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Advances in
IMAGING and ELECTRON PHYSICS

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Beate Meffert

DOGMA OF THE CONTINUUM AND THE CALCULUS OF FINITE DIFFERENCES IN QUANTUM PHYSICS

Volume 137

Advances in

Imaging and Electron Physics

Dogma of the Continuum and the Calculus of Finite Differences in Quantum Physics

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VOLUME 137

ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO
Academic Press is an imprint of Elsevier

Elsevier Academic Press
525 B Street, Suite 1900, San Diego, California 92101-4495, USA
84 Theobald's Road, London WC1X 8RR, UK

This book is printed on acid-free paper. ∞

For all information on all Elsevier Academic Press publications visit our Web site at www.books.elsevier.com

ISBN-13: 978-0-12-014779-3
ISBN-10: 0-12-014779-3

PRINTED IN THE UNITED STATES OF AMERICA
05 06 07 08 09 9 8 7 6 5 4 3 2 1

PREFACE

This is H. F. Harmuth's eighth long contribution to these Advances and it adds a new chapter to his many studies of Maxwell's equations and his more recent preoccupations with finite difference equations instead of differential equations, in which he is joined by B. Meffert. A first examination of these questions formed volume 129 of these Advances and here, the work on quantum mechanics is pursued more deeply. The Klein–Gordon equation is at the heart of this volume but chapters are also devoted to the many difficult and little-studied problems that arise when discreteness is imposed and finite difference equations must be solved.

I am delighted to include this work in these Advances and hope, by doing so, to provoke much discussion among the theoreticians of quantum mechanics.

Peter Hawkes

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FOREWORD

The ancient Greeks did not distinguish between mathematics as a science of the thinkable and physics as a science of the observable. This cast long shadows over the development of physics. As a first example we cite the dogma of the circle. Ptolemy expressed it as follows:

> ... we believe it is the necessary purpose and aim of the mathematician to show forth all the appearances of the heavens as products of regular and circular motion. (Ptolemy, 1952, Almagest, Book III, 1; p. 83, §2)

It is generally assumed that Kepler ended the dogma of the circle, but this is true only for astronomy. The superposition of deferents and epicycles of Ptolemy and Copernicus developed into the Fourier series in complex notation. We meet the old circle under the new name *exponential function* $e^{i\omega t}$ *in the complex plane*. Another circle in disguise is *the character group* $\{e^{iyx}\}$ *of the topologic group of real numbers*. The word *the* in the title provides the connection with Greek thinking. Mathematics justifies only the name *a character group*. ...

A second long shadow was cast by Euclid's geometry. Navigators had been using spherical trigonometry since about 1500 to chart their course across the oceans. But the greatest mathematicians struggled three centuries later with the question of whether Euclid's geometry was the only possible one.

Here we are concerned with a third long shadow, the dogma of the continuum of physical space and time. It can be traced back to the Eleatic school of the Greeks in southern Italy. Zeno of Elea (c. 490–c. 430 BCE) advanced the paradox of the race between Achilles and the turtle as well as that of the arrow that does not fly, to refute the continuum or the infinite divisibility of distances in space and time. Zeno's paradoxes were in turn refuted by Aristotle in his *Physica*. Aristotle's arguments in favor of a mathematical continuum for the physical space and time were so convincing that they were questioned rarely since. The physics of space and time became a branch of mathematics.

Newton demonstrated the perception of physics as a branch of mathematics when he wrote

> Absolute, true and mathematical time, of itself, and from its nature, flows equably without connection with anything external, ... (Newton, 1971, p. 6)

Newton and Leibniz carried the concept of infinite divisibility from the *denumerable infinite* of the Greeks to the *nondenumerable infinite* of differential calculus.

The development of non-Euclidean geometries and the experimental verification of the acoustic Doppler effect changed our thinking about time and space to the concepts used in the special and the general theory of relativity, and beyond. A quotation of Einstein from his later years shows this development:

> But to connect every instant of time with a number, by the use of a clock, to regard time as an one-dimensional continuum, is already an invention. So also are the concepts of Euclidean and non-Euclidean geometry, and our space understood as a three-dimensional continuum. (Einstein and Infeld, 1938, p. 311)

The straightforward proof of a continuum of physical space and time would be the observation of events at two spatial points x and $x + dx$ or two times t and $t + dt$. What is physically possible are observations at x and $x + \Delta x$ or t and $t + \Delta t$, where Δx and Δt may be very small but must be finite. Any finite interval Δx, Δt can be divided into nondenumerably many subintervals dx, dt, which means we are a long way from a mathematical continuum.

If we want to use finite differences Δx, Δt instead of differentials dx, dt we must use the calculus of finite differences instead of the differential calculus. This is a true generalization since no fixed values for Δx, Δt are specified at the beginning of the calculation. When solving for the eigenfunctions of a difference equation in relativistic quantum physics we typically get well-behaved functions if the spatial resolution Δx is large enough, but sequences of random numbers for too small values of Δx. This is how the calculation represents the Compton effect. The theory goes beyond Heisenberg's uncertainty relation since it puts a lower limit on Δx rather than on the product $\Delta x \Delta p$.

Consider elementary particles within the framework of differential calculus. We must match the physical situation to the mathematical method and we do so by defining elementary particles to be "point-like" to avoid giving them any spatial features. Using the calculus of finite differences we must demand only that an elementary particle is smaller than an arbitrarily small but finite distance Δx to avoid any observable spatial feature. The difference theory clearly offers the better choice.

A particle with mass m_0 can become an antiparticle with mass $-m_0$ without a quantum jump in the difference theory. Under certain conditions a finite spatial resolution Δx permits such a transition without violating any physical laws.

Generally, the theorem of Hölder stated in 1887 that the gamma function can be defined by a simple difference equation $\Gamma(x + 1) = x\Gamma(x)$ but by no algebraic differential equation. This implies that differential and difference equations define different classes of functions.

The calculus of finite differences predates the differential calculus since differentials are obtained as limits of finite differences. The success of differential calculus in science and engineering stimulated its enormous development. There were no comparable applications for the calculus of finite differences before its usefulness for relativistic quantum physics was discovered and there was thus little development. Our Bibliography lists only 10 mathematical books on the calculus of finite differences published in the twentieth century[1].

We want to thank Humboldt-Universität of Berlin for help with computer and library services.

Henning F. Harmuth

[1]Our search was limited to books in English, French, German, Russian, and Spanish. We would be grateful for information about books in Chinese or Japanese.

List of Frequently Used Symbols

$\mathbf{A}_e$	As/m	electric vector potential
$\mathbf{A}_m$	Vs/m	magnetic vector potential
$\mathbf{B}$	Vs/m^2	magnetic flux density
b_κ, b_κ^*	–	Eq. (2.5-28)
b_κ^-, b_κ^+	–	Eq. (2.5-29)
c	m/s	299 792 458; velocity of light (definition)
$D_{\kappa i}(\theta)$	–	Eqs. (6.3-17)–(6.3-21)
$\hat{D}_{\kappa i}(\theta)$	–	Eqs. (6.3-25)–(6.3-28)
$\tilde{D}_{\kappa i}(\theta)$	–	Eqs. (6.4-6)–(6.4-9)
$\mathbf{D}$	As/m^2	electric flux density
$d(\kappa)$	–	Eq. (2.5-18)
$d_{\kappa i}, d_{\kappa i}(\theta)$	–	Eqs. (6.3-47)–(6.3-50)
$\mathbf{E}, E$	V/m	electric field strength
E	VAs	energy
e	As	electric charge
$F(\zeta)$	–	Eqs. (2.3-7), (2.3-15)
$\mathbf{g}_e$	A/m^2	electric current density
$\mathbf{g}_m$	V/m^2	magnetic current density
$G_1(\zeta, \theta)$	–	Eq. (3.2-4)
$G_2(\zeta, \theta)$	–	Eq. (4.2-3)
$G_{c\kappa}(\theta)$	–	Eq. (3.2-32)
$G_{s\kappa}(\theta)$	–	Eq. (3.2-33)
$\mathbf{H}, H$	A/m	magnetic field strength
$H_{c\kappa}(\theta)$	–	Eq. (6.3-29)
$H_{s\kappa}(\theta)$	–	Eq. (6.3-1)
$\mathcal{H}$	–	Hamilton function
h	Js	$6.626\,075\,5 \times 10^{-34}$, Planck's constant
$\hbar = h/2\pi$	Js	$1.054\,572\,7 \times 10^{-34}$
I_T	–	Eq. (2.4-29)
m_0	kg	rest mass
N	–	$T/\Delta t$, Eq. (2.2-6)
$p_1(\zeta, \theta)$	–	Eq. (4.2-7)
p_C	–	Eq. (4.4-10)
$p_N(\zeta, \theta)$	–	Eq. (4.2-5)
Q	–	Eq. (1.1-45)
$\tilde{r}$	–	$\Delta\tilde{r}$, Eq. (5.5-2)
$S_\kappa(\theta)$	–	Eqs. (3.2-20), (3.2-54)

(Continued)

s	V/Am	magnetic conductivity
T	s	arbitrarily large but finite time interval
$T_\kappa(\theta)$	–	Eqs. (3.2-20), (3.2-55)
t	s	time variable
Δt	s	arbitrarily small but finite time interval
U	VAs	Eq. (2.5-2)
$\hat{u}(\zeta,\theta)$	–	Eqs. (3.2-8), (3.2-10)–(3.2-13)
V	m/s	velocity
$\hat{v}(\zeta,\theta)$	–	Eqs. (3.2-20), (3.2-21)
$Z = \mu/c$	V/A	376.730 314; wave impedance of empty space
Z	–	1, 2, ...; charge number
α	–	$Ze^2/2h7.297\ 535 \times 10^{-3}$, Eq. (1.1-45)
α_e	–	$ZecA_e/m_0c^2$, Eq. (1.1-45)
$\tilde{\alpha}$	–	Eq. (5.5-2)
β_κ	–	Eqs. (2.4-11), (2.4-13), (2.4-14)
$\tilde{\gamma}$	–	$4\pi Z\alpha$, Eq. (5.5-2)
$\tilde{\delta}$	–	Eq. (5.5-2)
$\epsilon = 1/Zc$	As/Vm	$1/\mu c^2$; permittivity
$\tilde{\Delta}$	–	symbol for difference quotient: $\tilde{\Delta}F/\tilde{\Delta}x$, Eq. (1.2-1)
$\tilde{\Delta}_l$	–	left difference quotient, Eq. (1.2-5)
$\tilde{\Delta}_r$	–	right difference quotient, Eq. (1.2-4)
Δ	–	symbol for finite difference: $x + \Delta x$
ζ	–	$x_j/c\Delta t$, normalized distance; Eqs. (2.2-6), (2.3-1)
θ	–	$t/\Delta t$, normalized time; Eq. (2.2-6)
ι	–	Eq. (2.3-18)
κ_0	–	Eq. (2.4-32)
λ_1	–	$ec\Delta tA_{m0x}/\hbar$; Eqs. (2.3-2), (3.2-36)
λ_2	–	Eqs. (2.3-2), (3.2-36)
λ_3	–	ϕ_{e0}/cA_{m0x}; Eqs. (2.3-2), (3.2-37)
λ_C	m	$h/m_0c = 8.89 \times 10^{-15}$ for π^+ and π^-, Eq. (1.1-45)
$\tilde{\lambda}$	–	Eq. (5.5-2)
$\mu = Z/c$	Vs/Am	$4\pi \times 10^{-7}$; permeability
ρ_e	As/m^3	electric charge density
ρ_m	Vs/m^3	magnetic charge density
ρ_r	–	$\tilde{\alpha}r$, Eq. (5.5-1)
ρ_κ	–	constant, Eq. (2.3-25)
σ	A/Vm	electric conductivity, Eq. (1.1-7)

ϕ_e	V	electric scalar potential
ϕ_m	A	magnetic scalar potential
φ_κ	–	Eq. (2.3-38)
Ψ_0, Ψ_1	–	Eq. (2.1-38)
Ψ_x	–	Eq. (2.1-8)
Ψ_{x0}, Ψ_{x1}	–	Eq. (2.1-8)
Ψ_{x0x_j}	–	Eq. (2.1-25), (2.2-26), (2.1-37)
Ψ_{x1x_j}	–	Eq. (2.1-37)