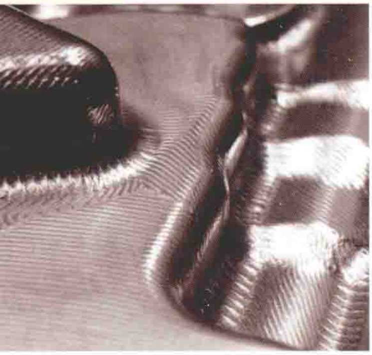
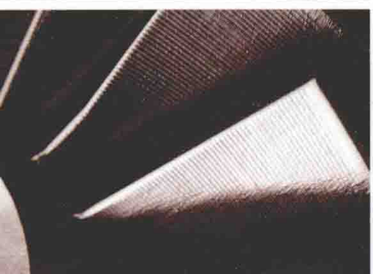
STEPHEN P. RADZEVICH

GEOMETRY OF SURFACES

A PRACTICAL GUIDE FOR MECHANICAL ENGINEERS





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A PRACTICAL GUIDE FOR MECHANICAL ENGINEERS

Stephen P. Radzevich
Principal Gear Engineer, USA

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GEOMETRY OF SURFACES

This book is dedicated to my wife Natasha

About the Author

Dr. Stephen P. Radzevich is a Professor of Mechanical Engineering and a Professor of Manufacturing Engineering. He received the M.Sc. (1976), Ph.D. (1982) and Dr.(Eng.)Sc. (1991) – all in mechanical engineering. Dr. Radzevich has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with CAD and CAM of precise gear finishing for a variety of industrial sponsors. His main research interest is in the kinematic geometry of surface generation, particularly focusing on (a) precision gear design, (b) high power density gear trains, (c) torque share in multi-flow gear trains, (d) design of special purpose gear cutting/finishing tools, (e) design and machining (finishing) of precision gears for low-noise/noiseless transmissions for cars, light trucks, etc. Dr. Radzevich has spent about 40 years developing software, hardware and other processes for gear design and optimization. Besides his work for industry, he trains engineering students at universities and gear engineers in companies. He has authored and co-authored over 30 monographs, handbooks and textbooks. Monographs entitled "*Generation of Surfaces*" (2001), "*Kinematic Geometry of Surface Machining*" (CRC Press, 2008), "*CAD/CAM of Sculptured Surfaces on Multi-Axis NC Machine: The DG/K-Based Approach*" (M&C Publishers, 2008), "*Gear Cutting Tools: Fundamentals of Design and Computation*" (CRC Press, 2010), "*Precision Gear Shaving*" (Nova Science Publishers, 2010), "*Dudley's Handbook of Practical Gear Design and Manufacture*" (CRC Press, 2012) and "*Theory of Gearing: Kinematics, Geometry, and Synthesis*" (CRC Press, 2012) are among recently published volumes. He has also authored and co-authored over 250 scientific papers, and holds over 200 patents on inventions in the field.

Preface

This book is about the geometry of part surfaces, their generation and interaction with one another. Written by a mechanical engineer, this book is ***not*** on the differential geometry of surfaces. Instead, this book is devoted to the application of methods developed in the differential geometry of surfaces, for the purpose of solving problems in mechanical engineering.

A paradox exists in our present understanding of geometry of surfaces: we know everything about ideal surfaces, which do not exist in reality, and we know almost nothing about real surfaces, which exist physically. Therefore, one of the main goals of this book is to adjust our knowledge of ideal surfaces for the purpose of better understanding the geometry of real surfaces. In other words: to bridge a gap between ideal and real surfaces. One of the significant advantages of the book is that it has been written ***not*** by a mathematician, but by a mechanical engineer for mechanical engineers.

Acknowledgments

I would like to share the credit for any research success with my numerous doctoral students, with whom I have tested the proposed ideas and applied them in industry. The many friends, colleagues and students who contributed are overwhelming in number and cannot be acknowledged individually – as much as they have contributed, their kindness and help must go unrecorded.

My thanks also go to those at John Wiley who took over the final stages and will have to manage the marketing and sale of the fruit of my efforts.

Glossary

We list, alphabetically, the most commonly used terms in engineering geometry of surfaces. In addition, most of the newly introduced terms are listed below as well.

Auxiliary generating surface R a smooth regular surface that is used as an intermediate (auxiliary) surface when an envelope for successive positions of a moving surface is determined.

***CA*-gearing** crossed-axis gearing, or a gearing having axes of rotation of the gear and of the pinion that are skewed in relation to one another.

Cartesian coordinate system a reference system comprised of three mutually perpendicular straight axes through the common origin. Determination of the location of a point in a Cartesian coordinate system is based on the distances along the coordinate axes. Commonly, the axes are labeled X, Y and Z. Often, either a subscript or a superscript is added to the designation of the reference system XYZ.

Center-distance this is the closest distance of approach between the two axes of rotation. In the particular case of hypoid gearing, the center-distance is often referred to as the offset.

Characteristic line this is a limit configuration of the line of intersection of a moving surface that occupies two distinct positions when the distance between the surfaces in these positions is approaching zero. In the limit case, a characteristic line aligns with the line of tangency of the moving surface and with the envelope for successive positions of the moving surface.

Darboux frame in the differential geometry of surfaces, this is a local moving Cartesian reference system constructed on a surface. The origin of a Darboux frame is at a current point of interest on the surface. The axes of the Darboux frame are along three unit vectors, namely along the unit normal vector to the surface, and two unit tangent vectors along principal directions of the gear tooth flank. The Darboux frame is analogous to the *Frenet–Serret* frame applied to surface geometry. A Darboux frame exists at any non-umbilic point of a surface. It is named after the French mathematician *Jean Gaston Darboux*.

Degree of conformity this is a qualitative parameter to evaluate how close the tooth flank of one member of a gear pair is to the tooth flank of another member of the gear pair at a point of their contact (or at a point within the line of contact of the teeth flanks).

Dynamic surface this is a part surface that is interacting with the environment.

Engineering surface this is a part surface that can be reproduced on a solid using for these purposes any production method.

Free-form surface this is a kind of part surface (see: *sculptured part surface* for more details).

Indicatrix of conformity this is a planar centro-symmetrical characteristic curve of fourth order that is used for the purpose of analytical description of geometry of contact of the gear tooth flank and of the pinion tooth flank. In particular cases, the indicatrix of conformity also possesses the property of mirror symmetry.

Natural kind of surface representation specification of a surface in terms of the first and second fundamental forms, commonly referred to as a *natural kind of surface representation.*

Part surface this is one of numerous surfaces that bound a solid.

Point of contact this is any point at which two tooth profiles touch each other.

***R*-gearing** a kind of crossed-axis gearing that features line contact between tooth flanks of the gear and pinion.

$\mathbb{R}$-mapping of the interacting part surfaces a kind of mapping of one smooth regular part surface onto another smooth regular part surface, under which normal curvatures at every point of the mapped surface correspond to normal curvatures at a corresponding point of a given surface.

Reversibly enveloping surfaces (or just R_e-surfaces) a pair of smooth regular part surfaces that are enveloping one another regardless of which one of the surfaces is traveling and which one of them is enveloping.

Rotation vector a vector along an axis of rotation having magnitude equal to the rotation of the axis. The direction of the rotation vector depends upon the direction of the rotation. Commonly, the rotation vector is designated $\boldsymbol{\omega}$. The magnitude of the rotation vector is commonly denoted ω. Therefore, the equality $\omega = |\boldsymbol{\omega}|$ is valid. The rotation vector of a gear is designated $\boldsymbol{\omega}_g$, the rotation vector of the mating pinion is designated $\boldsymbol{\omega}_p$, and the rotation vector of the plane of action is designated $\boldsymbol{\omega}_{pa}$.

Sculptured part surface this is a kind of part surface parameter of local geometry where every two neighboring infinitesimally small patches differ from one another. *Free-form surface* is another terminology that is used for part surfaces of this particular kind.

S_{pr}-gearing a kind of crossed-axis gearing that features base pitch of the gear, base pitch of the pinion, and operating base pitch, which are equal to one another under various values of the axis misalignment. Gearing of this kind is noiseless and capable of transmitting the highest possible power density.

Surface that allows for *sliding over itself* a smooth regular surface for which there exists a motion that results in the envelope for successive positions of the moving surface being congruent to the surface itself.

Vector of instant rotation a vector along the axis of instant rotation, either of the pinion in relation to the gear or of the gear in relation to the pinion. The direction of the vector of instant rotation depends upon the direction of rotation of the gear and of the pinion. Commonly, the rotation vector is designated $\boldsymbol{\omega}_{pl}$.

Notation

A_{P1}	apex of the base cone of the part surface P_1
A_{P2}	apex of the base cone of the part surface P_2
A_{pa}	apex of the plane of action, *PA*
C	center-distance
$C_{1.P1}$, $C_{2.P1}$	the first and second principal plane sections of the traveling part surface P_1
$C_{1.P2}$, $C_{2.P2}$	the first and second principal plane sections of the generated part surface P_2 (the enveloping surface)
$Cnf_R(P_1/P_2)$	indicatrix of conformity for two smooth regular part surfaces P_1 and P_2 at a current contact point K
$Cnf_k(P_1/R_2)$	indicatrix of conformity that is converse to the indicatrix $Cnf_R(P_1/P_2)$
$\mathscr{E}$	a characteristic line
E_{P1}, F_{P1}, G_{P1}	fundamental magnitudes of first order of the smooth regular part surface P_1
E_{P2}, F_{P2}, G_{P2}	fundamental magnitudes of first order of the smooth regular part surface P_2
$\mathscr{G}_P$	*Gaussian* curvature of a part surface P at a point m
$\mathscr{M}_P$	mean curvature of a surface P at a point m
K	point of contact of two smooth regular part surfaces P_1 and P_2 (or a point within a line of contact of the part surfaces P_1 and P_2)
L_c (or *LC*)	line of contact between two regular part surfaces P_1 and P_2
L_{P1}, M_{P1}, N_{P1}	fundamental magnitudes of second order of the smooth regular part surface P_1
L_{P2}, M_{P2}, N_{P2}	fundamental magnitudes of second order of the smooth regular part surface P_2
O_{P1}	axis of rotation of the part surface P_1
O_{P2}	axis of rotation of the part surface P_2
O_{pa}	axis of rotation of the plane of action, *PA*
PA	plane of action
P_{ln}	axis of instant rotation of two regular part surfaces P_1 and P_2 in relation to one another
$\mathbf{Rc}\,(PA \mapsto \mathscr{G})$	the operator of rolling/sliding (the operator of transition from the plane of action, *PA*, to the gear, $\mathscr{G}$, in crossed-axis gearing)
$\mathbf{Rc}\,(PA \mapsto \mathscr{P})$	the operator of rolling/sliding (the operator of transition from the plane of action, *PA*, to the pinion, $\mathscr{P}$, in crossed-axis gearing)

$\mathbf{Rl}_x(\varphi_y, Y)$	the operator of rolling over a plane (Y-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_y, Y)$	the operator of rolling over a plane (Y-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_x, X)$	the operator of rolling over a plane (X-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_x, X)$	the operator of rolling over a plane (X-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_x(\varphi_z, Z)$	the operator of rolling over a plane (Z-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_z, Z)$	the operator of rolling over a plane (Z-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rr}_u(\varphi, Z)$	the operator of rolling of two coordinate systems
$\mathbf{Rs}\,(A \mapsto B)$	the operator of the resultant coordinate system transformation, say from a coordinate system A to a coordinate system B
$\mathbf{Rt}\,(\varphi_x, X)$	the operator of rotation through an angle φ_x about the X-axis
$\mathbf{Rt}\,(\varphi_y, Y)$	the operator of rotation through an angle φ_y about the Y-axis
$\mathbf{Rt}\,(\varphi_z, Z)$	the operator of rotation through an angle φ_z about the Z-axis
$R_{1.P1}, R_{2.P1}$	the first and second principal radii of the gear tooth flank P_1
$R_{1.P2}, R_{2.P2}$	the first and second principal radii of the gear tooth flank P_2
$\mathbf{Sc}_x(\varphi_x, p_x)$	the operator of screw motion about the X-axis
$\mathbf{Sc}_y(\varphi_y, p_y)$	the operator of screw motion about the Y-axis
$\mathbf{Sc}_z(\varphi_z, p_z)$	the operator of screw motion about the Z-axis
$\mathbf{Tr}\,(a_x, X)$	the operator of translation at a distance a_x along the X-axis
$\mathbf{Tr}\,(a_y, Y)$	the operator of translation at a distance a_y along the Y-axis
$\mathbf{Tr}\,(a_z, Z)$	the operator of translation at a distance a_z along the Z-axis
U_{P1}, V_{P1}	curvilinear (*Gaussian*) coordinates of a point of a smooth regular part surface P_1
U_{P2}, V_{P2}	curvilinear (*Gaussian*) coordinates of a point of a smooth regular part surface P_2
$\mathbf{U}_{P1}, \mathbf{V}_{P1}$	tangent vectors to curvilinear coordinate lines on a smooth regular part surface P_1
$\mathbf{U}_{P2}, \mathbf{V}_{P2}$	tangent vectors to curvilinear coordinate lines on a smooth regular part surface P_2
$\mathbf{V}_\Sigma$	vector of the resultant motion of the smooth regular part surface P_1 in relation to a reference system that the smooth regular part surface P_2 will be associated with
$d_{cnf}^{\min}$	minimal diameter of the indicatrix of conformity $Cnf_R(P_1/P_2)$ for two smooth regular part surfaces P_1 and P_2 at a current contact point K
$k_{1.P1}, k_{2.P1}$	the first and second principal curvatures of the smooth regular part surface P_1
$k_{1.P2}, k_{2.P2}$	the first and second principal curvatures of the smooth regular part surface P_2
$\mathbf{n}_P$	unit normal vector to a smooth regular part surface P
p_{sc}	screw parameter (reduced pitch) of instant screw motion of the part surface P_1 in relation to the part surface P_2

$\mathbf{r}_{P1}$	position vector of a point of a smooth regular part surface P_1
r_{cnf}	position vector of a point of the indicatrix of conformity $Cnf_R(P_1/P_2)$ for two smooth regular part surfaces P_1 and P_2 at a current contact point K
$\mathbf{t}_{1.P1}, \mathbf{t}_{2.P1}$	unit tangent vectors of principal directions on the smooth regular part surface P_1
$\mathbf{t}_{1.P2}, \mathbf{t}_{2.P2}$	unit tangent vectors of principal directions on the smooth regular part surface P_2
$\mathbf{u}_{P1}, \mathbf{v}_{P1}$	unit tangent vectors to curvilinear coordinate lines on the smooth regular part surface P_1
$\mathbf{u}_{P2}, \mathbf{v}_{P2}$	unit tangent vectors to curvilinear coordinate lines on the smooth regular part surface P_2
$x_P y_P z_P$	local Cartesian coordinate system having its origin at a current point of contact of the part surfaces P_1 and P_2

Greek symbols

$\Phi_{1.P1}, \Phi_{2.P1}$	the first and second fundamental forms of the smooth regular part surface P_1
$\Phi_{1.P2}, \Phi_{2.P2}$	the first and second fundamental forms of the smooth regular part surface P_2
$\phi_{n.\omega}$	normal pressure angle
μ	angle of the part surfaces' P_1 and P_2 local relative orientation
$\boldsymbol{\omega}_{P1}$	rotation vector of the regular part surface P_1
$\boldsymbol{\omega}_{P2}$	rotation vector of the part surface P_2
$\boldsymbol{\omega}_{pl}$	vector of instant rotation of the part surfaces P_1 and P_2 in relation to one another

Subscripts

cnf	conformity
max	maximum
min	minimum
n	normal
opt	optimal

Introduction

The performance of parts depends largely on the geometry of the interacting surfaces. An in-depth investigation of the geometry of smooth regular part surfaces is undertaken in this book. An analytical description of the surfaces, and the methods of their generation, along with an analytical approach for description of the geometry of contact of the interacting part surfaces, is covered. The book comprises three parts, and appendices.

The specification of part surfaces in terms of the corresponding nominal smooth regular surface is considered in Part I of the book.

The geometry of part surfaces is discussed in Chapter 1. The discussion begins with an analytical description of ideal surfaces. Here, the ideal surface is interpreted as a zero-thickness film. The difference between *classical differential geometry* and *engineering geometry of surfaces* is analyzed. This analysis is followed by an analytical description of real part surfaces, based largely on an analytical description of the corresponding ideal surface. It is shown that while it remains unknown, a real part surface is located between two boundary surfaces. The said boundary surfaces are represented by two ideal surfaces, of upper tolerance and lower tolerance. The specification of surfaces ends with a discussion of the natural representation of a desired part surface. This consideration involves the first and second fundamental forms of a smooth regular part surface. For an analytically specified surface, the elements of its local geometry are outlined. This consideration includes but is not limited to an analytical representation of the unit tangent vectors, the tangent plane, the unit normal vector, the unit vectors of principal directions on a part surface, etc. Ultimately, the parameters of part surface curvature are discussed. Mostly, the equations for principal surface curvatures along with normal curvatures at a surface point are considered. In addition to the mean curvature, the *Gaussian* curvature, absolute curvature, shape operator and curvedness of a surface at a point are considered. The classification of local part surface patches is proposed in this section of the book. The classification is followed by a circular chart comprising all possible kinds of local part surface patches.

Chapter 2 is devoted to the analysis of a possibility of classification of part surfaces. Regardless of the fact that no scientific classification of smooth regular surfaces in a global sense is feasible in nature, local part surface patches can be classified. For an investigation of the geometry of local part surface patches, planar characteristic images are employed. In this analysis the *Dupin* indicatrix, curvature indicatrix and circular diagrams at a part surface point are covered in detail. Based on the results of the analysis, two more circular charts are developed. One of them employs the part surface curvature indicatrices, while the other is based on the properties of circular diagrams at a current part surface point. This section of the book ends with a brief consideration of one more useful characteristic curve, which can be helpful for analytical description of the geometry of a part surface locally.

In Part II the geometry of contact of two smooth regular part surfaces is considered. This part of the book comprises four chapters.

In Chapter 3 the discussion begins with a review of earlier works in the field of contact geometry of surfaces. This includes the order of contact of two surfaces, the local relative orientation of the surfaces at a point of their contact, and the first- and second-order analysis. The first-order analysis is limited just to the common tangent plane. The second-order analysis begins with the author's comments on the analytical description of the local geometry of contacting surfaces loaded by a normal force: *Hertz*'s proportional assumption. Then, the surface of relative normal curvature is considered. The *Dupin* indicatrix and curvature indicatrix of the surface of relative normal curvature are discussed. This analysis is followed by a discussion of the surface of relative normal radii of curvature, normalized relative normal curvature along with a characteristic curve $\Im_k(\mathscr{R})$ of novel kind.

This section of the book is followed by Chapter 4, in which an analytical method based on second fundamental forms of the contacting part surfaces is discussed. It is shown here that the resultant deviation of one of the contacting surfaces from the other contacting surface expressed in terms of the second fundamental forms of the contacting surfaces could be the best possible criterion for the analytical description of the contact geometry of two smooth regular surfaces. Such a criterion is legitimate, but computationally impractical. Thus, other analytical methods need to be developed for this purpose.

In Chapter 5 a novel kind of characteristic curve for the purpose of analytical description of contact geometry of two smooth regular part surfaces in the first order of tangency is discussed in detail. The discussion begins with preliminary remarks, followed by the introduction and derivation of an equation of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two part surfaces. Then, the directions of extremum degree of conformity of two part surfaces in contact are specified and described analytically. This analysis is followed by the determination and derivation of corresponding equations of asymptotes of the indicatrix of conformity $Cnf_R(P_1/P_2)$. The capabilities of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two smooth regular part surfaces P_1 and P_2 in the first order of tangency are compared with the corresponding capabilities of the *Dupin* indicatrix *Dup* $(\mathscr{R})$ of the surface of relative curvature $\mathscr{R}$. Important properties of the indicatrix of conformity of two smooth regular part surfaces are outlined. Ultimately, the converse indicatrix of conformity $Cnf_R^{cnv}(P_1/P_2)$ of two regular part surfaces in the first order of tangency is introduced and discussed briefly as an alternative to the regular indicatrix of conformity $Cnf_R(P_1/P_2)$.

In Chapter 6 more characteristic curves are derived on the premise of the *Plücker* conoid constructed at a point of a smooth regular part surface. Initially, the main properties of the surface of the Plücker conoid are briefly outlined. This includes, but is not limited to, the basics, analytical representation and local properties along with auxiliary formulae. This analysis is followed by an analytical description of the local geometry of a smooth regular part surface. Ultimately, expressions for two more characteristic curves are derived. These newly introduced characteristic curves are referred to as the *Plücker* curvature indicatrix and $\mathscr{A}n_R(P_1)$-indicatrix of a part surface. The analysis performed makes possible the derivation of equations for two more planar characteristic curves for analytical description of the contact geometry of two smooth regular part surfaces P_1 and P_2 at a point of their contact. One of the newly derived characteristic curves is referred to as the $\mathscr{A}n_R(P_1/P_2)$-*relative indicatrix of the first kind* of two contacting part surfaces P_1 and P_2. Another is a curve inverse to the characteristic curve $\mathscr{A}n_R(P_1/P_2)$. This second characteristic curve is referred to as the $\mathscr{A}n_k(P_1/P_2)$*-relative indicatrix of the second kind*. The main properties of both the characteristic curves are discussed briefly.

The feasible kinds of contact of two smooth regular part surfaces in the first order of tangency are discussed in Chapter 7. This analysis begins with an investigation of the possibility of implementing the indicatrix of conformity for the purpose of identification of the actual kind of contact of two smooth regular part surfaces. Then, the impact of accuracy of the computation of the parameters of the indicatrix of conformity $Cnf_R(P_1/P_2)$ of two part surfaces is investigated. Ultimately, a classification of all possible kinds of contact of two smooth regular part surfaces in the first order of tangency is developed.

Various kinds of mapping of one part surface onto another part surface are discussed in Part III. The discussion in this part of the book begins with a novel kind of surface mapping, the so-called $\mathbb{R}$-mapping of the interacting part surfaces.

In Chapter 8 a novel method of surface mapping, namely $\mathbb{R}$-mapping of the interacting part surface, is disclosed. The preliminary remarks on the developed approach are followed by a detailed consideration of the concept underlying the $\mathbb{R}$-mapping of the interacting part surfaces. Then, the principal features of $\mathbb{R}$-mapping of a part surface P_1 onto another part surface P_2 are disclosed. Because $\mathbb{R}$-mapping of a surface returns an equation of the mapped surface in a natural representation, namely in terms of the fundamental magnitudes of the first and second order, the derived equation of the mapped surface must be reconstructed and represented in a convenient reference system. This issue receives comprehensive coverage in this chapter. The chapter ends with a consideration of two examples of implementation of the discussed method of part surface mapping.

A general consideration of the generation of enveloping surfaces is discussed in Chapter 9. The consideration begins with the analysis of generation of an envelope for successive positions of a moving planar curve. Then, the discussion is extended to the generation of the enveloping surface for successive positions of a moving smooth regular part surface. Enveloping surfaces for one-parametric, as well as two-parametric, families of surfaces are covered in this section. Further, the *kinematic method* for generation of enveloping surfaces is introduced. This method was developed in the 1940s by *V.A. Shishkov*. Implementation of the kinematic method for generation of one-parametric enveloping surfaces is discussed. Then, the approach is extended to multi-parametric motion of a smooth regular part surface.

In Chapter 10 special cases of generation of enveloping surfaces are disclosed. For this purpose a concept of reversibly enveloping surfaces is introduced. For the generation of reversibly enveloping surfaces, a novel method is proposed. This method is illustrated by an example of the generation of reversibly enveloping surfaces in the case of tooth flanks for geometrically accurate (ideal) crossed-axis gear pairs. The performed analysis makes possible a conclusion that two *Olivier principles* of generation of enveloping surfaces

- in the general case are not valid, and
- in a degenerate case are useless.

Ultimately, there is no sense in applying *Olivier principles* for the purpose of generation of reversibly enveloping smooth regular part surfaces.

Part surfaces that allow for sliding over themselves are considered as a particular degenerate case of enveloping surfaces.

The appendices contain reference material that is useful in practical applications. The elements of vector algebra are briefly outlined in Appendix A. In Appendix B, the elements of coordinate system transformation are represented. This section of the book also includes direct transformation of the surface fundamental forms. The latter makes it possible to avoid calculation of the first and second derivatives of the part surface equation after the equation is represented in a new reference system. Formulae for changing surface parameters are represented in Appendix C.

A book of this size is likely to contain omissions and errors. If you have any constructive suggestions, please communicate them to the author via e-mail: radzevich@gmail.com.