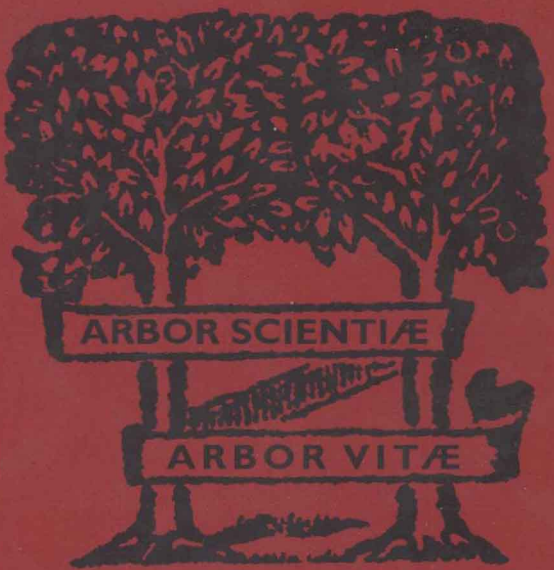
The International Library of Philosophy

LOGICAL SYNTAX OF LANGUAGE



Founded by C. K. Ogden

LOGICAL SYNTAX OF LANGUAGE

RUDOLF CARNAP

First published in 1937 by
Kegan Paul, Trench, Trubner & Co Ltd

Reprinted in 2000 by
Routledge
11 New Fetter Lane, London EC4P 4EE

Reprinted 2001

Routledge is an imprint of the Taylor & Francis Group

Printed and Bound in Great Britain

British Library Cataloguing in Publication Data
A CIP catalogue record for this book
is available from the British Library

Logical Syntax of Language
ISBN 0-415-22553-1
Philosophy of Mind and Language: 8 Volumes
ISBN 0-415-22576-0
The International Library of Philosophy: 56 Volumes
ISBN 0-415-21803-9

The International Library of Philosophy

LOGICAL SYNTAX OF LANGUAGE

Founded by C. K. Ogden

PREFACE TO THE ENGLISH EDITION

The present English edition contains some sections which are not found in the German original. These are §§ 16*a*, 34*a–i*, 38*a–c*, 60*a–d*, 71*a–e*. These twenty-two sections were included in the manuscript of the German original when it was sent for publication (in December 1933) but had to be taken out because of lack of space. The content of § 34*a–i* was, in a slightly different formulation, published in German in the paper *Ein Gültigskriterium für die Sätze der klassischen Mathematik*, and the content of §§ 60*a–d* and 71*a–d* in *Die Antinomien und die Unvollständigkeit der Mathematik*. § 60 of the original has been omitted here, since it was only a shortened substitute for § 60*a–d*.

In the Bibliography some less important publications have been deleted, and others, mainly of the last few years, have been added.

Several smaller additions and corrections have been made. The more important of these occur at the following points: § 8, regressive definition; § 12, RI 2 (see footnote); § 14, proofs added to Theorems 3 and 7; § 21, D 29; § 22, two insertions in D 64 (see footnote), D 83; § 29, footnote; § 30, PSII 4 (see footnote to § 12); PSII 19, condition added; § 48 (see footnote); § 51, definition of 'L-consequence'; § 56 (see footnote), Theorems 8 and 9 taken out; § 57, Theorems 2 and 3 corrected, and last paragraph added; § 62, explanation of '$\mathfrak{Q}_2[\mathfrak{S}_2]$'; §§ 65 and 66, definitions of 'extensional' restricted to closed partial expressions, and Theorem 65. 8*a* added; § 67, end of second paragraph. The majority of these corrections and a number of further ones have been suggested by Dr. A. Tarski, others by J. C. C. McKinsey and W. V. Quine, to all of whom I am very much indebted for their most helpful criticisms.

The problem of rendering the German terminology was naturally a most difficult one, in some cases there being no English word in existence which corresponded exactly to the original, in others the obvious equivalent being unavailable because of its special associations in some other system. It was necessary sometimes to appropriate for our purposes words which have not previously borne a technical significance, sometimes to coin entirely new ones. If at

first sight some of these seem ill at ease or outlandish, I can only ask the reader to bear in mind the peculiar difficulties involved, and assure him that no term was chosen without most careful consideration and the conviction that it would justify itself in use.

To facilitate discussion and reference, the German symbolic abbreviations have been retained in all the strictly formalized portions of the book. English equivalents have been substituted only where they occur in the non-formal text, as mere convenient abbreviations which are not properly symbolic (e.g. "TN" for "term-number" instead of the German "GZ"), or as incidental symbols introduced simply for purposes of illustration (e.g. "fa" for "father" instead of the German "Va"). Wherever a German abbreviation has been used for the first time, the full German word has been inserted in brackets; and in the case of the terms introduced by formal definitions, a complete key to the symbolization is given in a footnote at the beginning of the respective sections.

I wish to express my best thanks to the Countess von Zeppelin for the accomplishment of the difficult task of translating this book, further to Dr. W. V. Quine for valuable suggestions with regard to terminology, and to Dr. E. C. Graham, Dr. O. Helmer, and Dr. E. Nagel for their assistance in checking the proofs.

R. C.

Cambridge, Mass., May 1936

The International Library of Philosophy

PHILOSOPHY OF MIND AND LANGUAGE
In 8 Volumes

I	Dialectic	*Adler*
II	Communication	*Britton*
III	The Mind and its Place in Nature	*Broad*
IV	Logical Syntax of Language	*Carnap*
V	Methods and Criteria of Reasoning	*Crawshay-Williams*
VI	Bentham's Theory of Fictions	*Ogden*
VII	Philosophy of the Unconscious	*von Hartmann*
VIII	The Spirit of Language in Civilization	*Vossler*

FOREWORD

For nearly a century mathematicians and logicians have been striving hard to make logic an exact science. To a certain extent, their efforts have been crowned with success, inasmuch as the science of logistics has taught people how to manipulate with precision symbols and formulae which are similar in their nature to those used in mathematics. But a book on logic must contain, in addition to the formulae, an expository context which, with the assistance of the words of ordinary language, explains the formulae and the relations between them; and this context often leaves much to be desired in the matter of clarity and exactitude. In recent years, logicians representing widely different tendencies of thought have developed more and more the point of view that in this context is contained the essential part of logic; and that the important thing is to develop an exact method for the construction of these sentences about sentences. The purpose of the present work is to give a systematic exposition of such a method, namely, of the method of "logical syntax". (For further details, see Introduction, pp. 1 and 2.)

In our "Vienna Circle", as well as in kindred groups (in Poland, France, England, U.S.A., and, amongst individuals, even in Germany) the conviction has grown, and is steadily increasing, that metaphysics can make no claim to possessing a scientific character. That part of the work of philosophers which may be held to be scientific in its nature—excluding the empirical questions which can be referred to empirical science—consists of logical analysis. The aim of logical syntax is to provide a system of concepts, a language, by the help of which the results of logical analysis will be exactly formulable. *Philosophy is to be replaced by the logic of science*—that is to say, by the logical analysis of the concepts and sentences of the sciences, for *the logic of science is nothing other than the logical syntax of the language of science*. That is the conclusion to which we are led by the considerations in the last chapter of this book.

The book itself makes an attempt to provide, in the form of an exact syntactical method, the necessary tools for working out the problems of the logic of science. This is done in the first place by the formulation of the syntax of two particularly important types of language which we shall call, respectively, 'Language I' and

'Language II'. Language I is simple in form, and covers a narrow field of concepts. Language II is richer in modes of expression; in it, all the sentences both of classical mathematics and of classical physics can be formulated. In both languages the investigation will not be limited to the mathematico-logical part of language—as is usually the case in logistics—but will be essentially concerned also with synthetic, empirical sentences. The latter, the so-called 'real' sentences, constitute the core of science; the mathematico-logical sentences are analytic, with no real content, and are merely formal auxiliaries.

With Language I as an example, it will be shown, in what follows, how the syntax of a language may be formulated within that language itself (Part II). The usual fear that thereby contradictions—the so-called 'epistemological' or 'linguistic' antinomies—must arise, is not justified.

The treatment of the syntax of Languages I and II will be followed by the outline of a general syntax applicable to any language whatsoever (Part IV); and, although the attempt is very far from attaining the desired goal, yet the task is one of fundamental importance. The range of possible language-forms and, consequently, of the various possible logical systems, is incomparably greater than the very narrow circle to which earlier investigations in modern logic have been limited. Up to the present, there has been only a very slight deviation, in a few points here and there, from the form of language developed by Russell which has already become classical. For instance, certain sentential forms (such as unlimited existential sentences) and rules of inference (such as the Law of Excluded Middle), have been eliminated by certain authors. On the other hand, a number of extensions have been attempted, and several interesting, many-valued calculi analogous to the two-valued calculus of sentences have been evolved, and have resulted finally in a logic of probability. Likewise, so-called intensional sentences have been introduced and, with their aid a logic of modality developed. The fact that no attempts have been made to venture still further from the classical forms is perhaps due to the widely held opinion that any such deviations must be justified—that is, that the new language-form must be proved to be 'correct' and to constitute a faithful rendering of 'the true logic'.

To eliminate this standpoint, together with the pseudo-problems

and wearisome controversies which arise as a result of it, is one of the chief tasks of this book. In it, the view will be maintained that we have in every respect complete liberty with regard to the forms of language; that both the forms of construction for sentences and the rules of transformation (the latter are usually designated as "postulates" and "rules of inference") may be chosen quite arbitrarily. Up to now, in constructing a language, the procedure has usually been, first to assign a meaning to the fundamental mathematico-logical symbols, and then to consider what sentences and inferences are seen to be logically correct in accordance with this meaning. Since the assignment of the meaning is expressed in words, and is, in consequence, inexact, no conclusion arrived at in this way can very well be otherwise than inexact and ambiguous. The connection will only become clear when approached from the opposite direction: let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. By this method, also, the conflict between the divergent points of view on the problem of the foundations of mathematics disappears. For language, in its mathematical form, can be constructed according to the preferences of any one of the points of view represented; so that no question of justification arises at all, but only the question of the syntactical consequences to which one or other of the choices leads, including the question of non-contradiction.

The standpoint which we have suggested—we will call it the *Principle of Tolerance* (see p. 51)—relates not only to mathematics, but to all questions of logic. From this point of view, the task of the construction of a general syntax—in other words, of the definition of those syntactical concepts which are applicable to languages of any form whatsoever—is a very important one. In the domain of general syntax, for instance, it is possible to choose a certain form for the language of science as a whole, as well as for that of any branch of science, and to state exactly the characteristic differences between it and the other possible language-forms.

The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, that impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities.

In a number of places in the text, reference is made to the most important literature on the subject. A complete list has not, however, been attempted. Further bibliographical information may easily be obtained from the writings specified. The most important references are given on the following pages: pp. 96 ff., comparison of our Language II with other logical systems; pp. 136 ff., on the symbolism of classes; pp. 158 ff., on syntactical designations; pp. 253 f., on the logic of modalities; pp. 280 f. and 320 f. on the logic of science.

For the development of ideas in this book, I owe much to the stimulation I have received from various writings, letters and conversations on logical problems. Mention should here be made of the most important names. Above all, I am indebted to the writings and lectures of Frege. Through him my attention was drawn to the standard work on logistics—namely, the *Principia Mathematica* of Whitehead and Russell. The point of view of the formal theory of language (known as "syntax" in our terminology) was first developed for mathematics by Hilbert in his "metamathematics", to which the Polish logicians, especially Ajdukiewicz, Lesniewski, Lukasiewicz, and Tarski, have added a "metalogic". For this theory, Gödel created his fruitful method of "arithmetization". On the standpoint and method of syntax, I have, in particular, derived valuable suggestions from conversations with Tarski and Gödel. I have much for which to thank Wittgenstein in my reflections concerning the relations between syntax and the logic of science; for the divergences in our points of view, see pp. 282 ff. (Incidentally, *à propos* of the remarks made —especially in § 17 and § 67—in opposition to Wittgenstein's former dogmatic standpoint, Professor Schlick now informs me that for some time past, in writings as yet unpublished, Wittgenstein has agreed that the rules of language may be chosen with complete freedom.) Again, I have learned much from the writings of authors with whom I am not entirely in agreement; these are, in the first place, Weyl, Brouwer, and Lewis. Finally, I wish to express my gratitude to Professor Behmann and Dr. Gödel for having read the manuscript of this book in an earlier draft (1932), and for having made numerous valuable suggestions towards its improvement.

R. C.

Prague, May 1934

CONTENTS

PART III. THE INDEFINITE LANGUAGE II

A. Rules of Formation for Language II

B. Rules of Transformation for Language II

C. Rules of Consequence for Language II

INTRODUCTION

§ 1. What is Logical Syntax?

By the **logical syntax** of a language, we mean the formal theory of the linguistic forms of that language—the systematic statement of the formal rules which govern it together with the development of the consequences which follow from these rules.

A theory, a rule, a definition, or the like is to be called *formal* when no reference is made in it either to the meaning of the symbols (for example, the words) or to the sense of the expressions (e.g. the sentences), but simply and solely to the kinds and order of the symbols from which the expressions are constructed.

The prevalent opinion is that syntax and logic, in spite of some points of contact between them, are fundamentally theories of a very different type. The syntax of a language is supposed to lay down rules according to which the linguistic structures (e.g. the sentences) are to be built up from the elements (such as words or parts of words). The chief task of logic, on the other hand, is supposed to be that of formulating rules according to which judgments may be inferred from other judgments; in other words, according to which conclusions may be drawn from premisses.

But the development of logic during the past ten years has shown clearly that it can only be studied with any degree of accuracy when it is based, not on judgments (thoughts, or the content of thoughts) but rather on linguistic expressions, of which sentences are the most important, because only for them is it possible to lay down sharply defined rules. And actually, in practice, every logician since Aristotle, in laying down rules, has dealt mainly with sentences. But even those modern logicians who agree with us in our opinion that logic is concerned with sentences, are yet for the most part convinced that logic is equally concerned with the relations of meaning between sentences. They consider that, in contrast with the rules of syntax, the rules of logic are non-formal. In the following pages, in opposition to this standpoint, the view that logic, too, is concerned with the *formal* treatment of sentences will be presented and developed. We shall see that the