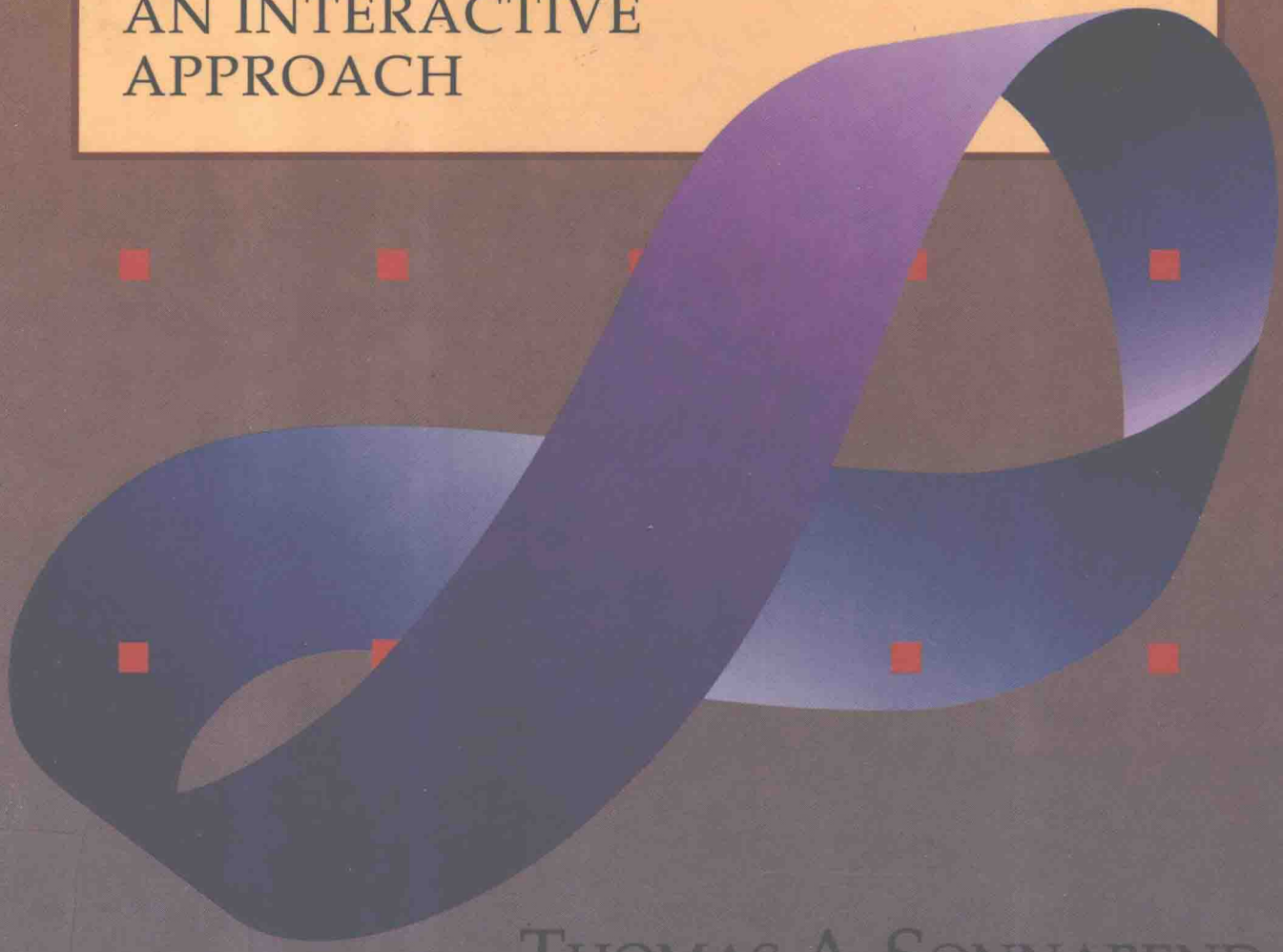
MATHEMATICS
for Elementary Teachers

AN INTERACTIVE
APPROACH



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Preface

The NCTM Standards state that "prospective teachers must be taught in a manner similar to how they are to teach—by exploring, conjecturing, communicating, reasoning, and so forth." This necessitates devoting more class time to discussing and discovering ideas and less time to lecturing.

Can a mathematics book actively involve students in developing and explaining mathematical concepts? Yes, by using a carefully organized, interactive lesson format that promotes student involvement and gradually leads the student to a deeper understanding of mathematical ideas. The interactive format also allows for more class discussion and small group work.

To implement the NCTM Standards, one needs a textbook that provides numerous opportunities for investigation and discourse. Rather than having a few special puzzle and investigation problems, this textbook presents a substantial collection of exercises in every lesson and homework set that involve reasoning, investigating, or communicating.

Most people do not enjoy reading mathematics textbooks. Can a mathematics textbook be interesting to read and study? Yes, if the mathematical presentation is straightforward and clear, and the lesson content is enriched with investigations, appropriate uses of technology, humor, history, and interesting applications. A text for this course should also make it clear how the topics in each chapter relate to both the NCTM Standards and the current school mathematics curriculum.

You are looking at the result of my effort to address all these concerns while covering new topics and examining underlying concepts and connections in elementary-school mathematics. This textbook is the culmination of 13 years of work with university, college, and community college students.

Teaching elementary school is one of the most important and challenging professions in our society. It is my hope that this textbook provides

material for a more interesting course that produces more competent teachers with a better sense of what elementary-school mathematics is all about.

Distinctive Features

This textbook places *greater emphasis* on

- discovery, discussion, and explanation of concepts that involve students and deepen their understanding
- investigations and activities that require higher-level thinking
- applications of mathematics that connect mathematical concepts to everyday life
- explaining a concept or solving a problem in more than one way so that future teachers can see a variety of approaches
- how material relates to the current elementary-school curriculum
- difficult concepts in the elementary-school mathematics curriculum that future teachers have trouble explaining
- connecting lessons, homework exercises, and chapter review exercises
- multiple representations of concepts so that students develop a broader understanding
- inductive and deductive reasoning, showing how these two processes are used throughout mathematics
- models for arithmetic operations that establish the connection between an operation and its everyday applications
- common student error patterns in arithmetic and measurement that prepare future teachers for diagnosing student difficulties
- spatial perception and perspective drawing so that students become more adept at three-dimensional geometry
- algebra as a mathematical language and a generalization of arithmetic
- important statistical applications including statistical deceptions, surveys, and standardized tests

In order to make time for these topics, this textbook places less emphasis upon (1) topics most students have studied or will study in other mathematics or education courses and (2) topics that are relatively more remote from the current elementary-school curriculum. Such topics include arithmetic and algebra skills, formal logic, geometry terminology, congruent triangle proofs, and isometries.

Other Features

- Students use relevant NCTM Standards to review each chapter and increase their awareness of specific standards.
- Charts at the end of each chapter show grade-level coverage of related elementary-school topics.
- Computer exercises throughout the textbook employ BASIC to study number theory, algebra, statistics, and probability; Logo and automatic drawers to study geometry and measurement; and MINITAB to study statistics.
- Extensive coverage of geometry and measurement topics strengthens students' spatial and analytical abilities.
- Extensive coverage of statistics and probability incorporates many realistic applications.
- Mental computation and estimation with whole numbers, fractions, decimals, and percents make use of properties of operations and complement the use of calculators.
- Humor and historical vignettes humanize the mathematics.

Teaching and Learning Aids

The following features make it easier for students and professors to make more effective use of this book.

- Chapter introductions set the stage for each new chapter.
- Subsection headings help students understand how each lesson is organized.
- Exercises within each lesson actively involve students in developing and reinforcing concepts.
- Lessons gradually lead students to a deeper understanding of difficult concepts in the elementary-school curriculum.
- Selected lesson exercises are marked D as especially suitable for discussion.
- Answers to most lesson exercises provide students with the opportunity to check their work as they develop new ideas.
- Important definitions, theorems, and properties are boxed with boldfaced headings.
- All mathematical terms are set boldface.
- Color is used to highlight mathematical ideas and add clarity to figures.
- Sample textbook pages show newer topics in elementary-school mathematics, thus increasing student awareness of recent changes.

- A "friendly" writing style and cartoons, drawings, and photographs stimulate student interest.
- Homework exercises are categorized as basic, extension, computer, and special so that instructors can select the appropriate level and type of homework exercises.
- Answers to most odd-numbered homework exercises and all chapter review exercises provide students with adequate feedback.
- Calculator and computer exercises are marked.
- Chapter study guides list important concepts and terms by section to help students recall major ideas.
- Chapter review exercises help students prepare for exams.
- Suggested reading lists for each chapter direct students to interesting resource material.

Chapter Organization and Features

Chapter 1 introduces mathematical reasoning processes and problem solving techniques that are used throughout the course. Its broader view of mathematical reasoning includes inductive and deductive reasoning as well as patterns and problem solving.

Chapter 2 covers set concepts that are used later in the course to clarify other concepts. Venn diagrams are used to solve problems in Chapters 1 and 2.

Chapters 3–7 cover the number systems from whole numbers to real numbers. Categories for each whole number operation (e.g., compare and take away) are studied in depth in Chapter 3 and used with other number systems in Chapters 5–7. Mental computation and estimation are also used with each number system. In number theory (Chapter 4), students do proofs and learn two different methods for solving certain problem types. In Chapters 5–7, students learn how to explain difficult procedures in integer, fraction, and decimal arithmetic using models and realistic applications. Students also compare different representations of real numbers.

Chapters 8–10 cover geometry and measurement. In Chapter 8, the van Hiele model is used to develop categories and definitions of quadrilaterals. Sections on space figures and spatial perception strengthen students' spatial abilities. In Chapter 9, students connect transformation geometry to congruence, symmetry, and similarity, and students analyze a series of constructions using congruence properties. In Chapter 10, students perform a series of activities to learn about the metric system and to develop area formulas in a logical sequence.

Chapter 11, Algebra and Coordinate Geometry, follows and extends arithmetic from Chapters 3–7 and geometry and measurement from Chapters 8–10. Students learn different ways to represent a function or relation.

Multiple representations (words, tables, graphs, and formulas) are used in solving realistic application problems. Algebra is studied as a mathematical language.

Chapter 12 is the ideal place for BASIC programming. This optional chapter connects introductory BASIC with algebraic formulas from Chapter 11. The hands-on approach leads beginners through the essential ideas of BASIC. Students then use the computer as another approach to solving algebra, number theory, and measurement problems.

For those who complete Chapter 12, BASIC branches and loops can then be used with statistics and probability in Chapters 13 and 14. Chapter 13 emphasizes choosing the most appropriate graph or statistic to summarize results. Choices include the relatively new stem-and-leaf plots and boxplots. The extensive applications of statistics include statistical deceptions, surveys, and standardized tests.

In Chapter 14, students learn the connection between theoretical and experimental probabilities and use simulations to generate experimental probabilities. Students see the importance of probability in insurance, drug testing, and gambling games.

Course Outlines

This textbook provides for some flexibility in organizing the course. The textbook contains more than enough material for two four-semester-hour mathematics courses for preservice elementary teachers. The material in Chapters 1–7 should be studied in sequence; however, a number of sections and parts of sections can be omitted.

In Chapters 1–7 you should cover the following material.

1. Chapter 1
2. In Section 2.1, review set notation and define whole numbers. In Section 2.2, cover "intersection and union."
3. In Section 3.1, cover "models for place value" and "rounding." Cover Sections 3.2 and 3.3. In Section 3.4, cover the properties. Cover Section 3.5.
4. In Section 4.1, cover "factors." In Section 4.2, cover "multiples," "divisibility tests for 2, 5, and 10," and "divisibility tests for 3 and 9." Cover Sections 4.3 and 4.4.
5. Cover Section 5.1 and "inverses" in Section 5.3.
6. Cover Chapter 6.
7. Cover Sections 7.1, 7.3, and 7.4.

After studying the basic material in Chapters 1–7, the major ideas of Chapters 8–14 can be studied independently of one another with the following exceptions.

Material	Prerequisite
Logo in Chapters 9 and 10	Sections 8.6, 8.7
Chapter 10	Chapter 8
Section 10.6	Section 9.4
Section 11.5	Chapters 8–10
Sections 12.2–12.4	Sections 7.3, 10.4, 11.3, 11.4
Section 14.4	Section 13.3
BASIC in Chapters 13 and 14	Chapter 12

One-Semester Course

Design a one-semester course that suits your needs, with the following conditions.

1. Cover the required sections and parts of sections in Chapters 1–7 as already outlined.
2. You can skip optional Sections 3.6 and 7.5, investigations, extension exercises, and special exercises.
3. Round out the course with additional material from Chapters 1–7, or select sections from Chapters 8–14. Depending upon the length of your course, the ability of your students, and the amount of time you spend on extension exercises, you might be able to cover about 5 to 15 more sections. Suggestions for additional material would include Sections 8.1–8.5, 9.1, 10.1–10.3, 13.1–13.3, 14.1 and 14.2.

Two-Semester Course

In two four-semester one-hour courses, one can cover most of the sections in the book, depending upon the number of homework exercises covered. One could skip some of Sections 3.6, 7.5, 9.2, 10.6, 11.5, Chapter 12, and Section 14.4 or 14.5, or optional subsections that are listed in the *Instructor's Manual*.

Supplement for Students

The *Student Resource Manual* provides additional hands-on activities for each chapter. These activities utilize material cards that come with the manual. Also included are activities that focus on calculators, critical thinking, connections, and the NCTM standards.

Supplements for Teachers

The *Instructor's Manual* contains teaching suggestions, overhead transparency masters, and exercise answers that are not given in the textbook. The *Test Bank* contains sample test questions for each chapter. The *Computerized Test Bank,* available in IBM and Macintosh formats, contains the questions from the *Test Bank.*

Manipulatives Kit

This kit will consist of Cuisenaire mathematics manipulatives. The kit includes geometric solids, connecting cubes, and the following materials for the overhead projector: a geoboard, fraction tiles, base-ten blocks, and a tangram puzzle. This kit will be provided free to professors upon adoption of the text.

Sample Elementary Textbook Kit

This kit consists of Instructor's Editions of the Harcourt Brace Jovanovich 1992 publications *Mathematics Plus,* Grades 3 and 6. This kit will be provided free to professors upon adoption of this book.

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Principal Reviewers

Judy Bergman, *University of Houston*
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Contents Overview

Contents

Introduction

Why Learn Mathematics?

As a teacher, you will introduce the next generation of students to mathematics, teaching them arithmetic, geometry, and statistics. Your students will solve problems, think logically, find patterns, and apply their knowledge to practical situations.

But why does anyone teach children mathematics as part of their education? Mathematics offers a unique way of looking at the world. Mathematics gives simple, abstract descriptions that illuminate general relationships among quantities or shapes while simplifying or ignoring qualities. Mathematics can show that, ounce for ounce, Brand A is a better buy than Brand B, focusing upon the amount and the price while ignoring other differences in the products and the companies that produce them. Such mathematical information can help one decide which brand to buy.

Furthermore, the logical, objective approach of mathematics is applied to many situations. Mathematics has had an impact on nearly every area of life, from philosophy to economics to art. Philosophers apply the logic of mathematics to ultimate questions. Economists employ quantitative methods to describe financial trends. Artists use geometry when they represent our three-dimensional world on canvas.

Finally, mathematics is the language of technological societies. Businesses use mathematics in record-keeping and analysis. As a consumer and citizen, each of us needs mathematics to make financial decisions and to interpret statistics in political and economic news.

What have your mathematical experiences been like? Is it one of your favorite subjects? Or have you felt like Peppermint Patty?