Exploring Chaos

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Exploring Chaos

A Guide to the New Science of Disorder

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Introduction

We are all aware of how small events can drastically change the course of history: the assassin's bullet that triggers a revolution, a chance meeting with a stranger at a party, the impulsive decision to catch a plane doomed to crash. Romantic novels are full of such stories. We expect life to be complicated and uncertain, scattered with random events that make the future difficult to predict. For this reason, many people find the 'pure' predictability of traditional science unattractive and difficult to relate to their own lives.

Recently, however, a new line of scientific inquiry called 'chaos theory' has caught the popular imagination. It seems to link our everyday experiences to the laws of nature by revealing, in an aesthetically pleasing way, the subtle relationships between simplicity and complexity and between orderliness and randomness.

Scientists have always searched for simple rules, or laws, that govern the Universe. For example, Isaac Newton could explain how the stars appeared to move across the sky with his simple laws of motion and theory of gravitation. At the beginning of the 19th century, the famous French mathematician Pierre Simon de Laplace believed firmly in a Newtonian universe that worked on clockwork principles. He proposed that if you knew the position and velocities of all the particles in the Universe, you could predict its future for all time.

This deterministic view received its first blow in the 1920s, when quantum mechanics was developed to describe the world of the very small. It explained how fundamental particles such as the electron behaved. But quantum mechanics is a statistical theory based on probability, and it is impossible to measure the position and momentum of a particle at the same time. This so-called uncertainty principle of quantum theory seems to be inherent in the laws of nature.

Nevertheless, physicists have used quantum mechanics to construct a reasonably robust theoretical framework for describing the fundamental properties of matter and the forces at work in the Universe. They hope to explain how the Universe has evolved and even how it came into existence. The 'reductionist' view is that once they formulate 'a theory of everything', it should be possible to explain the more complicated natural phenomena – how atoms and molecules behave (chemistry) and how they organize into self-replicating entities (biology). It is just a matter of time and effort. Indeed, human beings are subject to the same laws of nature as the galaxies, so eventually we may be able to make predictions about human concerns, such as the fluctuations of the stock market or the spread of an epidemic. In theory, life is supposed to be predictable.

Is this approach useful? There are considerable shortcomings. At the moment, scientists cannot even use the fundamental laws of nature to predict when the drips will fall from a leaking tap, or what the weather will be like in two weeks' time. In fact, it is difficult to predict very far ahead the motion of any object that feels the effect of more than two forces, let alone complicated systems involving interactions between many objects.

Recently, researchers in many disciplines have begun to realize that there seem to be inbuilt limits to predicting the future at all levels of complexity. It is here that chaos theory steps in to shed some light on the way the everyday world works.

Chaos theory has resulted from a synthesis of imaginative mathematics and readily accessible computer power. It presents a universe that is deterministic, obeying the fundamental physical laws, but with a predisposition for disorder, complexity and unpredictability. It reveals how many systems that are constantly changing are extremely sensitive to their initial state – position, velocity, and so on. As the system evolves in time, minute changes amplify rapidly through feedback. This means that systems starting off with only slightly differing conditions rapidly diverge in character at a later stage. Such behaviour imposes strict limitations on predicting a future state, since the prediction depends on how accurately you can measure the initial conditions. In fact, if you model such a system on a computer, by feeding in equations and numbers, just rounding off the decimal points in a different way can radically change the future behaviour of the system.

The computer revealed the subtle behaviour of chaotic systems because it can follow their trajectories over many millions of steps. This approach has exposed the abstract geometrical nature of chaos theory, in the form of computer graphics. Within the overall shape, there lies a repetitive pattern whose exquisite substructure characterizes the nature of chaos, indicating when predictability breaks down.

It is the beautiful graphics associated with chaotic systems that has made the subject so appealing to everybody who sees them. They show how quite simple equations, when fed into a computer, can produce breathtaking patterns of ever increasing complexity. Within the often lifelike forms, there are shapes that repeat themselves on smaller and smaller scales – a phenomenon called 'self-similarity'. Benoit Mandelbrot coined the word 'fractal' for such shapes. And here we have an immediate link with nature, for trees and mountains are examples of fractals.

But there is more to it than that. Scientists now have an interpretive tool for describing many of the complexities of the world, from brain rhythms to gold futures. Recently, *New Scientist* published a series of features on chaos theory and its implications. This is the complete series in book form. The articles, which have been written by some of the world's leading experts, explain what chaos is, the mathematics behind chaos, how chaos can be found in virtually every discipline from astronomy to population dynamics, and how chaos theory can be applied practically to areas such as engineering and economics. The series also looks at the role of fractal geometry in chaos theory, and how computers are now changing the way science is being done.

Computing theory is spawning ways of modelling complexity and disorder by describing information in algorithmic forms. In this way, chaos is revealing fundamental limits to human knowledge in an uncomfortable way. Because chaos implies a delicate dependence on initial conditions, a complete knowledge of a chaotic system demands knowing the position and velocity of every particle in the Universe. Even if it were possible to carry out such a task, such a measurement would disturb the system it is measuring anyway. It seems that the macroworld may have its own uncertainty principle, a result of the nature of chaotic dynamics rather than quantum probability. Chaos may even provide the Universe with an arrow of time.

Chaos also seems to be responsible for maintaining order in the natural world. Feedback mechanisms not only introduce flexibility into living systems, sustaining delicate dynamical balances, but also promote nature's propensity for self-organization. Even the beating heart relies on feedback for regularity.

Chaos also triggers our aesthetic responses. The young, in particular, have taken to the complex graphics that seem to teeter wondrously between order and randomness. Everyone enjoys that stunning mathematical object, the Mandelbrot set. It now appears on posters, T-shirts, record sleeves and pop videos. Such colourful iterations have linked mathematics with art and nature in a stimulating way. We appreciate how the spontaneous complexity generated in self-organizing systems makes a tree more beautiful than a telegraph pole. Chaos has made mathematics come alive.

I should like to thank Nigel Hawtin and Neil Hyslop for the artwork, Richard Fifield and Ian Percival for helping me to coordinate the series, and Liz Else and Peter Wrobel for help with the editing.

NINA HALL

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Chaos: a science for the real world

IAN PERCIVAL

Traditionally, scientists have looked for the simplest view of the world around us. Now, mathematics and computer power have produced a theory that helps researchers to understand the complexities of nature. The theory of chaos touches all disciplines.

If you watch from a bridge as a leaf floats down a stream, you may see it trapped by a small whirlpool, circulate a few times, and escape, only to be trapped again further down the stream. Trying to guess what will happen to a leaf as it comes into view from under the bridge is an idle pursuit in more senses than one: the tiniest shift in the leaf's position can completely change its future course.

Small changes lead to bigger changes later. This behaviour is the signature of chaos.

Chaos is found everywhere in nature, sometimes even in the beating of the human heart. Under certain circumstances, the human heart can beat chaotically. It is controlled by natural pacemakers, which normally give it a steady, regular beat, but sometimes they do not work together properly, so that there are alternate long and short gaps between the beats. In yet more extreme conditions, the rhythm becomes irregular. A small change in the timing of one beat makes a bigger change in the next. The beating becomes chaotic, and may threaten survival. This is a good example of how regular motion makes the transition to chaos when the conditions are changed.

You can hear this transition from regular to chaotic motion in your home, by listening to a dripping tap. If you hold a fragment of kitchen foil on the sink or wash basin below the tap, while it is dripping slowly, you will hear a regular beat on the foil. Now turn on the tap very slightly, and, under the right conditions, you will hear alternate long and short beats. Give the tap another tiny turn and it will never settle down to regular stable behaviour; it has become chaotic.

Chaos is persistent instability.

Instability is part of our own environment and our culture. The situation that is balanced on a knife-edge and the straw that can break the camel's back are metaphors for life's instabilities.

Chaotic motion contrasts with the regularity that we see on a grander scale in the cosmos. People have always wondered at the order in the seasons, at the way night follows day, and at the precision with which the stars and planets appear to move across the sky. Such celestial events all have their origin in the regularity of the motion of the Earth and the other planets, explained more than 300 years ago by Isaac Newton with his laws of motion and theory of gravitation. According to these laws, the present positions and velocities of the Sun and the planets determine the positions and velocities for all past and future times.

Newton's laws of motion are the classical example of determinism, in which the future is uniquely determined by the past. When scientists look for this kind of order in the Universe, they are often rewarded. But, as we know, order is not universal; we also need to understand disorder.

One of the first mathematicians to study disorder was Pierre Simon de Laplace. He was born in Normandy and survived the French Revolution by flattering those in power. Laplace had a thoroughly Newtonian view of the Universe, yet he helped to found the theory of disorder, or probability, which describes how large numbers of events can behave in a typical way, even when the individual events are unpredictable. This happens in gambling, which was as popular in his time as it is today. Laplace even applied his thoughts on probability to the law courts. The theory of probability now helps us to estimate the spread of AIDS without understanding the detail of how it works.

So during the 19th century, there were two kinds of theory for changing systems, deterministic theories and theories of probability. The two approaches appeared to be incompatible. In the first, the future is determined from the past, with no apparent need for probability. In the second, the future depends in some random way on the past, and cannot be determined from it.

The first challenge to this picture came with the quantum theory in the 1920s and the 1930s, which is also based on calculating probabilities – theorists describe the behaviour of an electron in terms of a 'probability wave'. The second challenge came from the theory of chaos. Simple mathematical analysis shows that even in simple systems, which obey Newton's laws of motion, you cannot always predict what is going to happen next. The reason is that there is persistent instability. This often arises when an object feels the effect of more than one force.

A well-known example is a pendulum with a bob that is attracted equally to two magnets below it. When the bob moves slowly near to a point midway between the magnets, it is affected almost equally by the force from each magnet. Its future motion becomes extremely sensitive to small changes in its present position and velocity, so the motion is chaotic. Suppose the sensitivity is so great that the error in measuring its position increases by 10 times in one swing between the magnets, which is not at all exceptional. In that case, predicting its position to within a centimetre after one swing entails measuring the position to within a millimetre. To make the same prediction after four swings, its position would have to be measured to within the size of a bacterium, and after nine swings, to within less than the size of an atom. The pendulum obeys Newton's deterministic laws, but any attempt to predict its future behaviour over long times will be defeated.

This does not mean, however, that we can say nothing about the motion of this pendulum. For some initial conditions, the motion is regular, and not chaotic at all, so we can predict a long time ahead. And we can understand many properties of the chaotic motion with the help of probability theory (see 'Determinism and probability', below). But the need to use probabilities for such simple systems came as a great surprise to those who were brought up in the Newtonian tradition.

Determinism and probability: the pinball and the word

There are few better illustrations of chaos and the connection between determinism and probability than the pinball machine illustrated in Figure 1.1.

Suppose the ball is released from just above the top pin P1, and just to the left of it. Then it bounces to the left, and hits the pin P2 on the next level down. If the size of the ball and the distance between the pins are chosen well, the ball will then have an equal chance of going to the left and hitting P4, or of going to the right and hitting P5. Suppose it goes to the right. Then it has an equal chance of hitting the pins on either side on the next level, and so on, down to the lowest level. The motion of the pinball is chaotic.

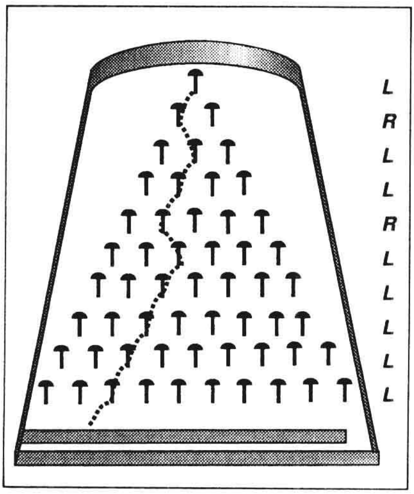

Figure 1.1

In Newtonian dynamics, the paths are usually distinguished by their 'initial conditions', which in this case means the position of the ball when it is dropped on to the first pin. But because the motion is chaotic, the path of the ball through the pins is extraordinarily sensitive to that position, so initial conditions are not very helpful.

In chaos theory, we distinguish between different paths by 'words' made up of letters. For the pinball we write an 'L' for a bounce to the left, and an 'R' for a bounce to the right.

If the numbers of layers of pins is 10, then every possible path of the ball from the top to the bottom is labelled by a 'word' of 10 letters, made up entirely of Ls and Rs, with the first letter giving the direction of the first bounce from P1, the second letter the direction of the second bounce from a pin on the second level, and so on. The word for the path shown in the Figure is LRLLRLLLLL. There are 1024 different possible paths between the pins from the top to the bottom, which is the number of such 10-letter words. In the theory of chaos, each letter and each word has a probability, just like the probability of a head or a tail in the throw of a coin, and it is this that makes the surprising connection between determinism and probability.

Distinguishing such paths or 'orbits' by words instead of initial conditions is known as symbolic dynamics. Wherever there is chaos there are words to describe it, just as for the pinball, but it is often very difficult to find them. It is worth the effort, because symbolic dynamics can tame the science of chaos.

The theory of deterministic chaos mixes determinism and probability in a totally unexpected way. Understanding the subtle unfolding of chaos in a system is helping us to describe not only the behaviour of the floating leaf, the irregular heartbeat and the dripping tap, but many aspects of our complex Universe, on both a small and a grand scale. Chaos stalks every scientific discipline.

Astronomers now use theories of chaos to model the pulsation of the early Universe and the motion of stars in galaxies, as well as that of the planets, satellites and comets of the Solar System. Nearer home, chaos helps us to study the charged particles that are trapped by the Earth's magnetism, whose escape into the atmosphere gives us the aurorae. And one of the most exciting applications of chaos theory is in studying the movements of that atmosphere that give us our weather.

Biologists also see chaos in the changing populations of insects and birds, in the spreading of epidemics, the metabolism of cells, and the propagation of impulses along our nerves. Physicists come across chaos in the motion of electrons in atoms, and of atoms in molecules and gases, and in the theory of elementary particles. Even engineers have to think about chaos because it intervenes in their designs. Chaos can frustrate those who design electrical circuits. It can lead to the loss of particles from a particle accelerator or a plasma, or to the capsize of a moored ship in a rough sea.

Some of the most beautiful examples of chaos are in mathematics, where the solutions of apparently simple problems show extraordinarily complicated behaviour. In the past, before the days of computers, this made scientists shy away from these problems, but now that we have computers to help us, the beauty of these complications is one of the subject's main sources of attraction. Chaos is a science of the computer age. And some of the elegant mathematics used to model chaos has important applications in many fields.

Nevertheless, you cannot use the theory of chaos everywhere. Science takes words and shapes their meanings to its own ends, and 'chaos' is no exception. The state of Eastern European politics may look chaotic, but you cannot study a subject of this type using chaos theory. There are many other situations that are chaotic in the ordinary sense, but not in the scientific sense of chaos.

The science of chaos is like a river that has been fed from many streams. Its sources come from every discipline – mathematics, physics, chemistry, engineering, medicine and biology, astronomy and meteorology, from those who study fluids and those who study electrical circuits, from strict and rigorous provers of theorems, and from swashbuckling computer experimenters.

Many of the new ideas in chaos were discovered independently in different fields. Researchers realized that although they were working on vastly different problems, they were employing the same kinds of mathematical techniques to deal with them. Much credit for bringing unity to the infant science belongs to those like Joseph Ford of the Georgia Institute of Technology in Atlanta, who recognized the common features early on.

The study of chaos, however, is not completely new. One early contribution came in the 19th century with a Russian