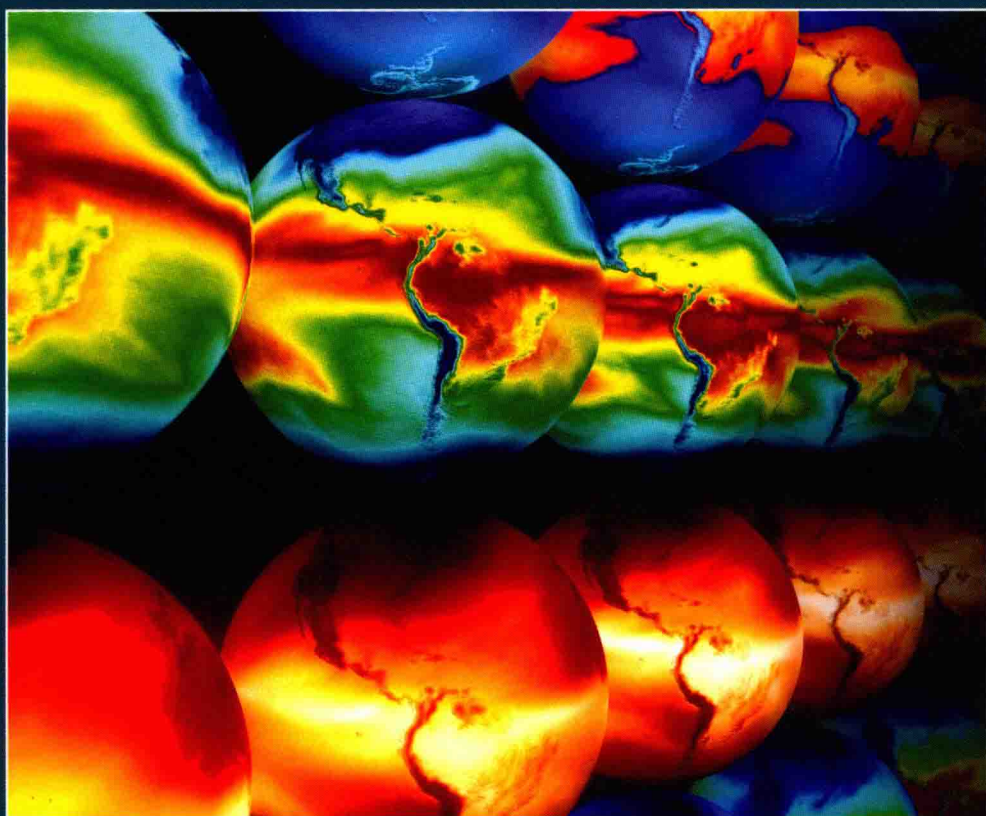


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Climate Modeling for Scientists and Engineers



John B. Drake

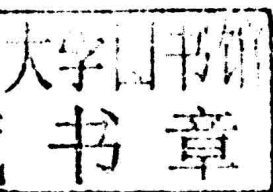
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Mathematical Modeling and Computation

Climate Modeling for Scientists and Engineers

John B. Drake

University of Tennessee
Knoxville, Tennessee



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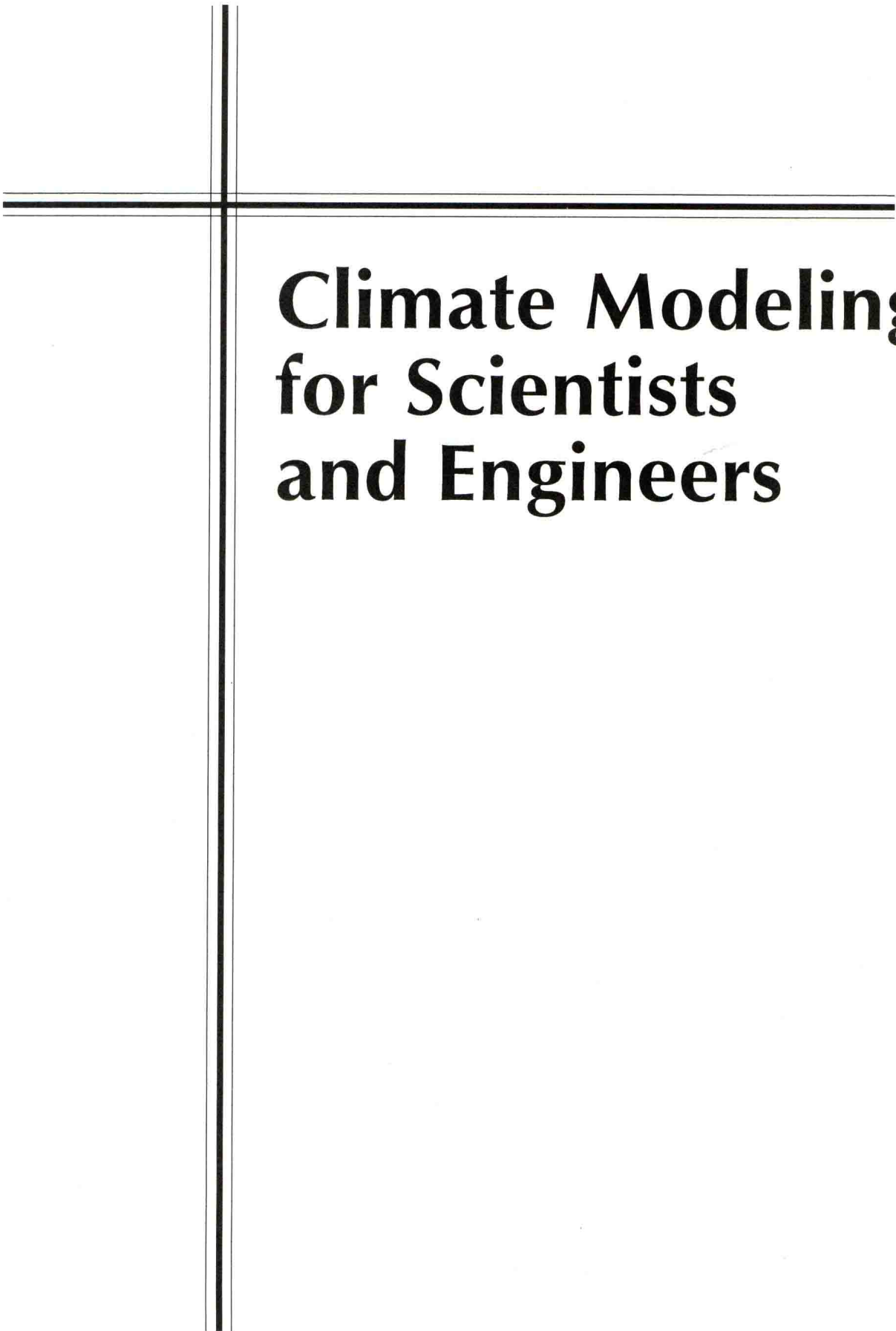
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Climate Modeling for Scientists and Engineers

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Preface

Many excellent textbooks describe the physics of climate and give an introduction to the processes that interact and feedback to produce the earth's weather and climate [178, 79, 133, 94, 83, 134]. In this book we approach the subject from another direction, admitting from the outset that the definition of climate is nebulous and, in fact, still evolving. The climate will be viewed as multifaceted but always as the solution of a particular mathematical model. That all climate models are incomplete is a consequence of our lack of understanding of the physics as well as the incompleteness of mathematical knowledge.

The understanding of the nature of the earth's climate has improved and changed as the mathematical notions of what constitutes a solution to a set of partial differential equations has changed.¹ The development of theories of deterministic chaos and the existence of global attractors as invariant subsets of the time evolution associated with the Navier-Stokes equations have had a profound influence on climate research [165]. The invariants of flows, what is conserved, and what coherent structures persist inform the theory of climate. How oscillations, bifurcations, and singularities arise in the solutions of partial differential equations is fundamental to drawing a line between natural climate variability and human induced climate change. A SIAM focus for 2013 on the "Mathematics of Planet Earth" was marked with the notable publication of Kaper and Engler's text [100] treating many of the current conceptual models of climate.

The role of general circulation models and high-end computer simulation in the understanding of climate should not be underestimated. Describing the principles and practice of this type of modeling is the primary focus of this book. The first chapter is devoted to the observations of weather phenomena and the special programs to collect climate data that provide a wealth of information for calibration, validation, and verification. The data themselves, however, do not provide the interpretation of climatic events or give a means of projecting future climate responses. Only high-end models show how the processes interact and feedback upon one another culminating in weather phenomena and climate. Chapter 2 introduces the governing equations of geophysical flow that model the circulations of the atmosphere and ocean. Chapter 3 introduces numerical methods for solving these equations starting from a simplified subset known as the shallow water equations. High performance computing is a uniquely powerful tool to probe the solutions of the equations that constitute the model, and this is introduced in Section 3.9. Numerical methods and algorithms are the backbone of simulations based on general circulation models. Attention is given to parallel algorithms and promising numerical methods that will likely figure in the next generation of climate models.

Chapter 4 describes what has been learned from climate simulations, and a few case studies are presented with the hope that interested readers and students will pursue other

¹The mathematical theory for atmospheric and ocean flows is not complete [26, 164], so there is still room for growth.

simulation studies following their own interests. Finally, a brief chapter introduces some of the methods and mathematical basis for the analysis of climate. These methods must be used to summarize simulation results and, of course, are useful in analyzing weather data to extract climate statistics and trends.

Exercises are scattered throughout the text as well as references to MATLAB codes that are part of these exercises and are described in supplemental material available online at www.siam.org/books/MM19. Since methods and simulation are a thread throughout the material, students wishing to master the material should gain experience with computer simulation and programming through these exercises. Full-fledged simulations using parallel computers requires more sophisticated programming than the MATLAB environment offers. Usually simulation codes are written in FORTRAN and C++. But access to the full code and simulation system of the Community Climate System Model is available to the ambitious reader. For analysis, Python or the NCAR Command Language (NCL) are the languages of choice.

Reference is also made to Supplemental Lectures [49], provided in a separate online volume at www.siam.org/books/MM19. These lectures each start with something we know fairly well, usually some piece of mathematics, but then branch out to things we do not understand well. The supplemental lectures serve as a somewhat light-hearted introduction to research areas where open questions still exist and important perspectives are emerging. Students seem to appreciate the change of pace offered in these lectures.

The book is the result of a course sequence taught at the University of Tennessee-Knoxville, in the Civil and Environmental Engineering graduate studies department. I am grateful to all the graduate students who have asked questions and provided input on my lectures. In particular, thanks to Dr. Nicki Lam, Dr. Yang Gao, Dr. Abby Gaddis, Ms. Melissa Allen, Dr. Evan Kodra, Mr. Scott DeNeale, Dr. Abdoul Oubeidillah, Mr. Jian Sun, and my colleagues Dr. Joshua Fu and Dr. Kathrine Evans. I am indebted to the many exceptional researchers that I have worked with, learned from, and been inspired by at the Oak Ridge National Laboratory Climate Change Science Institute and at the National Center for Atmospheric Research. I owe a particular debt to Dr. David Williamson and Dr. Warren Washington of the National Center for Atmospheric Research in Boulder. My admiration for Dr. Washington's book [178] will be evident throughout this text. In the DOE National Laboratories, my colleagues Dr. Patrick Worley, Dr. David Bader, and Dr. Jim Hack have been pioneers in the development of this computational science discipline. Finally, I wish to thank my wife, Frances, for supporting me in this project and providing editorial suggestions.

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Chapter 1

Earth Observations

Few moments in history stand in such sharp contrast as the moment captured in Figure 1.1: the natural world in plane view from the most unnatural environment of space. The questioning of science's ability to provide answers for our future is part of the post-modern social fabric in which the climate change discussion is taking place and climate modelers are questioned intensely. It has been said that if you believe models, you will believe anything. Yet in the quiet eye of the storm the discussion hinges on things we know and believe, on physical, chemical, and biological processes, on cause and effect. If we take an engineering and scientific perspective, we begin with the principles behind the dynamics and physics of the climate system. The implications of climate change, as they are currently understood, are strong motivation for the study that leads ultimately to the question, "So, what are we going to do about it."

Geologic time scales indicate large variations in earth's climate. The recent (geologic) past is known from ice core data obtained by drilling deep into the Antarctic ice sheet. The research sites at Vostok Station and the European Project for Ice Coring in Antarctica (EPICA) Dome C, have produced a record extending back for the past 800,000 years. What the cores reveal is a series of eight ice ages and interglacials (the warm periods between the ice expansions) (Figure 1.2). During these variations the temperature ranges from -10°C to $+4^{\circ}\text{C}$ from the modern baseline. The concentration of atmospheric carbon dioxide, CO_2 , varies from 180 parts per million (ppm) to 300 ppm. Our present warm period began to develop 30,000 years ago and, looking at the frequency of past variation, a signal operating on the 23,000 year period emerges. According to this signal, we are due for a cooling trend and should soon be heading into another ice age.

The timing of the observed variations are consistent with Milankovich cycles [89] and the main theory of climate change—that the climate is forced by variations in the earth's orbit and the intensity and orientation of the solar input. If summer and winter are caused by changes in the solar angle and nearness of the earth to the sun, then orbital precession, which has a 23,000 year cycle, and orbital eccentricity, with 100,000 year cycles, are likely causes of the longer term variations in Figure 1.3.² Calculations of the amount of change in the solar insolation given the variations of the orbital parameters suggest that the climate is quite sensitive to these changes [100, 131].

Looking more deeply into the past, we know that other forces have also been at work. Some 600 million years ago, a single super continent, Pannotia, accounted for the earth's

²See [178, Figure 2.3] for the tilt angle (23.45°) of the earth and the picture of the wobbling top that points the axis of rotation at the North Star. The axis will not point at the North Star in the future.



Figure 1.1. NASA Apollo 8, the first manned mission to the moon, entered lunar orbit on Christmas Eve, December 24, 1968. Said Lovell, “The vast loneliness is awe-inspiring and it makes you realize just what you have back there on Earth.” At the height of the technical achievement of space travel, a question is asked. Reprinted courtesy of NASA.

land mass. In what appears to be an oscillation between single and dispersed continents, this super continent broke up and reassembled 250 million years ago to form Pangea with the Appalachian mountains in its geographic center. The present locations of the continents are a result of plate tectonics from the breakup of Pangea [35]. Possibly caused by climate stresses in the Permian–Triassic period 251 million years ago, a mass extinction occurred. The high latitude temperatures were 10–30 °C above present temperatures and recovery took 30 million years. The Cretaceous–Tertiary extinction of 65 million years ago, possibly the result of a large asteroid impact or increased volcanism, is responsible for the disappearance of dinosaurs [177].

Climate changes that have occurred as earth warmed from the last ice age are also responsible for some familiar extinctions. The Younger Dryas event 12,900 years ago saw the extinction of mammoths and the North American camel and the disappearance of the Clovis culture from North America. Warm periods are often called optimums, and the Holocene climatic optimum occurred from 9,000–5,000 years before the present. During this time the Sahara was green. With a temperature increase of +4 °C in the high latitudes, the United States Midwest was a desert. The Medieval Warm period occurred from 800–1300 CE and what has been called the Little Ice Age (–1 °C cooling) occurred soon after. Without systematic records or temperature proxy data it is hard to tell the

extent of regional climate change and the Little Ice Age may have been a localized cooling of the European region not reflecting global conditions.

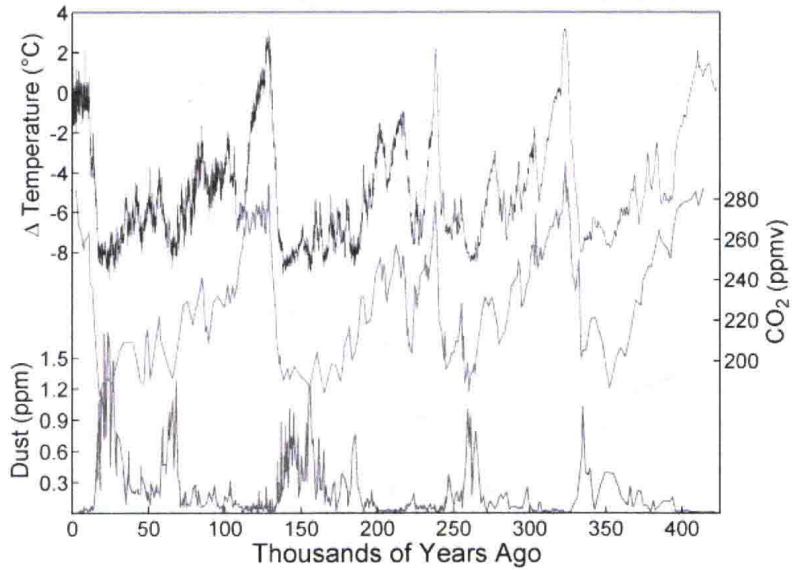


Figure 1.2. Vostok Temperature, CO₂ and dust from ice cores. Reprinted courtesy of NOAA (www.ngdc.noaa.gov/paleo/icecore/antarctica/vostok/).

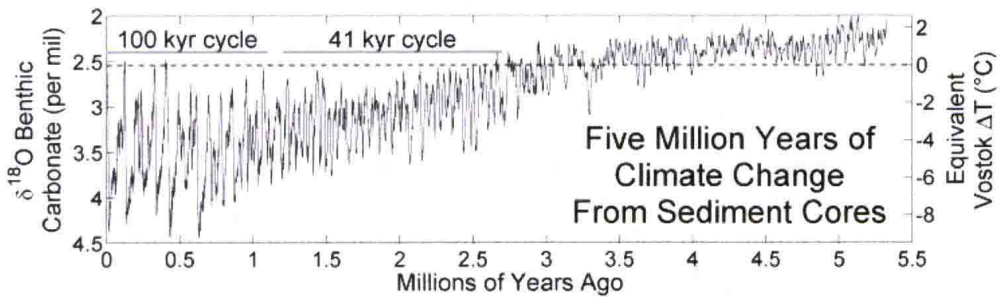


Figure 1.3. Five million years from climate record constructed by combining measurements from 57 globally distributed deep sea sediment cores. Reprinted with permission, Robert A. Rhode, *Global Warming Art*.

Exercise 1.0.1 (Younger Dryas). What was the Younger Dryas event? When did it start and how long did it last? How was the event characterized? What are the theories about its cause?

Exercise 1.0.2 (Vostok ice cores). The Vostok ice core data shows what period? How can you get the data? After obtaining the data plot CO₂ versus ΔT and dust versus ΔT . According to

the data, what should the present ΔT be based on a current CO_2 concentration of 400 ppm? What about for 450 ppm? How would you characterize the “error bound” on the estimate?

1.1 ■ Weather and Climate Records

The historical period of direct observations begins around 1850. After World War II, observation networks started to develop from twice daily rawinsondes³ launched at each major airport. The World Meteorological Organization (WMO)⁴ was formed in 1950. A variety of measurement campaigns have been launched to advance our understanding of weather and climate phenomena, for example, the Global Atmospheric Research Program (GARP), the First GARP Global Experiment (FGGE), the Tropical Ocean and Global Atmosphere Program (TOGA), the International Satellite Cloud Climatology Project (ISCCP), the World Ocean Climate Experiment (WOCE), the Global Energy and Water Cycle Experiment (GEWEX), the International Geosphere-Biosphere Program (IGBP), and the International Polar Year (IPY).

1.1.1 ■ Ground-based Weather Data

The National Climatic Data Center in Asheville, North Carolina⁵, is responsible for collecting and storing all weather data for the United States. A typical weather station reports the following:

- air temperature including daily maximum and minimum ($^{\circ}\text{C}$ or $^{\circ}\text{F}$),
- barometric pressure (inches or mm of mercury or hPa or atmospheres),
- surface wind speed and direction (mph , knots or m/sec),
- dew point and relative humidity⁶,
- precipitation, and
- snowfall and depth.

The data from over 40,000 sites worldwide is available through the Global Historical Climate Network (GHCN).⁷ The GHCN-Daily dataset serves as the official archive for daily data from the Global Climate Observing System (GCOS) Surface Network (GSN). It is particularly useful for monitoring the frequency and magnitude of extremes. The dataset is the world’s largest collection of daily weather data.

The highest concentration of observations are from continental land masses with relatively fewer historical measurements for the oceans. Ocean going vessels log meteorological observations including⁸

³A weather balloon that measures T , p , and humidity from 0 to 30km height. Wind velocity is deduced from the drift path of the balloon.

⁴www.wmo.int

⁵www.ncdc.noaa.gov/oa/climate/stationlocator.html

⁶ $RH = \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2\text{O}}^*}$, where the pressures are partial pressure and saturation pressure of water. This may be approximated using the temperature and dew point temperature. Dew point is the temperature at which the water vapor condenses under constant temperature and is measured with a wet bulb thermometer.

⁷www.ncdc.noaa.gov/oa/climate/ghcn-daily

⁸www.sailwx.info/index.html

- sea surface temperature,
- air temperature,
- barometric pressure,
- surface wind speed and direction, and
- wave height.

Since shipping routes do not offer a uniform coverage of the ocean and do not monitor any single point continuously through time, the data must be processed to provide maps of temperature at a given time.

The weather maps and other products derived from the surface network observing system may be obtained from a number of sources. The following is a list of a few relevant links from the United States:

- Unisys weather⁹,
- Weather Underground¹⁰,
- National Center for Atmospheric Research¹¹,
- Geophysical Fluid Dynamics Laboratory¹²,
- Earth System Research Laboratory¹³, and
- University of Wisconsin Space Science and Engineering Center¹⁴.

Comprehensive datasets are also collected by countries other than the United States. For example, the Climate Research Unit at East Anglia University¹⁵ maintains an independent, comprehensive collection of weather and climate observations. Multiple sets of observations and processing techniques provide a scientific check on the reliability of any single source.

1.1.2 • Climate Data

Creating a consistent set of climate data from weather observations is not a simple matter. Several sources provide their best estimates of conditions on a latitude-longitude (lat-lon) grid for daily and monthly averaged periods. These “reanalysis” datasets are particularly useful in verification of climate models. The following two datasets are widely used:

- the National Centers for Environmental Prediction (NCEP) Reanalysis Data.¹⁶

⁹weather.unisys.com/

¹⁰www.wunderground.com/.

Also see Ricky Rood’s climate change blog www.wunderground.com/blog/RickyRood/.

¹¹www.ncar.ucar.edu

¹²www.gfdl.noaa.gov

¹³www.esrl.noaa.gov

¹⁴www.ssec.wisc.edu/data/

¹⁵www.cru.uea.ac.uk/data/

¹⁶www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.html

- the ERA-40 archive¹⁷ from the European Centre for Medium Range Weather Forecasting (ECMWF) spans forty years starting in the mid-1950s at 2.5° horizontal resolution. Data at different atmospheric heights are part of this record and the derived variables offer more comprehensive coverage than the available measurements.

Climatologies and summaries of weather and satellite observations for the US are available from the Earth System Research Laboratory (ESRL).¹⁸ Climate science relies heavily on the available observations. Familiarity with available data holdings provides a rich source of insight and fertile ground for research ideas.

1.1.3 • Interpolation Basics

How does randomly distributed data (in space, time, and instrumental method) get manipulated to a usable format for analysis? The answer, in one form or another, is *approximation*. We want to approximate a function f by another function g that is “close” to f but more usable. Two functions are close as measured by a functional norm $\|\cdot\|$, so approximation is summarized by the following statement: given f find g so that

$$\|f - g\| < \epsilon, \quad (1.1)$$

where $\epsilon > 0$ is a relatively small error in the approximation. If we think of the function f as the “actual” surface temperature of the earth, then observations sample this function in space and time. Of course, the measurement has some error in it but we will ignore this. Over a day’s time, the temperature at any given point will vary quite a bit (*the diurnal variation*). A more usable function is the daily or monthly average temperature. The function g used as an approximation, in this case, is piecewise constant over a day or a month although f is presumed to be continuous in time.

Spatial approximations have some of the same characteristics. For example, the values of the temperature on a regular grid of lat-lon coordinates are convenient for analysis, but since the observation network is not regular, the values of f at the grid points must be approximated, for example, as an average of observed values near the grid point. A simple linear interpolation among grid points could fill in all the places where no observations exist. This is the process used in coloring contour maps. What should be noted is that for some approximate functions g , no observation data coincide exactly in time or space. An error or tolerance in the approximation, ϵ , is thus unavoidable.

One-dimensional linear interpolation fits a set of points $(x_i, f(x_i))$ with a piecewise linear interpolant g such that $g(x_i) = f(x_i)$. Then g may be evaluated at any point $g(x)$. Interpolation is a specialized form of approximation where the norm is the discrete maximum norm, $\|f - g\| = \max_i |f(x_i) - g(x_i)|$ and $\epsilon = 0$. This says nothing about how good the approximation is at points not included in the discrete set.

A more accurate interpolant is obtained by choosing the interpolant from a basis set of functions that span the space that the function f lives in. For example, we may interpolate single variable functions using a polynomial basis $[1, x, x^2, x^3, x^4, \dots, x^n]$. The function $g(x) = \sum_{k=0}^n c_k x^k$ shows the form of a higher order interpolant with polynomial coefficients c_k . The interpolation condition requires that g match f at given points. The following is a set of simultaneous equations results that determine the values of the c_k :

$$c_0 + c_1 x_i + c_2 x_i^2 + c_3 x_i^3 + \dots + c_n x_i^n = f(x_i) \text{ for each } i = 0, n. \quad (1.2)$$

¹⁷ www.ecmwf.int/research/era/do/get/era-40

¹⁸ www.esrl.noaa.gov/psd/data/usclimate

Once the c_k 's are determined, g may be evaluated at any point of interest.

The choice of powers of x as the basis set is far from optimal and the solution of the linear system (1.2), called the Vandermonde system, will run into numerical difficulties. This system is particularly ill-conditioned. A better conditioned choice of basis functions for the polynomials are the Lagrange polynomials,

$$l_k(x) = \prod_{i=0(i \neq k)}^n \frac{(x - x_i)}{(x_k - x_i)}. \quad (1.3)$$

The function $g(x) = \sum_{k=0}^n c_k l_k(x)$ and the solution to the Vandermonde system is simply $c_k = f(x_k)$. The resulting Lagrange interpolant is

$$g(x) = \sum_{k=0}^n f(x_k) \prod_{i=0(i \neq k)}^n \frac{(x - x_i)}{(x_k - x_i)}. \quad (1.4)$$

Combining this approach with piecewise interpolation, where an interpolant is constructed for each sub-interval of the domain, gives efficient and accurate interpolants.

But this is for one-dimensional data and we are interested in functions on the sphere. Interpolation on a lat-lon grid can use two-dimensional methods, but care must be taken at the poles to avoid unnecessary oscillations, since the pole is a single point rather than a line, as it appears on most maps. Interpolating velocities is particularly tricky since the velocity (or any vector quantity on a lat-lon grid) has a singularity at the poles. This leads to the question, what are the natural or optimal basis for approximating functions on the sphere? This question will be answered in section 3.5.2.

Exercise 1.1.1 (NCEP observational data). *Interpolate the NCEP monthly mean data to produce a time series for Knoxville, TN or a location of interest to you. Compare with station data. What are the reasons for the differences?*

1.2 • Satellite Observations Since 1979

The first weather satellite, the Vanguard 2, was launched on February 17, 1959, but it primarily provided pictures of cloud patterns. As future missions added other instruments and sensors, the data became useful for weather and climate purposes. The NASA Nimbus satellites began launching in 1964 and were the first to collect data on the earth's radiation budget, that is the basic forcing that drives climate. The data since 1979 includes sea surface temperatures and ice extent. The next few subsections are devoted to some particular datasets derived from satellite data; some are supplemented with ground-based observations. One of the important tasks of the last few decades has been to reconcile these two records where they are ambiguous. Since the satellite sees things from the top of the atmosphere and ground-based observations are from the bottom of the atmosphere, the accounting for differences is at the intersection of atmospheric and space science. Much of the NASA satellite data are available online¹⁹ along with an excellent image library.

1.2.1 • The Sea Surface Temperature

For many years the Levitus dataset²⁰ has been the gold standard for ocean temperature and salinity data. Figure 1.4 shows the surface temperature climatology.

¹⁹<http://earthobservatory.nasa.gov>

²⁰http://gcmd.nasa.gov/records/GCMD_LEVITUS_1982.html

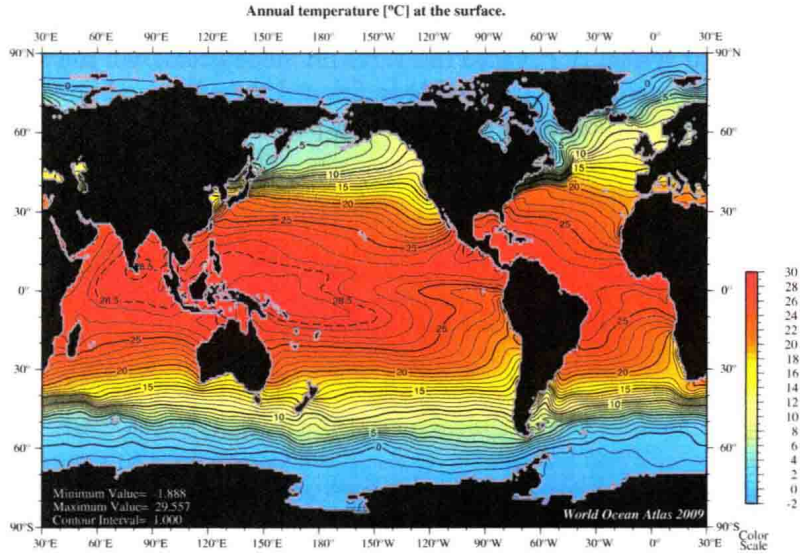


Figure 1.4. Annual average ocean surface temperature based on Levitus data. Note the warm pools in the western Pacific and the Indian ocean. Reprinted courtesy of NOAA.

Many products have been derived from this dataset and it is used to initialize and provide SST boundary conditions for climate simulations and weather forecasts. For example, the Program for Climate Model Diagnosis and Intercomparison (PCMDI)²¹ compares atmospheric models (atmospheric model intercomparison projects (AMIPs)) and coupled models (coupled model intercomparison projects (CMIPs)) for the international community. The standard comparisons require that simulations use the same boundary conditions and forcing. A data archive of SSTs used to drive the models for a standard reality check experiment is provided.

The satellite-based records have been supplemented by measurements at depth from Argo floats since 2000.²² The float data provide the instantaneous state of the ocean for the first time. This new observational record is the basis for a better understanding of ocean circulation patterns and of the southern oceans in particular. Coupled ocean and atmosphere climate models will use this new information to produce better forecasts on a seasonal to decadal timescale.

1.2.2 • Biosphere and Land-Use Data

Travelers to a distant planet would no doubt be curious about the temperature, the composition of the atmosphere, and the geography of the continents and oceans, but they would be most curious about the vegetation covering the land and the life in the seas. In the 1990s, NASA started a program called *Mission to Planet Earth* that began to survey the globe from space, building an Earth Observing System (EOS). The observing satellite Terra successfully launched in December 1999 and the Aqua spacecraft launched in May 2002. The Moderate Resolution Imaging Spectroradiometer (MODIS) is one of the instruments aboard the Terra (EOS AM) and Aqua (EOS PM) satellites. Terra and

²¹www.pcmdi.llnl.gov/projects/amip/

²²www-argo.ucsd.edu