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ELECTRICITY AND MAGNETISM

An introduction to the theory
of electric and magnetic fields



NEW YORK

Appleton-Century-Crofts

EDUCATIONAL DIVISION

Meredith Corporation

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6128 - 3

Library of Congress Card Number: 65-12058

PRINTED IN THE UNITED STATES OF AMERICA

390-48025-8

ELECTRICITY AND MAGNETISM



Series in Physics

Richard M. Sutton
Editor

PREFACE

This book has been written for an advanced undergraduate course in electricity and magnetism offered to students majoring in physics and in related fields. It presupposes a year's course in general physics and one in calculus. It is based on the lectures in electricity and magnetism given by the author for the past nine years and is designed to be readily understood by even the student who will receive only minimal guidance from his instructor.

The book has three main objectives. The first objective is a presentation of the fundamentals of electromagnetic theory reflecting recent developments and applications of the subject. To achieve this objective, considerable amount of modern material is included in the book; operational definitions are introduced for all fundamental electric and magnetic quantities; current and voltage are used as the basic measurables (*mksva* system of units)*; vector analysis is used as a standard mathematical tool; and, which is most important, the theory is presented in a logical rather than in a historical sequence.

The second objective of the book is a rigorous but simple presentation of electromagnetic theory, with emphasis on the internal unity and harmony of the mathematical description of electric and magnetic phenomena. To achieve this objective, the basic structure of the theory is first determined. With the aid of general physical considerations it is made plausible that the theory must be based upon three types of experimental laws: the field laws, the energy laws, and the constitutive laws. At the same time it is deduced from Helmholtz's theorem of vector analysis that a complete set of electric or magnetic field laws need not contain more than two experimentally established correlations, which may be either in a differential form (curl and divergence laws) or in an integral form (circulation and flux laws). On the basis of these considerations the theory is then presented rigorously and simply in a systematic, coherent, and logical manner.

The third objective of the book is to develop in the student a creative ability in the application of electromagnetic theory. For this purpose,

*The formulation of the concepts of electric current, voltage, charge, and electric and magnetic fields is based on ideas developed by R. W. Pohl in his famous lectures on general physics. The electricity and magnetism section of the lectures is described in R. W. Pohl, "Elektrizitätslehre," XIX Auflage, Springer, Berlin (1964).

detailed solutions to a large number of illustrative examples demonstrating various methods and applications of the theory have been incorporated in the book. Furthermore, each chapter, except Chapter 3, has been supplemented by a number of carefully selected problems which should help the student to build up his skill and initiative in practical application of the presented material.

In agreement with modern curricula, the book deals primarily with a detailed exposition of the theory of macroscopic electric and magnetic fields. The book is, however, sufficiently flexible to allow the instructor to add supplementary topics to the course. With this in mind, much subordinate material has been relegated to starred sections, which may be omitted without loss of continuity, and to illustrative examples. The instructor can easily substitute additional material for these sections and examples.

In writing the book, the author has attempted a complete rethinking of the subject matter. The book contains therefore an appreciable amount of original material, most of which has evolved in the process of developing the theory in accordance with the principles outlined in connection with the second objective of the book.

The author is grateful to many of his former students for their encouraging attitude and helpful suggestions. He owes a great debt to his wife Valentina, who patiently typed and proofread the numerous drafts of the manuscript and assisted in the preparation of the lines-of-force photographs appearing in this book.

OLEG D. JEFIMENKO

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I

MATHEMATICAL INTRODUCTION

1

PHYSICAL QUANTITIES AND PROPERTIES OF PHYSICAL EQUATIONS

In physics extensive use is made of the possibility of mathematical representation of physical phenomena: physical concepts are designated by symbols, the relationships between concepts are expressed by formulas, and the correlations between phenomena are represented by equations. Physical formulas and equations are characterized by special properties and form a special class of mathematical expressions. The knowledge of these properties is essential for an accurate formulation and intelligent application of physical theories. In the field of electricity and magnetism this knowledge is also needed for the understanding of the relations between different systems of electric and magnetic measurables used in scientific literature. We shall start therefore with a brief discussion of the nature and properties of physical formulas and equations.

1-1. Physical Quantities and Physical Equations

The properties of physical formulas and equations are closely connected with four preliminary procedures which constitute the starting point for a quantitative study of physical phenomena. These procedures are:

(1) selection of *basic*, or *primary*, *measurables* (basic objects of measurements) and specification of properties to be used for their identification

(2) selection of instruments for the measurement of basic measurables

(3) selection of standards and units for the calibration of these instruments and

(4) selection of *derived*, or *secondary*, *measurables* and specification of rules for their measurement.

The first of these procedures defines the conceptual contents of the *basic*, or *primary*, *quantities*, while the second and third procedures make it possible to associate a definite magnitude with each of these quantities, thus completing their definition. The fourth procedure consists in selecting certain groups of primary measurements in combination with specified mathematical operations to be performed upon the results of these measurements and defines the *derived*, or *secondary*, *quantities*.

With the aid of these four procedures it is possible to describe various physical phenomena in terms of a few primary quantities (results of single measurements) and a few secondary quantities (results of certain groups of measurements). The experimentally observed correlations between phenomena can then be expressed as correlations between these quantities in the form of algebraic equations.

It is clear that such equations reflect two different things. On one hand, they reflect correlations inherent in the physical phenomena. On the other hand, they reflect our approach to the quantitative description of these phenomena—in particular, our selection of measurables, standards, and units. This selection involves a considerable degree of arbitrariness. In principle, one can express the same set of correlations by using one, two, or any other number of basic measurables of any reasonable kind. The division of measurables into primary and secondary is also arbitrary. The choice of standards and units of measurements is, of course, arbitrary too. As we shall see, this arbitrariness in the selection of measurables, standards, and units is responsible for several remarkable properties of the physically meaningful mathematical expressions.

1-2. Ratio Requirement. Dimensions of Physical Quantities

Let us investigate how physical quantities, formulas, and equations are affected by the possibility of choosing different standards and units of measurements.