obtain desired system operation.

Typical problems arising during the system definition phase which the designer may be expected to solve are:

(1) Determine the system design offering the highest reliability subject to various constraints such as cost, weight, and volume.

(2) Determine the minimum number of components of each type needed to meet a certain performance requirement.

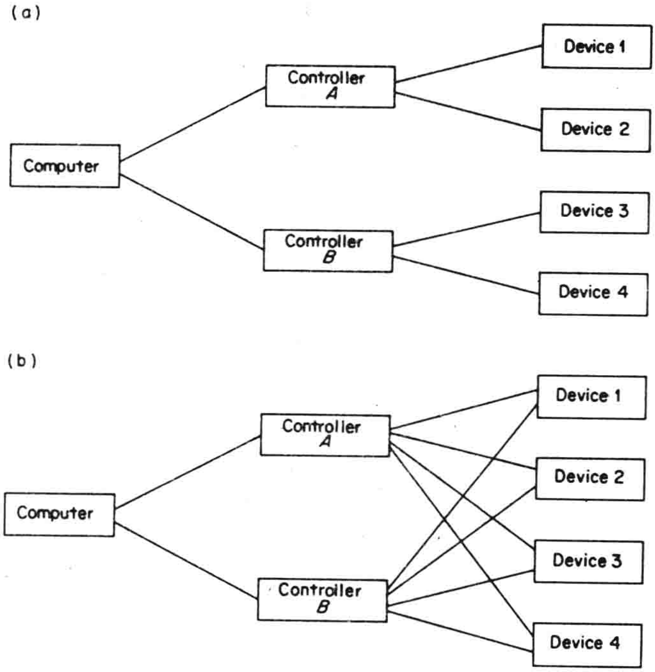

FIGURE 1.2 CONTROLLER ALLOCATION FOR A COMPUTER SYSTEM.

(3) For fixed cost, which system design yields the highest effectiveness?

(4) For fixed effectiveness, which system design has lowest cost?

In the system development phase, the final system design must be formalized making sure that the design includes adequate service features, and to make provisions for a rational basis for product improvement and redesign. The system must be designed for convenience in use (human factors aspects), operational economy, and adequate duration of service. The major tasks of this phase, however, are the planning and implementing of production and distribution. The production aspects include: detailed planning of the manufacturing processes as required for every part, subassembly, and final assembly; specification of the raw material required; design of tools and fixtures; planning, specifying, or designing new production and plant facilities; planning the quality control system, work schedules, and the inventory control policies. Distribution aspects include designing the packaging of the product (the outer shape of the product may be influenced by the need to effect economy in shipping costs) and planning the warehousing system (determining economically favorable locations for warehouses, designing the warehousing facilities, and determining the size of inventories for both system spares and component spares).

Typical problems arising during the system development phase are:

(1) How should system production be scheduled so as to minimize elapsed time and parts inventory costs?

(2) How should tasks associated with the production process be assigned to both men and machines, and in what order?

(3) How many spares should be produced to meet specified logistics requirements?

(4) How many parts should be ordered and when so as to minimize costs for ordering or setup, shortage or delay, and for changing the level of production or purchasing?

In the system evaluation phase, the main task is to determine whether or not the system is performing as designed, are adequate spares and maintenance facilities provided, and is the system physical life going to match the designed service life. Typical problems that may be considered are:

(1) How frequently should preventive or periodic maintenance be performed?

(2) When should the system be replaced so as to minimize costs involved?

(3) If the system is not performing up to par, then where can improvements be made in the most feasible and economic manner?

(4) For a given wear-out or failure pattern of the system, where can improvements be made in maintaining the system so as to extend its operational life and usefulness?

2 OPTIMAL SYSTEM DESIGN SELECTION

The ability to optimize a system depends on the availability of alternate means of meeting the system requirements and objectives. These alternatives include the means, approaches, or techniques that can be employed to meet the objectives within the constraints of the resources. The task of identifying alternatives consists basically of examining the system objectives together with the basic system concept proposed and listing all possible alternative means of meeting the objectives. The alternatives thus listed should then be screened against the available resources and stated constraints to insure that they are in fact feasible. The next step is to select the alternative or combination of alternatives which is optimal. This process consists of applying mathematical and probabilistic tools to determine how well the system objectives and requirements are met for each of the selected alternatives. Because of the possibility that no one alternative may meet all of the objectives better than any other alternative, it then sometimes becomes a difficult task to select the optimal alternative. When this situation arises, the system designer is usually faced with reevaluating the system requirements and objectives and, perhaps, even attaching weighting factors to them relative to their importance. The result is that the system alternative selected is the one which is optimal relative to these weightings.

Because, in general, the over-all objectives of a particular system are very complex, vary widely, and depend upon specific operational requirements, simple measures of how well the system objectives and requirements are met, such as performance, reliability, or cost, alone are not always sufficient. Actually, all of these factors must be considered in the design of any system, however other factors that should not be ignored are risk, schedule requirements, maintainability, availability, power consumption, weight, volume, and life expectancy. Some systems may stress performance, others reliability, others schedule, and still others cost. For example, the design of a single, high-performance positioning system for a land-based tracking radar used in conjunction with the reentry of a space vehicle, would stress performance and reliability, placing less emphasis on the importance of cost and weight. However, the design of a commercial hydraulic positioning system used in road construction and which is to be manufactured in very large quantities, would stress cost and relegate the importance of reliability and performance to lesser roles. Because of these basic different requirements, it is necessary to consider each system separately and determine the relative importance of the various objectives in their proper perspective.

For example, the performance of an information processing system can be measured by its processing rate or message handling capability. If the system is basically a fixed cost system which has two computers editing input messages and three computers processing these input messages, suppose analysis reveals that the three processors are used 100% of the time, but the edit computers are used only 40% of the time. Let E be the editing rate of each of the edit computers in messages per second and let P be the processing rate of a processing computer. Since the total number of messages passing through the editing and processing computers is the same, if it is assumed that the messages divide equally and independently among the computers, then the following steady state relation exists:

$$(2)(.4E) = 3P, \tag{1.1}$$

or

$$E = 3.75P. \tag{1.2}$$

This means each edit computer can edit messages for 3.75 processing computers. Using this information, it appears that the system could be improved by using one computer for editing and four for processing. Then the processing rate of the system is limited by that of the edit computer. If it is busy 100% of the time, then, in the modified configuration, the steady-state equation is

$$E = 4PX, \tag{1.3}$$

Where $X = \%$ time each of the four processing computers is kept busy on the average when the edit computer is fully loaded. Thus,

$$3.75P = 4PX \tag{1.4}$$

or, equivalently,

$$X = 93.75\%. \tag{1.5}$$

This represents a more uniform distribution of computer usage; in fact, it represents a 25% increase in the total message handling capability. This particular system design alternative thus provides an improvement in system performance and clearly is the optimal system design if only five computers are used.

In designing for system reliability, consider a system with n components in which the i^{th} component has reliability p_i, and suppose it is possible to increase each p_i by an amount Δp_i but at a cost increase from C_i to $C_i + \Delta C_i$ for $i = 1, 2, \cdots, n$. Suppose the design objective is to determine which component should have its reliability increased in order to maximize the ratio of the increase in system reliability to the increase in system cost. If the system is a simple parallel system, its reliability is given by the expression

$$R = 1 - \prod_{j=1}^{n}(1 - p_j). \tag{1.6}$$