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«ENRICO FERMI»

XCIII CORSO

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Introduction.

The School on «Frontiers in Physical Acoustics» discussed some aspects of recent research which have relevant importance in both acoustics and related fields.

The topics treated were concerned mainly with nonlinear acoustics and the subharmonic path to chaos, sound propagation in special conditions (in porous materials, at rough surfaces and in space), acoustic engines, acoustic microscopy and some recent developments in acoustic transduction.

D. G. CRIGHTON presented the basic theoretical treatment of sound propagation, mainly in unbounded fluids with some reference to propagation in tubes, when local or global effects of weak nonlinearity are present. The analysis considers in detail the validity of the solutions of differential equations in the various conditions of nondissipative, dispersive, dissipative media, and for various types of waves. Particular effort was devoted to illustrate the Burgers' differential equation, valid for the wave evaluation when weak nonlinear and dissipative terms are comparable. This famous canonical equation is the only nonlinear parabolic equation which has an exact linearization (Hopf-Cole transformation) and an exact general solution for arbitrary initial data. The fact that no other parabolic equation has these properties indicates clearly the difficulties to be found in the solution of nonlinear acoustic problems.

The author considers, in detail, the solutions of the Burgers' equation for periodic plane waves and of the so-called generalized Burgers' equation when geometrical spreading effects are included so as to treat propagation along a «ray tube» or horn. The particular difficulties found in the theoretical treatment of propagation over long ranges, where various conditions may strongly affect the evaluation, are also examined. The effect of the presence of other processes are described, such as relaxation in the media, or the influence of nonlinearity on beam diffraction (which is of great importance in the theory of parametric array). The author indicated the position of the theoretical problems which present research is facing.

The physical origins of noise were discussed by Prof. J. E. FROWES WILLIAMS. The problem of source definition in the aerodynamic context was presented pointing out, at the start, the ambiguity intrinsically connected with a determination of the source from a knowledge of the wave field. Various source distributions (monopole, dipole, quadrupole, multipole) were considered.

The multipole expansion, which indicates how point sources may duplicate the radiation field of a different source distribution, is mainly useful for compact sources smaller than the acoustic wave length. The cases of noncompact source distribution, or of sources with variable compactness ratios were discussed. The author then continued on to consider the situation where the sources are moving. The treatment becomes much more difficult in comparison with the case of the source at rest, even when the hypothesis of uniform motion can be made. In such a case, it is possible to choose among different co-ordinate systems for the computation of the radiated field. Such a possibility fails when the motion is nonuniform. Important cases were treated for both situations.

Research conducted in the last few years has made evident that deterministic systems can present erratic and seemingly irregular motion. This behaviour, different from erratic motion generated by random external forces, is classified as chaotic behaviour. Its appearance within a dynamical system requires, as a prerequisite, the existence of nonlinearities in the equation of motion. Most nonlinear systems exhibit, at certain critical values of a relevant parameter, a transition from regular to chaotic behaviour, *i.e.* the regular orbits of the system may pass to a practically unpredictable random process. Such a course of behaviour has great importance in the understanding of turbulence (and noise) in dissipative systems and in the thermodynamic behaviour of conservative systems.

The contribution of R. H. G. HELLEMAN, «Chaotic and turbulent behavior of simple nonlinear systems», dealt with such an important subject. The author introduced a general model, «period-doubling transition to chaos», which is able to describe the growth in complexity of mechanical orbits in nonlinear systems when a characteristic parameter (energy, Reynolds number, etc.) is increased. It predicts the existence of a finite critical value of such a parameter beyond which the spectral (noise) intensity of the orbits increases so greatly that unpredictable chaotic behaviour is established. The predictions of this model, which allows random processes from deterministic equations of motion to be obtained, have been experimentally tested in various fields. They have been confirmed by numerical simulations for many other nonlinear systems in which experiments have not as yet been performed.

In the present School the lectures of M. Giglio and W. Lauterborn discussed some of the more relevant experimental indications. M. GIGLIO dealt with pioneering experiments on fluid dynamics instabilities, in particular the Rayleigh-Benard instability where the approach to chaos via period-doubling bifurcation has been clearly demonstrated. He discussed also the experimental data obtained in the chaotic region.

W. LAUTERBORN presented some brilliant experiments on the dynamics of acoustic cavitation showing how the cavitation noise is of deterministic origin. The results can be found from a simple theoretical model of acoustic turbulence based on nonlinear resonances and bifurcation structures.

Acoustic cavitation is a research field which has received increasing attention in recent years. The contribution of A. PROSPERETTI to the present School was a thorough discussion of the subject. The various physical processes which are involved in determining acoustic cavitation were discussed as well as the theoretical models and the most recent experimentation results. Special attention was given to the basic process of forced oscillations of simple bubbles, to their stability and to the aspects of their large-amplitude motion when successive subharmonic bifurcations occur.

Another contribution to the study of the path to chaos of oscillating nonlinear systems was given by R. KEOLIAN and I. RUDNICK, who have studied parametrically excited surface waves (gravity being the major restoring force) in a one-dimensional trough containing liquid He (at 1.22 K). As the driving force is increased, the surface wave becomes modulated at a frequency apparently incommensurate with the driving frequency. The modulation itself becomes successively modulated.

J. WU, R. KEOLIAN and I. RUDNICK reported another experiment in a nonlinear system formed by a trough resonator partially filled with water. A new type of nonpropagating solitary wave (soliton) can be established when the resonator is parametrically driven in a direction normal to its length and at an appropriate frequency and amplitude.

The study of the scattering of waves from rough surfaces is of great interest in various fields of physics (acoustics, optics, geophysics) because it allows problems of real practical interest to be considered. I. TOLSTOY considered the subject of long-wavelength acoustical scatter from rough surfaces by developing a smoothed boundary-condition technique. The theory, initially suggested by BIOT for hard surfaces, has been extended to rough two-fluid interfaces. It demonstrates the existence of a boundary mode which originates by the trapping of energy near the boundary through a process of multiple scattering between the roughness elements. Such an existence and the properties of the mode have been confirmed by model experiments with hard rough surfaces in air in the (1÷30) kHz band. Many applications can be foreseen in the entire frequency range from infrasound to ultrasound. They may involve problems in such fields as geophysics, geoacoustics, material science and processing devices.

Prof. H. N. V. TEMPERLEY considered the reflection of positive pressure pulses at the free surface of water. Even under laboratory conditions, the free surface of water is not a perfect reflector. The author's analysis leads to the suggestion that the reflecting surface has to be considered as molecularly rough and «loaded». The last effect could be explained by imagining that the reflection actually occurs at a small distance from the geometrical surface from a mixture of drops and vapour bubbles. Such an interpretation is capable of explaining the discrepancies between results of static and dynamic determinations of the tensile strength of water.

Recent developments in the acoustics of porous media were the subject

of the lectures presented by D. L. JOHNSON. The porous fluid-filled solid media may be considered as a two-component system where each component forms its own infinite cluster connected throughout the sample. Experimentally and theoretically, it can be shown that two distinct longitudinal modes exist in the long-wavelength limit. A semi-phenomenological theory has been given by BIOT where the average motions of both solid and fluid parts are considered separately as in two different, although interpenetrating, media. Such a relatively simple theory makes use of a few phenomenological parameters which may be related to measurable quantities. It is successful in accounting for the results of many experiments such as the diffusive mode seen in gels, 4th sound in superfluid/superleak systems, the diffusion of a fluid pressure pulse through a porous medium and the «slow» compressional wave in water-saturated samples. The author also presented theoretical considerations on the input parameters in order to show the way in which the physical properties of porous, fluid-saturated solids determine the propagation of sound in them. This allows the theory from microscopic considerations, such as effective-medium theories and multiple-scattering theory, to be discussed.

Various experiments to be performed inside space vehicles are presently programmed to take advantage of almost zero gravity and of no need for containers. The experiments are of interest for research in physics and material science. T. G. WANG, a scientist involved in these experiments, reviewed the recent progress in some of the space technologies and facilities with regard to acoustics. A novel acoustic method has been developed for controlling liquid samples without physical contact. It makes use of the static acoustic pressure generated by three mutually orthogonal standing waves. The same apparatus allows for the rotation of the sample if suitable phase differences between drivers of the field in orthogonal directions are introduced. Acoustical methods for inducing oscillation in a liquid drop and for levitating samples are described. The nature of the physical acoustic research which should be allowed by these technologies was briefly discussed.

The problem of determining in great detail the field radiated by a source, a complex vibrator in general, is of great importance in order to correlate the properties of the source (structure, mode of vibration, etc.) with those of the radiated field (power, far field pattern, vector intensity field, etc.). J. D. MAYNARD presented the features of a newly developed experimental method, the near-field acoustic holography system, which, by fully exploiting the principles of holography, allows computer graphic displays to be obtained of 1) the sound pressure field and the particle velocity field from the source to far field, 2) the modal structure of a vibrating surface, 3) the vector intensity field, 4) the far-field radiation pattern and 5) the total power radiated.

The technique uses an open array of microphones on a two-dimensional surface close to the source. This allows a large area to be covered and a large solid angle from the source to be subtended. The technique involves a simple

noncontact measurement and, therefore, requires only a short time for carrying out the calculations. It lends itself to follow the time evolution of sound fields.

The subject of acoustic imaging has progressed largely in recent years due to the work of the Stanford Group, led by C. F. QUATE, which has developed the scanning acoustic microscope. This subject was presented and discussed at the School by J. E. HEISERMAN and C. F. QUATE. The heart of the microscope is a spherical lens created on the surface of a sapphire rod with a radius of the order of $12\text{ }\mu\text{m}$ which focuses in a liquid the sound emitted by a piezoelectric source placed at the other end of the rod. The object to be examined is placed in the focal plane, and it is mechanically moved in two directions, so that it is periodically examined point by point by the sound beam. The apparatus can be arranged as a reflection microscope (using short pulses) or as a transmission microscope (using continuous waves). Resolutions as high as $2000\text{ }\text{\AA}$ have been achieved at room temperature by operating the microscope in a special nonlinear mode at 4.2 GHz . Still higher resolutions can be achieved by using liquid ^4He at a very low temperature. A microscope operating at 8 GHz and 0.1 K has reached a resolution of $200\text{ }\text{\AA}$. According to theoretical and experimental studies, the ultimate limit has not yet been reached.

The possible applications of the method in fields such as material science, integrated-circuit technology, medicine and biology are enormous. They can exceed the limits of the optical microscope, being the method capable of observation in opaque media and of having the high resolutions quoted above. Moreover, it has been possible to operate in a very wide frequency range with wavelengths exceeding the ratio $10\,000$ to 1 : from 3 MHz (in water $\lambda = 500\text{ }\mu\text{m}$) to 8 GHz (in helium, $\lambda = 300\text{ }\text{\AA}$).

A very promising new field of research concerns the study of the thermoacoustic effect and of some types of intrinsically irreversible engines, i.e. of thermodynamic engines whose operation rests on the irreversible nature of the processes: their efficiency would be zero if all processes involved were reversible. J. C. WHEATLEY, who is conducting pioneering work in this advanced field, presented a thorough discussion on the thermoacoustic effect. This effect consists in the establishment of a temperature difference between the ends of plates conveniently placed near the closed end of a tube where sound waves (of wavelength much larger than the longitudinal length of the plates) are present. Such a temperature difference, which may reach high values, is caused by the pressure change in the gas due to the wave and to the intrinsically irreversible process of heat exchange between the gas and the plates. The thermodynamics of the effect is discussed as well as its possible application to acoustic engines (heat-driven cooler, cryo-cooler and the thermoacoustic oscillator). Demonstration experiments on the basic elements and the concepts and calculations for the engines were discussed.

The influence of nonlinearities in the processes and the development of nonlinear acoustic elements for studying two-media-dependent phenomena are

considered. To stress the very fundamental nature of the treatment, it is to be noted that the intrinsically irreversible engines, which are here studied in an acoustic context, are not limited to this field (one can consider, for instance, magnetic engines and plasma engines).

E. F. Carome's contribution, «Acousto-optic transduction in optical fibers and fiber optic acoustic devices», examined the applications of the low-loss optical-fiber technology to the development of acoustic sensors and of acousto-optics devices. The relevant properties of optical fibers and the general characteristics of interferometric sensors were reviewed before discussing the acoustically induced phase transduction in fiber. The phase variation of the light, produced by the propagation through a fiber length, is changed if an acoustic wave impinges on the fiber as a consequence of the acoustic pressure on the length itself and on the refractive index. Such an effect may lead to an acoustic detector when used in interferometric schemes. The acoustic sensitivity of a fiber strongly depends upon the elastic properties of the fiber coating and on the frequency range (*i.e.* the ratio of sound wavelength to the linear dimensions of the fiber section and of the device). Birefringence can also be induced in special arrangements. Various devices have been produced, such as gradient hydrophones, optical phase modulators and acousto-optical frequency shifters. Many other are foreseen in the near future.

D. SETTE

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Basic Theoretical Nonlinear Acoustics.

D. G. CRIGHTON (*)

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1. - Introduction.

In these notes I want to give an introductory account of the basic theoretical developments in nonlinear acoustics. I take a very limited view of the scope, understanding *acoustics* to refer to the propagation of disturbances in the linear regime (implying a limitation on the amplitude and the space-time domain considered), at « moderate » frequencies, through *bulk fluid media* of a fairly « normal » kind (in particular, through air and water in normal conditions); and *nonlinear acoustics* to enlarge that scope by introducing the local or global effects of *weak nonlinearity* in the governing equations. Nonlinear acoustics of solid media will be completely excluded, as will surface wave phenomena, aspects requiring quantum-mechanical explanation, and interactions with optical and electromagnetic waves. Effects of strong local nonlinearity will not be considered (as in strong shock waves, blast waves, detonations and deflagrations), nor will mention be made of all the work on nonlinear acoustic propagation in tubes (dealing both with pure propagation, including mean flow and possible transonic effects, and with reflexion at closed or open ends and the establishment of resonant oscillations). In an introductory course I see it as preferable to go carefully into the basic issues, rather than to skate over such a wide field with diverse applications in science and engineering. And when all the above exclusions are made, a corpus of material remains that epitomises nonlinear acoustics and explains why there is an identifiable scientific discipline called nonlinear acoustics (and why much was missed when the subject was thought to be merely a trivial limit of gas dynamics).

I want to emphasize that, although much has been achieved in this field, developments have not been possible at the rate which has been attained in other fields of nonlinear wave theory in the last 20 years. Leaving aside prog-

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ress in dynamical systems theory for multicomponent nonlinear systems evolving under ordinary differential equations, developments in nonlinear waves governed by partial differential equations have been related very largely to soliton theory for dispersive waves. An essential feature of acoustic waves (from the theoretical and practical point of view) is that they propagate in ordinary unbounded media with *virtually no (frequency) dispersion*. Genuine intrinsic dispersion is found occasionally, as in the case of the bubbly liquid, to which we shall come, and then soliton behaviour is possible, has been seen in experiments and can sometimes be treated with all the apparatus which now exists for integrable nonlinear partial differential systems. Dispersion can be introduced by geometrical constraints, as, for example, in waves in tubes, where a given mode propagates axially in a dispersive fashion; but the fundamental absence of dispersion makes itself felt when nonlinear effects are considered. Nonlinear acoustics is, therefore, the prototype for an important class of nonlinear wave phenomena; dissipation, rather than dispersion, is the most important linear mechanism, whose conflict with nonlinearity is the identifying theme of the subject. As a result, not only is it not possible to transfer techniques and results of soliton theory over to nonlinear acoustics—it has been *proved* in some cases that no extensions of anything like soliton theory as we know it can provide exact solutions of the equations of nonlinear acoustics. This in turn implies, on the one hand, that work is needed at the most fundamental level of nonlinear wave theory, and, on the other, that approximate analytical and numerical methods are all we are likely to have in nonlinear acoustics for a long time to come. We must, therefore, exploit whatever solutions we have as fully as possible, and attempts to do that will form part of these notes.

The subject is fortunate in having two recent textbooks [1, 2]. Reference [2] is outstanding in providing a wide-ranging discussion of the topic on the basis of extensive and authoritative work by the authors and their immediate colleagues. Other books in which appropriate material is covered are those in ref. [3-5]. The conflict between nonlinearity and dissipation was extensively discussed in the celebrated article [6] by Lighthill, while a catalogue of equations generalizing the conflict analysed by Lighthill was provided by the author in ref. [7].

In the following notes, specific references will not be given for much of the basic material; for this, adequate references will be found in the books and articles just cited.

2. — Inviscid linear and nonlinear plane waves.

If the fluid is taken as inviscid and non-heat-conducting, and is initially of uniform entropy, then the exact Eulerian equations of motion in one-di-