Holger Hermanns

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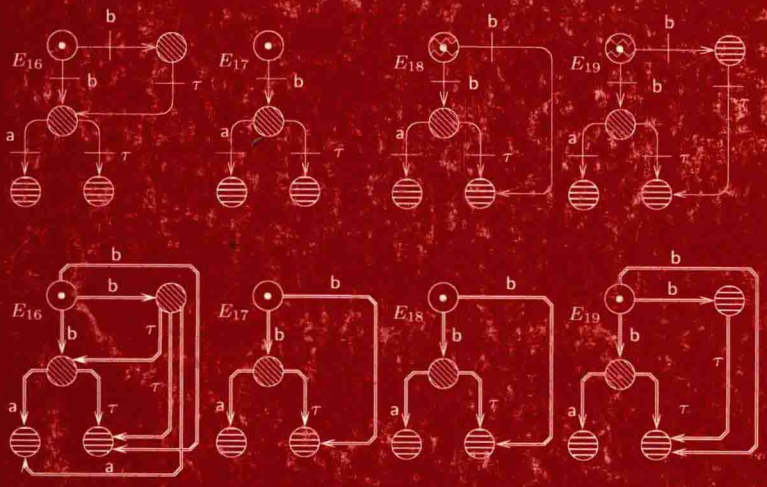
Interactive Markov Chains

And the Quest for Quantified Quality



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Interactive Markov Chains

Markov chains are widely used as stochastic models to study a broad spectrum of system performance and dependability characteristics. This monograph is devoted to compositional specification and analysis of Markov chains.

Based on principles known from process algebra, the author systematically develops an algebra of interactive Markov chains. By presenting a number of distinguishing results, of both theoretical and practical nature, the author substantiates the claim that interactive Markov chains are more than just another formalism: Among other topics, an algebraic theory of interactive Markov chains is developed, algorithms to mechanize compositional aggregation are devised, and state spaces of several million states resulting from the study of an ordinary telephone system are analyzed.

This monograph serves as both a source of inspiration and reference for the growing number of R&D professionals interested in performance analysis and formal methods.

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Foreword

To devise methods for the construction of high quality information processing systems is a major challenge of computer science. In most contexts, however, the definition of what constitutes (high) quality in a more concrete sense is problematic, as invariably any definition seems to fall short of its essence. Computer science proves no exception to the rule, and its quest for quality in relation to the analysis of system designs has given birth to two main interpretations: quality as correctness, and quality as performance.

The first interpretation assesses quality by showing formally that (a model of) a system satisfies the functional requirements of its formal specification. Its methods are rooted in logic and discrete mathematics, and are based on the all-or-nothing game imposed by the Boolean lattice: unless satisfaction has been demonstrated completely, nothing can be said. This is both the strength and the weakness of the approach: results have the utmost precision, but are hard to obtain.

The second interpretation aims to assess quality on a continuous scale that allows for quantification: using stochastic system models one tries to calculate system properties in terms of mathematical expectation, variation, probability, etc. The strong point of this approach is that it allows for quality in other than absolute terms, e.g. a message loss of less than 0.01%, service availability of more than 99.99%, etc. Its weaker side is that it cannot handle very well system properties that are not directly related to repeatable events, including many functional system properties, such as e.g. absence of deadlock, reachability of desirable system states, etc.

It is clear that the analysis of the quality of system designs must ultimately encompass both of the approaches above. A first step in this direction was the development of stochastic Petri net models, which combine a classical functional model for (concurrent) systems with stochastic features. The latter allow the derivation of performance models in the form of continuous-time Markov chains directly from a system description using such nets. Thus the formalism in principle allows functional and performance analysis of systems in terms of an integrated model and perspective.

This potentially great leap forward from the existing practice of studying correctness and performance through unrelated models (and by different scientific communities) proved harder to materialise than was initially hoped for. One of the main causes was the infamous state space explosion: the fact that the number of global states of a system grows exponentially with the number of components of the system. Because of this, non-trivial system designs give rise to large Petri net models, which in turn yield huge Markov models that can no longer be effectively manipulated, even with the aid of computers.

In the early 1990s this observation motivated the study of what is now referred to as *stochastic process algebras*. In the preceding decade process algebras had proven an effective means for the modelling and analysis of the functionality of concurrent systems. They address the problem of state space explosion by a powerful formalisation of system composition by process algebraic operators, combined with the study of *observational congruence* of behaviours. The latter allows for a *compositional* control of state space complexity: replacing components with observationally congruent but simpler components the state space can be reduced without explicitly generating it first.

The study of stochastic process algebra has for a considerable part been driven by the non-trivial question of how best to add stochastic features to process algebra, combining sufficient stochastic expressivity with compatibility with existing process algebraic theory. The present LNCS volume by Holger Hermanns contains his answer to this question for Markovian process algebra, i.e., where the stochastic model of interest is that of continuous-time Markov chains. Written in a clear and refreshing style it demonstrates that it is not only Hermanns' answer, but really 'the' answer.

Where others before him treated stochastic delay as attributes of system actions, Hermanns saw the enormous advantages of a completely different approach: treating delays as actions in their own right that silently consume exponentially distributed amounts of time, and treating system actions as instantaneous actions. This separation of concerns bears all the signs of a great idea: it is (retrospectively) simple and leads to very elegant results. A complication that mars the other approaches, viz. the synchronisation of delays as a by-product of synchronising actions, is completely avoided. Only system actions are subject to synchronisation, and delays in different components of a system are interleaved. Due to the memoryless nature of exponential distributions this yields a perfectly natural interpretation of the passage of (stochastic) time. It is the Platonic discovery that interleaving process algebra and Markov chains are a perfect couple. Another advantage of the decoupling of system actions and delays is that there can be more than one delay preceding an action. This extends the class of (implicit) action delays far beyond that of the exponential distribution, viz. to the (dense) class of phase-type distributions.

The author must be commended for the technical skills with which he has reaped the full benefits of this idea. In addition to defining and applying his formalism, he has also firmly embedded it in standard process algebraic theory by providing full axiomatisations for the stochastic varieties of observation congruence which are conservative extensions of the non-stochastic cases. Also, the link between the concepts of lumpability in Markov chains and bisimilarity in process algebra that was first observed by Hillston, comes to full fruition in the hands of Hermanns. Based on standard algorithms for bisimulation a low complexity algorithm is devised for lumping (Markov

chain) states that can be applied compositionally. The latter is a fine example of the advantages of interdisciplinary research, as such an algorithm was not available in the standard theory of Markov chains.

We believe that this monograph by Holger Hermanns represents an important step in the quest for the integrated modelling and analysis of functional and performance properties of information processing systems. It is also written in a very accessible and, where appropriate, tutorial style, with great effort to explain the intuition behind the ideas that are presented. With a growing number of researchers in the performance analysis and formal methods communities that are interested in combining their methods, we think that this monograph may serve both as a source of inspiration and a work of reference that captures some vital ingredients of quality.

May 2002 Ed Brinskma, Ulrich Herzog

Preface

Markov chains are widely used as stochastic models to study and estimate a broad spectrum of *performance and dependability characteristics*. In this monograph we address the issue of *compositional specification* and analysis of Markov chains. Based on principles known from *process algebra*, we develop an algebra of *Interactive Markov Chains* (IMC) arising as an *orthogonal* extension of both continuous-time Markov chains and process algebra. In this algebra the interrelation of delays and actions is governed by the notion of *maximal progress*: Internal actions are executed without letting time pass, while external actions are potentially delayed. IMC is more than 'yet another' formalism to describe Markov chains. This claim is substantiated by a number of distinguishing results of both theoretical and practical nature. Among others, we develop an *algebraic theory* of IMC, devise *algorithms* to mechanise *compositional aggregation* of IMC, and successfully apply these ingredients to analyse state spaces of several million states, resulting from a study of an ordinary telephone system.

The contents of this monograph is a revised version of my PhD thesis manuscript [96] which I completed in spring 1998 at the University of Erlangen, Germany. I am deeply indebted to Ulrich Herzog and Ed Brinksma for their enthusiastic support when preparing its contents, and when finalising this revision at the University of Twente, The Netherlands.

Many researchers had inspiring influence on this piece, or on myself in a broader context, and I take the opportunity to express my gratitude to all of them. I am particularly happy to acknowledge enjoyable joint research efforts with Christel Baier, Salem Derisavi, Joost-Pieter Katoen, Markus Lohrey, Michael Rettelbach, Marina Ribaudo, William H. Sanders, and Markus Siegle which have led to various cornerstones of this book. Henrik Bohnenkamp, Salem Derisavi, and Marielle Stoelinga read the manuscript carefully enough to spot many flaws, and gave me the chance to iron them out in this monograph. Cordial thanks go to Alfred Hofmann at Springer-Verlag for his support in the process of making the manuscript a part of the LNCS series. And finally, there is Sabine and the tiny crowd. Those who know her are able to assess how perfectly happy I account myself.

June 2002 Holger Hermanns

Contents

1. Introduction

1.1 Performance and Dependability Estimation with Markov Chains

The purpose of this book is to provide a *compositional* methodology of modelling and analysis with *Markov chains*. Markov chains are widely used as simple and yet adequate models in many diverse areas, not only in mathematics and computer science but also in other disciplines such as operations research, industrial engineering, biology, demographics, and so on. Markov chains can be used to estimate performance and dependability characteristics of various nature, for instance to quantify throughputs of manufacturing systems, locate bottlenecks in communication systems, or to estimate reliability in aerospace systems.

It is often possible to represent the behaviour of a system by specifying a discrete number of states it can occupy and by describing how the system moves from one state to another as time progresses. If the future evolution of the system only depends on its present state, the system may be represented as a (time homogeneous) Markov chain. If the future evolution depends in addition on some *external* influence, we fall into the basic model class considered within this monograph. We take the view that the evolution of a system can be the result of *interaction* among different parts of the system. We provide means to specify these parts, as well as combinators to compose parts. In this way, complex Markov models can be built in a compositional, hierarchical way. Since the inherent structure of nowadays and tomorrows systems is becoming more and more complex, the possibility to specify Markov chains in a compositional way is a significant advantage.

During the last two decades *process algebra* has emerged as *the* mathematical framework to achieve compositionality. Process algebra provides a formal apparatus for reasoning about structure and behaviour of systems in a compositional way. The theoretical basis developed in this monograph will therefore be a process algebraic one. It will turn out that compositionality is not only favourable to specify complex situations but also facilitates the analysis of such models.

1.2 The Challenge of Compositional Performance and Dependability Estimation

It is worth to have a look at the historical development of performance and dependability evaluation methodology. From the very beginning, *queueing systems* have been used as intuitive means for describing system and analysing their performance [60, 129]. However, in the late 1970-ies it has been recognised that different real world phenomena could not be expressed satisfactorily by means of queueing systems. In particular the need to model *synchronisation* and *resource contention* was recognised, as a consequence of the (still ongoing) trend towards distributed systems [45]. Thus, a bunch of extensions has been proposed for queueing systems in order to reflect these issues in an intuitive way. The unfortunate result was that the exact semantics of such extensions was unclear, due to a lack of formal meaning of the queueing approach. Even more severe, the interference among different extensions was confusing.

Instead of adding more and more symbols to a more and more ambiguous notation, the feeling grew that (extended) queueing systems, developed from an engineer's perspective, could benefit a lot from a scientific analysis of the core concepts inherent to distributed systems [110].

Petri nets turned out to be rather close to an abstract view on (extended) queueing systems [45]. In contrast to queueing systems, Petri nets are very parsimonious with respect to their basic ingredients [162]. This is beneficial in order to develop a precise theory. Nowadays, at least in the setting of Markov chains, there is a common agreement that many kinds of extended queueing system can be represented as a generalised stochastic Petri net (GSPN), an extension of Petri nets with exponentially timed and immediate transitions [4, 3]. In particular, various add-ons to queueing systems can be concisely expressed in terms of Petri nets.

Queueing and *scheduling disciplines* are exceptions. They have been incorporated into the Petri net terminology in the same informal way as in queueing systems, namely by adding a remark, say 'LIFO' or 'JSQ', to the respective entity of the net. ('LIFO' stands for 'last in first out' scheduling while 'JSQ' describes 'join the shortest queue' queueing strategy.)

The problem of this informality is more severe than it appears to be at first glance. Consider, for instance, a small system with a Markovian arrival stream and two queues, each having a few places and each connected to a private server. Assume further that the two servers have drastically different (exponentially distributed) service times and 'JSQ' queueing strategy. This system is depicted in Figure 1.1. Nevertheless, we point out that such a description, does not at all give rise to an *unambiguous* Markov chain.

Ambiguity arises whenever both queues are equally occupied. Then, the remark 'JSQ' does *not determine* where the next arrival will be scheduled. This phenomenon, known as *nondeterminism* or *underspecification*, has an important impact on performance estimates of such systems, when service