



Numerical Techniques in Electromagnetics

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Numerical Techniques in Electromagnetics

*To my teacher
Carl A. Ventrice
and my parents
Ayisat and Solomon Sadiku*

Preface

For too long, the electromagnetic (EM) community has suffered without a suitable text on computational techniques commonly used in solving EM-related problems. Although there have been monographs on one particular technique or the other, the monographs are written for the experts rather than being student-oriented. There has not been a single text that covers all the major techniques and does so in a manner suitable for classroom use. It seems experts in this area are familiar with one or few techniques, and no one seems to be familiar with all the common techniques. This text attempts to fill the gap.

The text is intended for seniors or graduate students and may be used for a one-semester or two-semester course. The main requirements for students taking a course based on this text are an introductory EM course and a knowledge of a high-level computer language, preferably FORTRAN. Although familiarity with linear algebra and numerical analysis is useful, it is not required.

In writing this book, three major objectives were kept in mind. First, the book is intended to teach the students how to pose, numerically analyze, and solve EM problems. Second, it is designed to give them the ability to expand their problem-solving skills using a variety of available numerical methods. Third, it is meant to prepare graduate students for research in EM. The aim throughout has been simplicity of presentation so that the text can be useful for both teaching and self-study. In striving for simplicity, however, the reader is referred to the references for more information. Toward the end of each chapter, the techniques covered in the chapter are applied to real life problems. Since the application of the technique is as

vast as EM and the author's experience is limited, the choice of application is selective.

The first chapter covers some fundamental concepts in EM. For the sake of clarity, EM problems are classified in terms of the equations describing them (differential or integral or both), the boundary conditions (Dirichlet, Neumann, or mixed), and the form of the solution regions (interior/closed or exterior/open).

Chapter 2 is intended to put numerical methods in a proper perspective. Analytical methods such as separation of variables and series expansion are covered. A practical problem of attenuation and phase shift of EM wave due to raindrops is presented to illustrate the concepts developed in the chapter. Chapter 3 on the finite difference methods begins with the derivation of difference equation from a partial differential equation (PDE) using forward, backward, and central differences. Various methods of solving elliptic partial differential equations (iteration, relaxation, and band matrix) are considered. Stability of finite difference schemes and discretization error are discussed. The finite-difference time-domain technique involving Yee's algorithm is presented and applied to scattering problems. Numerical integration is covered using trapezoidal, Simpson's, Newton-Cotes rules, and Gaussian quadratures.

The fourth chapter on variational methods serves as a preparation for the next two major topics: moment methods and finite element methods. Basic concepts such as inner product, self-adjoint operator, functionals, and the Euler equation are covered. Derivation of a functional from a given partial differential equation and vice versa are presented. The classical Rayleigh-Ritz method with its limitations is considered as a direct variational method. Along with the concepts of expansion and weighting functions, the weighted residual methods (collocation/point matching, least squares, and Galerkin) are covered. A simple application of determining the characteristic impedance of a strip transmission line is used in illustrating the concepts developed in the chapter.

Chapter 5 on moment methods focuses on the solution of integral equations. The interrelationship between differential and integral equation, Green's functions for two- and three-dimensional problems, and some typical applications in antenna, scattering, and EM absorption by human body are presented.

Chapter 6 covers the basic steps involved in using the finite element method. Solutions of Laplace's, Poisson's, and wave equations using the finite element method are covered. The problems of proper node numbering, bandwidth reduction, and mesh generation are treated. Higher-order and three-dimensional elements are briefly discussed.

Chapter 7 is devoted to transmission-line matrix or modeling (TLM). The method is applied to diffusion problem and scattering problems.

The last chapter is on Monte Carlo methods. The technique of generating random numbers is presented. Typical applications of the method to electrostatic problems are presented.

The appendices contain useful vector relations, a brief review of FORTRAN, some hints on how to develop good code, various techniques for solving simultaneous equations, and answers to some problems. A solutions manual is available from the publisher.

Acknowledgments

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A Note to Students

Before you embark on writing your own computer program or using the ones in this text, you should try to understand all relevant theoretical background. A computer is no more than a tool used in the analysis of a program. For this reason, you should be as clear as possible what the machine is really being asked to do before setting it off on several hours of expensive computations.

It has been well said by A. C. Doyle that "It is a capital mistake to theorize before you have all the evidence. It biases the judgment." Therefore, you should never trust the results of a numerical computation unless they are validated, at least in part. You validate the results by comparing them with those obtained by previous investigators or with similar results obtained using a different approach which may be analytical or numerical. For this reason, it is advisable that you become familiar with as many numerical techniques as possible.

The references provided at the end of each chapter are by no means exhaustive but are meant to serve as the starting point for further reading.

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Chapter 1

Fundamental Concepts

"Thy friend has a friend, and thy friend's friend has a friend; be discreet."
The Talmud

1.1 Introduction

Scientists and engineers use several techniques in solving continuum or field problems. Loosely speaking, these techniques can be classified as experimental, analytical, or numerical. Experiments are expensive, time consuming, sometimes hazardous, and usually do not allow much flexibility in parameter variation. However, every numerical method, as we shall see, involves an analytic simplification to the point where it is easy to apply the numerical method. Notwithstanding this fact, the following methods are among the most commonly used in electromagnetics (EM).

- A. Analytical Methods (exact solutions)
 - (1) Separation of variables
 - (2) Series expansion
 - (3) Conformal mapping
 - (4) Integral solutions, e.g., Laplace and Fourier transforms
 - (5) Perturbation methods
- B. Numerical methods (approximate solutions)
 - (1) Finite difference method
 - (2) Method of weighted residuals
 - (3) Moment method
 - (4) Finite element method
 - (5) Transmission-line modeling
 - (6) Monte Carlo method