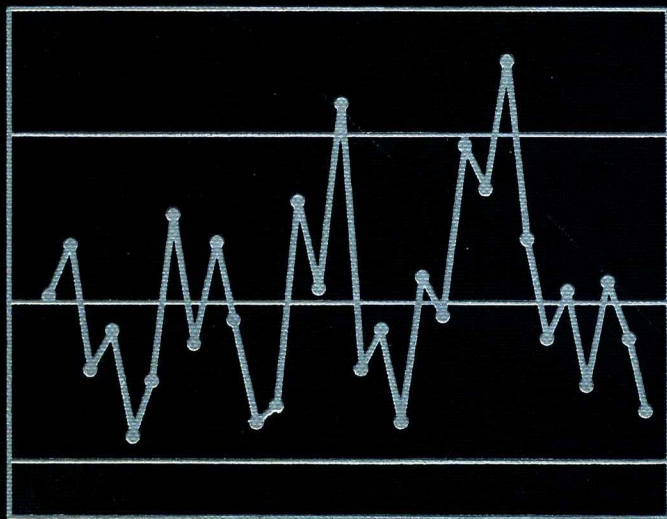


INTRODUCTION TO **Statistical**



**Quality
Control**
SECOND EDITION

**Douglas C.
Montgomery**

Introduction to Statistical Quality Control

Second Edition

Douglas C. Montgomery

Arizona State University



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Introduction to Statistical Quality Control

About the Author

Douglas C. Montgomery professor of industrial and management systems engineering at Arizona State University since 1988, received his B.S., M.S., and Ph.D. degrees from Virginia Polytechnic Institute, all in engineering. From 1969 to 1984 he was on the faculty of the School of Industrial & Systems Engineering at the Georgia Institute of Technology, and from 1984 to 1988 he was at the University of Washington, where he held the John M. Fluke Distinguished Chair of Manufacturing Engineering, was professor of mechanical engineering, and director of the program in industrial engineering.

In addition to his work in quality engineering, Dr. Montgomery has interests in experimental design, regression modeling and time series analysis, and the application of operations research methodology to problems in manufacturing systems. He has authored and coauthored many technical papers in these fields and is an author of six other books. Dr. Montgomery is a Fellow of the American Society for Quality Control, a Senior Member of the Institute of Industrial Engineers, a Senior Member of the Operations Research Society of America, a Senior Member of the Society of Manufacturing Engineers, and a Member of the American Statistical Association.

In addition to his academic activities, Dr. Montgomery is a partner in Statistical Productivity Consultants, a Seattle-based consulting organization specializing in quality and productivity improvement. Dr. Montgomery has served as a consultant in these areas to many major corporations in the electronics, aerospace, automotive, chemical, and process industries.

Preface

This book is about the modern practice of statistical quality control. It provides comprehensive coverage of the subject from basic principles to state-of-the-art concepts and applications. The objective is to give the reader a sound understanding of the principles and the basis for applying those principles in a wide variety of both product and nonproduct situations. While statistical techniques are used throughout, the book has a strong engineering and management orientation. Extensive knowledge of statistics is not a necessary prerequisite for using this book. Readers whose background includes a basic course in statistical methods will find the book easily accessible.

This book is an outgrowth of over 20 years of teaching, research, and consulting in the fields of quality engineering and quality improvement. It is designed as a textbook for students enrolled in colleges and universities, who are studying engineering, management, statistics, and related fields and are taking a first course in statistical quality control. Such courses are often taught at the junior or senior level. I have also used the text materials extensively in training programs for professional practitioners, including quality and reliability engineers, manufacturing and development engineers, managers, procurement specialists, marketing personnel, technicians and laboratory analysts, inspectors, and operators. Many professionals have also used the material for self-study.

Chapter 1 is an introduction to quality improvement in the modern business environment. It deals with the facts that quality has become a major business strategy and that organizations with successful quality-improvement programs can

increase their productivity, enhance their market penetration, and achieve greater profitability and a strong competitive advantage.

Following the introductory chapter, the book is divided into four parts. Part I presents a description of statistical methods useful in quality improvement. Topics covered included sampling and descriptive statistics, the basic notions of probability and probability distributions, point and interval estimation of parameters, and statistical hypothesis testing. These topics are usually covered in a basic course in statistical methods; however, their presentation in this text is from the quality-engineering viewpoint. My experience has been that even readers with a strong statistical background will find the approach to this material useful and somewhat different from that used in a standard statistics textbook.

Part II contains seven chapters on statistical process control. While the entire range of statistical process-control tools are extensively discussed, the primary focus is on the control chart. The concept of control charts is certainly not new, but its use in modern-day business and industry is of tremendous value. Furthermore, as sensing and measurement technology develops, along with the widespread availability of powerful microcomputers, the implementation of statistical process control at the workplace is becoming a commonplace activity in many businesses. Statistical process control will play an even greater role in U.S. industry over the next 20 years than it has in the last 50.

Part III contains two chapters that show how statistical experiment-design methods can be used for process improvement. Chapter 11 presents the fundamental concepts of designed experiments and introduces the reader to some of the data analysis methods employed. While the treatment of the subject is not extensive and is no substitute for a formal course in experimental design, it will enable the reader to appreciate more sophisticated examples of experimental design. Chapter 12 illustrates factorial and fractional factorial designs, response surface designs, and gives an overview of Taguchi's contributions to quality engineering. I've tried to present my view that Taguchi has made many valuable contributions to quality improvement philosophy, but that his technical methods are often ineffective and inefficient, and can be improved. These points are discussed in Chapter 12. Throughout both Chapters 11 and 12, the important interrelationship between statistical process control and experimental design for process improvement is emphasized.

Part IV contains three chapters dealing with acceptance sampling. The focus is on lot-by-lot acceptance sampling, although there is some discussion of continuous sampling and MIL STD 1235B in Chapter 15. Other sampling topics presented include details of the design of acceptance-sampling plans; a discussion of MIL STD 105D, MIL STD 414, and their civilian counterparts, ANSI/ASQC Z1.4 and ANSI/ASQC Z1.9; and other important techniques useful in a near zero defects manufacturing environment, such as chain sampling and skip-lot sampling.

Throughout Parts II and IV guidelines are given for selecting the proper type of statistical process-control or sampling technique to use in a wide variety of product and nonproduct situations. There are also extensive references to journal articles and other technical literature that should assist the reader in applying the methods described.

CHANGES IN THE SECOND EDITION

I have made extensive changes in this edition of the book. A major change is a more detailed discussion of the basic SPC problem-solving tools including cause-and-effect analysis, Pareto analysis, defect concentration analysis, and check sheets for data collection. These topics are discussed in Chapter 4. This chapter also contains two case studies illustrating the implementation of SPC in a manufacturing and nonmanufacturing setting. Chapter 4 also contains a much broader and deeper coverage of the philosophy underlying SPC and the approach that must be taken to successfully implement these techniques in industry.

The material on variables control charts has also been greatly expanded. Chapter 6, which discusses the basic variables control charts, contains several new examples. Chapter 7 is now devoted exclusively to cumulative-sum control charts and exponentially weighted moving-average control charts. Chapter 8 discusses a variety of topics including how to implement control charts in the short production-run environment, along with expanded coverage of group control charts, acceptance control charts, control charts with modified limits, some new material on multivariate quality control, SPC with autocorrelated data, and several other topics. The coverage of process-capability analysis in Chapter 9 has been expanded to include more information on capability indices and techniques for assessing gage capability.

Part III on process improvement with designed experiments is new. I have added this material at the request of many users of the textbook, who wish to introduce students in their basic quality control courses to the fundamentals of experimental design, and show them several examples of using these techniques for quality and process improvement.

All of the examples in this book utilize data from real applications. In some cases, I have disguised the data or the application situation so that proprietary information will be protected.

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Many people have generously contributed their time and knowledge of quality improvement to this book. A complete list of colleagues with whom I have interacted in various academic or consulting projects over the years would be too extensive to include here. However, some of the major contributors and their professional affiliations are as follows: Dr. William W. Hines, Dr. Lynwood A. Johnson, Dr. Russell G. Heikes, Dr. David E. Fyffe, and Dr. H. M. Wadsworth, Jr., Georgia Institute of Technology; Dr. Richard L. Storch, University of Washington; Dr. J. Burt Keats, Dr. Dwayne A. Rollier, and Dr. Norma F. Hubele, Arizona State University; Dr. Joseph J. Moder, University of Miami; Dr. Erwin M. Saniga, University of Delaware; Dr. John S. Ramburg, University of Arizona; Dr. Frank B. Alt, University of Maryland; Dr. Kenneth E. Case, Oklahoma State University; Mr. John A. Butora, Mr. Leon V. Mason, Mr. Lloyd K. Collins, Mr. Dana D. Leshner, Mr. Roy E. Dent, Mr. Mark Fazey, Ms. Kathy Schuster, Mr. Dan Fritze,

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I am also indebted to two organizations, the Office of Naval Research and the IBM Corporation. These organizations have sponsored my research activities in quality improvement for a number of years. Both organizations have provided an open and stimulating environment for conducting research and have generated many opportunities to work with their personnel on actual quality engineering problems. It has been my particular pleasure to be associated with Mr. Seymour M. Selig of ONR and Dr. Paul A. Tobias of IBM. They have given me a great deal of insight regarding quality engineering and quality improvement in their business environments.

Finally, I would like to thank the many users of the first edition of this book including students, practicing professionals, and my academic colleagues. Many of the changes and (hopefully) improvements in this edition of the book are the direct result of feedback from you.

Douglas C. Montgomery

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