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3+1 Formalism in General Relativity: Bases of Numerical Relativity

广义相对论的3+1形式
——数值相对论基础

(影印版)

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序　　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会　主任

中国科学院院士，北京大学教授

王恩哥

2010 年 5 月于燕园

Éric Gourgoulhon

3+1 Formalism in General Relativity

Bases of Numerical Relativity

Springer

To the memory of
Jean-Alain Marck (1955-2000)

Preface

This monograph originates from lectures given at the General Relativity Trimester at the Institut Henri Poincaré in Paris [1]; at the VII Mexican School on Gravitation and Mathematical Physics in Playa del Carmen (Mexico) [2]; and at the 2008 International Summer School on Computational Methods in Gravitation and Astrophysics held in Pohang (Korea) [3]. It is devoted to the 3+1 formalism of general relativity, which constitutes among other things, the theoretical foundations for numerical relativity. Numerical techniques are not covered here. For a pedagogical introduction to them, we recommend instead the lectures by Choptuik [4] (finite differences) and the review article by Grandclément and Novak [5] (spectral methods), as well as the numerical relativity textbooks by Alcubierre [6], Bona, Palenzuela-Luque and Bona-Casas [7] and Baumgarte and Shapiro [8].

The prerequisites are those of a general relativity course, at the undergraduate or graduate level, like the textbooks by Hartle [9] or Carroll [10], or part I of Wald's book [11], as well as track 1 of the book by Misner, Thorne and Wheeler [12]. The fact that the present text consists of lecture notes implies two things:

- the calculations are rather detailed (the experienced reader might say *too* detailed), with an attempt to made them self-consistent and complete, trying to use as infrequently as possible the famous phrases "as shown in paper XXX" or "see paper XXX for details";
- the bibliographical references do not constitute an extensive survey of the literature on the subject: articles have been cited in so far as they have a direct connection with the main text.

The book starts with a chapter setting the mathematical background, which is differential geometry, at a basic level (Chap. 2). This is followed by two purely geometrical chapters devoted to the study of a single hypersurface embedded in spacetime (Chap. 3) and to the foliation (or slicing) of spacetime by a family of spacelike hypersurfaces (Chap. 4). The presentation is divided in two chapters to distinguish between concepts which are meaningful for a single hypersurface and those that rely on a foliation. The decomposition of the Einstein equation relative to the foliation is given in Chap. 5, giving rise to the Cauchy problem with constraints, which constitutes the core of the 3+1 formalism. The ADM Hamiltonian formulation of general relativity is also introduced in this chapter. Chapter 6 is devoted to the decomposition of the matter and electromagnetic field equations, focusing on the astrophysically relevant cases of a perfect fluid and a perfect conductor (ideal MHD). An important technical chapter occurs then: Chap. 7 introduces some conformal transformation of the 3-metric on each hypersurface and the corresponding rewriting of the 3+1 Einstein equations. As a by-product, we also discuss the Isenberg-Wilson-Mathews (or conformally flat) approximation to general relativity. Chapter 8 details the various global quantities associated with asymptotic flatness (ADM mass, ADM linear momentum and angular momentum) or with some symmetries (Komar mass and Komar angular momentum). In Chap. 9, we study the initial data problem, presenting with some examples two classical methods: the conformal transversetraceless method and the conformal thin-sandwich one. Both methods rely on the conformal decomposition that has been introduced in Chap. 7. The choice of spacetime coordinates within the 3+1 framework is discussed in Chap. 10, starting from the choice of foliation before discussing the choice of the three coordinates in each leaf of the foliation. The major coordinate families used in modern numerical relativity are reviewed. Finally Chap. 11 presents various schemes for the time integration of the 3+1 Einstein equations, putting some emphasis on the most successful scheme to date, the BSSN one. Appendix A is devoted to basic tools of the 3+1 formalism: the conformal Killing operator and the related vector Laplacian, whereas Appendix B provides some computer algebra codes based on the Sage system.

A web page is dedicated to the book, at the URL

http://relativite.obspm.fr/3p1

This page contains the errata, the clickable list of references, the computer algebra codes described in Appendix B and various supplementary material. Readers are invited to use this page to report any error that they may find in the text.

I am deeply indebted to Michał Bejger, Philippe Grandclément, Alexandre Le Tiec, Yuichiro Sekiguchi and Nicolas Vasset for the careful reading of a preliminary version of these notes. I am very grateful to my friends and colleagues Thomas Baumgarte, Michał Bejger, Luc Blanchet, Silvano Bonazzola, Brandon Carter, Isabel Cordero-Carrión, Thibault Damour, Nathalie Deruelle, Guillaume Faye, John Friedman, Philippe Grandclément, José Maria Ibáñez, José Luis Jaramillo, Jean-Pierre Lasota, Jérôme Novak, Jean-Philippe Nicolas, Motoyuki Saijo, Masaru Shibata, Keisuke Taniguchi, Koji Uryu, Nicolas Vasset and Loïc Villain, for the numerous and fruitful discussions that we had about general relativity and the 3+1 formalism. I also warmly thank Robert Beig and Christian Caron for their invitation to publish this text in the Lecture Notes in Physics series.

Meudon, September 2011 Éric Gourgoulhon

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Acronyms

ADM	Arnowitt–Deser–Misner
BSSN	Baumgarte-Shapiro-Shibata-Nakamura
CMC	Constant mean curvature
CTS	Conformal thin sandwich
CTT	Conformal transverse traceless
IWM	Isenberg–Wilson–Mathews
MHD	Magnetohydrodynamics
PDE	Partial differential equation
PN	Post-Newtonian
TT	Transverse traceless
XCTS	Extended conformal thin sandwich

Contents